MA123 — Elem. Calculus	Fall 2014	Name: SOLUTIONS	Sec.: ________
Exam 3	2014-11-20		

Do not remove this answer page — you will turn in the entire exam. You have two hours to do this exam. No books or notes may be used. You may use an ACT-approved calculator during the exam, but NO calculator with a Computer Algebra System (CAS), networking, or camera is permitted. Absolutely no cell phone use during the exam is allowed.

The exam consists of multiple choice questions. Record your answers on this page. For each multiple choice question, you will need to fill in the box corresponding to the correct answer. For example, if (a) is correct, you must write

(a) (b) (c) (d) (e)

Do not circle answers on this page, but please circle the letter of each correct response in the body of the exam. It is your responsibility to make it CLEAR which response has been chosen. You will not get credit unless the correct answer has been marked on both this page and in the body of the exam.

GOOD LUCK!

1. (a) **(b)** (c) (d) (e)
2. (a) **(b)** (c) (d) (e)
3. **(a)** (b) (c) (d) (e)
4. (a) **(b)** (c) (d) (e)
5. (a) (b) **(c)** (d) (e)
6. (a) (b) (c) (d) **(e)**
7. (a) (b) (c) **(d)** (e)
8. (a) (b) (c) (d) **(e)**
9. (a) (b) (c) **(d)** (e)
10. (a) (b) (c) **(d)** (e)
11. (a) (b) **(c)** (d) (e)
12. **(a)** (b) (c) (d) (e)
13. (a) (b) **(c)** (d) (e)
14. (a) (b) (c) (d) **(e)**
15. **(a)** (b) (c) (d) (e)
16. (a) **(b)** (c) (d) (e)
17. (a) (b) **(c)** (d) (e)
18. (a) (b) (c) (d) **(e)**
19. **(a)** (b) (c) (d) (e)
20. (a) (b) (c) **(d)** (e)

For grading use:

Number Correct	
	(out of 20 problems)

Total	
	(out of 100 points)

Multiple Choice Questions

Show all your work on the page where the question appears.
Clearly mark your answer both on the cover page on this exam
and in the corresponding questions that follow.

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1. Find the largest value of A such that the function $f(t) = t^3 - 9t^2 - 48t + 1$ is decreasing for all t in the interval $(0, A)$.

$f'(t) = 3t^2 - 18t - 48 = 3(t^2 - 6t - 16)$

$= 3(t-8)(t+2) \leftarrow f'(t) = 0$ for $t = 8$, $t = -2$.

Possibilities:

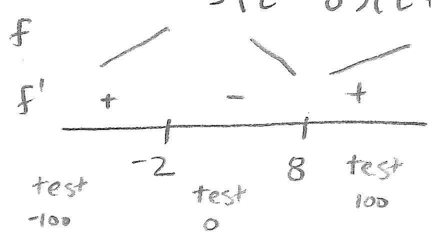
(a) 2

(b) 8

(c) -2

(d) ∞

(e) 3



$f(t)$ is decreasing for t in $(-2, 8)$. Since 0 is in this interval, we want $A = 8$.

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2. Suppose $g'(t) = (t-7)(t-8)^2(t-9)$. Find the largest value of A such that the function $g(t)$ is decreasing on the interval $(7, A)$.

Possibilities:

(a) 8

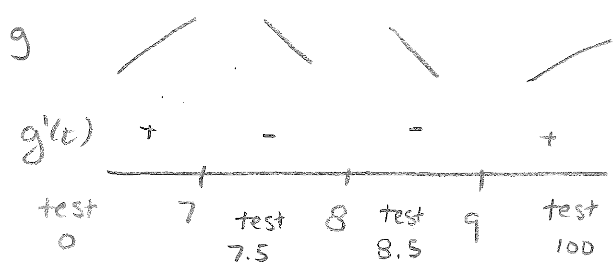
(b) 9

(c) ∞

(d) 504

(e) 7

$g'(t) = 0$ for $t = 7, 8, 9$.



$g'(8) = 0$, so $g'(8)$ exists: g is continuous at $x = 8$.

g is decreasing for x in $(7, 9)$.

Thus, we want $A = 9$.

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3. Suppose the derivative of $H(s)$ is given by $H'(s) = -(s^2 + 9)(s^2 + 1)$. Find the value of s in the interval $[-10, 10]$ where $H(s)$ takes on its minimum.

Possibilities:

(a) 10

(b) 9

(c) 1

(d) -9

(e) -10

Notice $H'(s)$ is never zero.
$H'(s)$ is negative for all values of s.
Thus $H(s)$ is always decreasing.
The minimum value of $H(s)$ on $[-10, 10]$ will occur farthest to the right, at $s = 10$.

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4. Suppose the derivative of $g(t)$ is $g'(t) = 7(t-2)(t-4)$. For t in which interval(s) is g concave up?

PRODUCT RULE :

$$g''(t) = 7(t-2)(1) + 7(t-4)(1)$$
$$= 7t - 14 + 7t - 28 = 14t - 42$$
$$= 14(t-3).$$

$g''(t) = 0$ for $t = 3$.

Possibilities:

(a) $(-\infty, 3)$

(b) $(3, \infty)$

(c) $(2, 4)$

(d) $(-\infty, 2) \cup (4, \infty)$

(e) $(7, 2) \cup (3, 4)$

g ∩ ∪

g'' − +

test 0 | 3 | test 100

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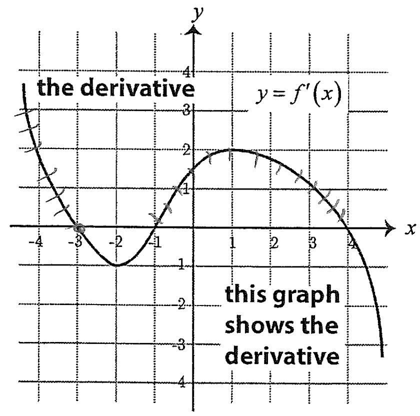
5. The following is the graph of the derivative, $f'(x)$, of the function $f(x)$.
Where is the regular function $f(x)$ increasing?

$f'(x) = 0$ for $x = -3, -1$ and 4



Possibilities:

(a) $(-\infty, -2)$ and $(1, \infty)$

(b) $(-3, -1)$ and $(4, \infty)$

(c) $(-\infty, -3)$ and $(-1, 4)$

(d) $(-1, 2)$

(e) $(-2, 1)$

f

f' + − + −

-3 -1 4

f is increasing where f' is positive (above the axis).

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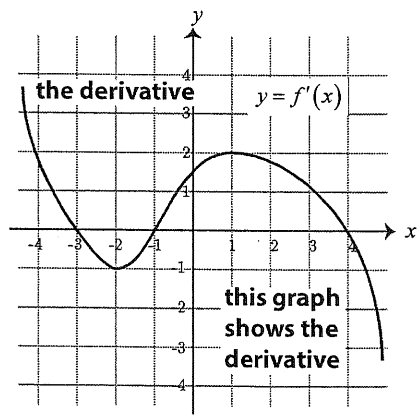
6. The following is the graph of the derivative, $f'(x)$, of the function $f(x)$.
Where is the regular function $f(x)$ concave down?



Possibilities:

(a) $(-2, 1)$

(b) $(-\infty, -3)$ and $(-1, 4)$

(c) $(-3, -1)$ and $(4, \infty)$

(d) $(-1, \infty)$

(e) $(-\infty, -2)$ and $(1, \infty)$

f ∩ ∪ ∩

f'' − + −

f'

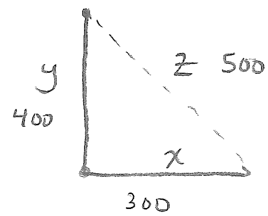
-2 1

[f is concave down where $f'' < 0$.
$f'' < 0$ where f' is decreasing]

7. Two trains leave the same station at different times, one traveling due East, and the other traveling due North. At 2pm the eastbound train is traveling at 45 mph and is 300 miles from the station, while the northbound train is traveling at 60 mph and is 400 miles from the station. At what rate is the distance between the trains increasing?

x = distance from Eastbound train to station
y = dist. from Northbound train to station
z = dist. between the trains.

Possibilities:



(a) 105 mph

(b) 6 mph

(c) 75000 mph

(d) 75 mph

(e) 500 mph

$$x^2 + y^2 = z^2$$

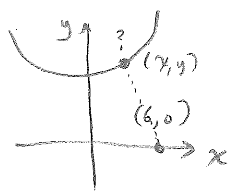
$$2x \cdot \frac{dx}{dt} + 2y\frac{dy}{dt} = 2z \cdot \frac{dz}{dt}$$

At 2pm: $2(300)\cdot 45 + 2(400)(60) = 2(500)\cdot \frac{dz}{dt}$

Solve for $\frac{dz}{dt}$: $1000\frac{dz}{dt} = 27000 + 48000 = 75000 \Rightarrow \frac{dz}{dt} = \frac{75000}{1000} = 75$

8. Find the point in the first quadrant that lies on the hyperbola $y^2 - x^2 = 3$ and is closest to the point $(6, 0)$.

Possibilities:



(a) $(6, \sqrt{39})$

(b) $(2, \sqrt{7})$

(c) $(6, 1)$

(d) $(0, \sqrt{3})$

(e) $(3, 2\sqrt{3})$

$$D = \sqrt{(x-6)^2 + (y-0)^2} = \sqrt{(x-6)^2 + y^2} = \sqrt{(x-6)^2 + 3 + x^2}.$$

$y^2 - x^2 = 3 \Rightarrow y^2 = 3 + x^2$

← easier to work with D^2 than D!

Distance will be smallest when the quantity under the radical is smallest: $D^2 = (x-6)^2 + 3 + x^2$.

$(D^2)' = 2(x-6)(1) + 0 + 2x = 2x - 12 + 2x = 4x - 12$.

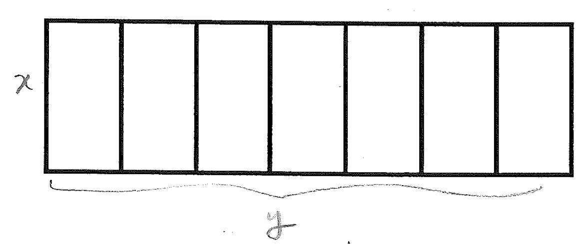
$(D^2)' = 0$ when $4x - 12 = 0 \Rightarrow x = 3$. (should verify, but, it works!)

$y^2 = 3 + x^2 \Rightarrow y^2 = 3 + 3^2 = 3 + 9 = 12$, so $y = \sqrt{12}$

$\boxed{(3, \sqrt{12})}$

9. A farmer builds a rectangular pen with 7 vertical partitions (8 vertical sides) using 800 feet of fencing. What is the maximum possible total area of the pen?

Possibilities:



(a) $\frac{80000}{9}$

(b) 800

(c) 16000

(d) 10000

(e) 40000

$$A = xy$$

$$8x + 2y = 800$$

$$4x + y = 400$$

$$y = 400 - 4x$$

domain interval: $[0, 100]$
$A(0) = A(100) = 0$.

$$A = x(400 - 4x) = 400x - 4x^2$$

$A' = 400 - 8x$, so $A' = 0$ if $8x = 400 \Rightarrow x = 50$

If $x = 50$, $A = 50(400 - 4\cdot 50) = 50(200) = 10000$ maximum.

10. Boyle's Law states that when a sample gas is compressed at a constant temperature, the pressure P and volume V satisfy the equation $PV = c$, where c is a constant. Suppose that at a certain instant the volume is 59 cubic centimeters, the pressure is 5 kPa, and the pressure is increasing at a rate of 4 kPa/min. At what rate is the volume decreasing at this instant?

Possibilities:

(a) $\frac{233}{5}$ cubic centimeters per minute
(b) $\frac{234}{5}$ cubic centimeters per minute
(c) 47 cubic centimeters per minute
(d) $\frac{236}{5}$ cubic centimeters per minute
(e) $\frac{237}{5}$ cubic centimeters per minute

$PV = c$

derivative with product rule:

$P \cdot \frac{dV}{dt} + V \cdot \frac{dP}{dt} = 0$ ← deriv. of constant is 0.

Fill in given values:

$5 \cdot \frac{dV}{dt} + 59 \cdot 4 = 0$

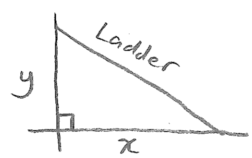
$\frac{dV}{dt} = -59(4)/5 = -236/5$

negative indicates decreasing

11. A ladder 20 feet long rests against a vertical wall. If the bottom of the ladder slides away from the wall at a rate of 3 feet per second, how fast is the top of the ladder sliding down the wall (in feet per second) when the bottom of the ladder is 16 feet from the wall?

Possibilities:

(a) 5 feet per second
(b) $\frac{4}{3}$ feet per second
(c) 4 feet per second
(d) $\frac{9}{5}$ feet per second
(e) $\frac{12}{5}$ feet per second



$x^2 + y^2 = 20^2$

$2x \cdot \frac{dx}{dt} + 2y \cdot \frac{dy}{dt} = 0$

when $x = 16$, $y^2 + 16^2 = 20^2 \Rightarrow y^2 = 144 \Rightarrow y = 12$.

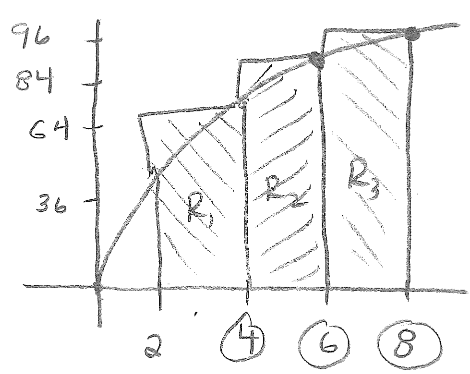
Fill in values: $2(16) \cdot 3 + 2(12) \cdot \frac{dy}{dt} = 0$

$24 \frac{dy}{dt} = -96 \Rightarrow \frac{dy}{dt} = \frac{-96}{24} = \boxed{-4 \text{ ft/sec}}$

12. Estimate the area under the graph of $-x^2 + 20x$ for x between 2 and 8, by using a partition that consists of 3 equal subintervals of $[2, 8]$ and use the right endpoint of each subinterval as a sample point.

Possibilities:

(a) 488
(b) 244
(c) 560
(d) 432
(e) 368



$\Delta x = \frac{b-a}{n} = \frac{8-2}{3} = \frac{6}{3} = 2$

R_1 height $= f(4) = -4^2 + 20(4) = 64$

R_2 height $= f(6) = -6^2 + 20(6) = 84$

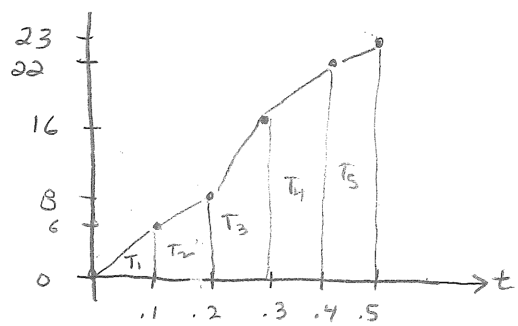
R_3 height $= f(8) = -8^2 + 20(8) = 96$

$A \approx 64(2) + 84(2) + 96(2) = 128 + 168 + 192 = 488$

13. A train travels in a straight westward direction along a track. The speed of the train varies, but it is measured at regular time intervals of 1/10 hour. The measurements for the first half hour are:

time	0	.1	.2	.3	.4	.5
speed	0	6	8	16	22	23

Estimate the total distance (in miles) traveled by the train during the first half hour by assuming the speed is a linear function of t on the subintervals. The speed in the table is given in miles per hour. Use all six speed measurements in your estimate.



$T_1: (.1)\left(\frac{0+6}{2}\right) = (.1)(3) = .3$

$T_2: (.1)\left(\frac{6+8}{2}\right) = (.1)(7) = .7$

$T_3: (.1)\left(\frac{8+16}{2}\right) = (.1)(12) = 1.2$

$T_4: (.1)\left(\frac{16+22}{2}\right) = (.1)(19) = 1.9$

$T_5: (.1)\left(\frac{22+23}{2}\right) = (.1)(22.5) = 2.25$

add these together

Possibilities:

(a) 8.00 miles

(b) 3.00 miles

(c) 6.35 miles

(d) 11.50 miles

(e) 7.50 miles

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14. One way to approximate $\int_3^8 e^{7-2x}\,dx$ is with the sum $\sum_{k=1}^{10}\left((\Delta x)\cdot\left(e^{7-2(3+k\Delta x)}\right)\right)$. What is the best value of Δx to use?

b

a

Since we add 10 things, $n = 10$

$\Delta x = \frac{b-a}{n} = \frac{8-3}{n} = \frac{5}{n}$

$= \frac{5}{10} = .5 = \frac{1}{2}$

Possibilities:

(a) 1.359079209

(b) 10

(c) 0.01

(d) 3

(e) $\frac{1}{2}$

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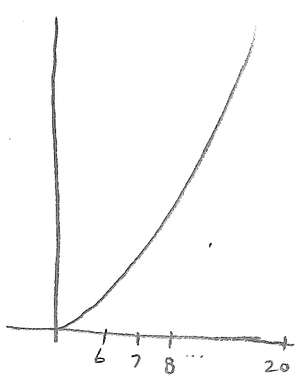
15. Suppose you estimate the area under the graph of $f(x) = x^3$ from $x = 6$ to $x = 26$ by adding the areas of the rectangles as follows: partition the interval into 20 equal subintervals and use the right endpoint of each interval to determine the height of the rectangle. What is the area of the 6th rectangle?



	x-value	height $= x^3$	width	area
R1	7	7^3	1	7^3
R2	8	⋮		
R3	9			
R4	10			
R5	11			
# R6	12	12^3	1	$12^3 = 1728$

Possibilities:

(a) 1728

(b) $\frac{6095}{4}$

(c) 1331

(d) 26

(e) 122760

16. Evaluate the difference of sums

identical inside

$$\left(\sum_{k=1}^{6000}(5k^3+4)\right) - \left(\sum_{k=3}^{6000}(5k^3+4)\right)$$

Possibilities:

(a) 1080000000004

(b) 53

(c) 0

(d) ∞

(e) 18003000

If we add 6000 terms, and then take away all but the 1st two terms, what is left is

$(k=1) \quad + \quad (k=2)$

$5\cdot 1^3+4 \qquad 5\cdot 2^3+4 \qquad = (5+4)+(5\cdot 8+4)$

$= 9 + 44 = 53$

17. Evaluate the sum

$$\sum_{k=1}^{N}(5k^2) = 5\sum_{k=1}^{N} k^2$$

Possibilities:

(a) $5\dfrac{N(N+1)}{2}$

(b) $5N^2-5$

(c) $5\dfrac{N(N+1)(2N+1)}{6}$

(d) $5N^2$

(e) $5N^2+5$

$= 5\cdot \dfrac{N(N+1)(2N+1)}{6}$

formula provided on the exam

18. Evaluate the sum $5+10+15+20+25+30+\cdots+95+100$.

Possibilities:

(a) 150

(b) 5

(c) 5050

(d) 4

(e) 1050

Factor out 5

$= 5(1+2+3+4+5+6+\cdots+19+20)$

$= 5\cdot\sum_{k=1}^{20} k = 5\cdot\dfrac{20(21)}{2} = 5(210) = 1050$

rewrite with notation

apply formula

19. Evaluate the sum $\sum_{k=7}^{200}(5+3k)$. $= \underbrace{\sum_{k=1}^{200}(5+3k)} - \underbrace{\sum_{k=1}^{6}(5+3k)}$

to take away the 1st six terms, either use formulas (shown) or calculate all six terms.

Possibilities:

(a) 61207

(b) 61300

(c) 605

(d) 60305

(e) 26

$$\sum_{k=1}^{200}(5+3k) = \sum_{k=1}^{200} 5 + 3\sum_{k=1}^{200} k = 5(200) + 3 \cdot \frac{200(201)}{2}$$

$$= 61300$$

$$\sum_{k=1}^{6}(5+3k) = \sum_{k=1}^{6} 5 + 3\sum_{k=1}^{6} k = 5(6) + 3 \cdot \frac{6(7)}{2}$$

$$= 93$$

$$61300 - 93 = \boxed{61207}$$

20. Evaluate the sum $\sum_{k=3}^{n}(11k)$. $= \sum_{k=1}^{n} 11k - (\underset{k=1}{11 \cdot 1} + \underset{k=2}{11 \cdot 2})$

Possibilities:

(a) $\frac{33}{2} + \frac{11}{2}n$

(b) $11n$

(c) $\frac{11}{2}n(n+1) - 66$

(d) $\frac{11}{2}n(n+1) - 33$

(e) $\frac{11}{2}n(n+1)$

$$= 11\sum_{k=1}^{n} k - (11+22)$$

$$= 11 \cdot \frac{n(n+1)}{2} - 33$$

Some Formulas

1. Summation formulas:

$$\sum_{k=1}^{n} k = \frac{n(n+1)}{2}$$

$$\sum_{k=1}^{n} k^2 = \frac{n(n+1)(2n+1)}{6}$$

2. Areas:

(a) Triangle $A = \dfrac{bh}{2}$

(b) Circle $A = \pi r^2$

(c) Rectangle $A = lw$

(d) Trapezoid $A = \dfrac{h_1 + h_2}{2} b$

3. Volumes:

(a) Rectangular Solid $V = lwh$

(b) Sphere $V = \dfrac{4}{3}\pi r^3$

(c) Cylinder $V = \pi r^2 h$

(d) Cone $V = \dfrac{1}{3}\pi r^2 h$

4. Distance:

(a) Distance between (x_1, y_1) and (x_2, y_2)

$$D = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$