MA 123 — Elem. Calculus Spring 2011 **Name:** Solutions. **Sec.:** _____
EXAM 3 4/13/2011

Do not remove this answer page — you will turn in the entire exam. You have two hours to do this exam. No books or notes may be used. You may use a graphing calculator during the exam, but NO calculator with a Computer Algebra System (CAS) or a QWERTY keyboard is permitted. Absolutely no cell phone use during the exam is allowed.

The exam consists of multiple choice questions. Record your answers on this page. For each multiple choice question, you will need to fill in the box corresponding to the correct answer. For example, if (a) is correct, you must write

■ b c d e

Do not circle answers on this page, but please circle the letter of each correct response in the body of the exam. It is your responsibility to make it CLEAR which response has been chosen. You will not get credit unless the correct answer has been marked on both this page and in the body of the exam.

GOOD LUCK!

1. a b c ■ e
2. a b ■ d e
3. a b c d ■
4. a b c d ■
5. ■ b c d e
6. a b c d ■
7. a b c ■ e
8. ■ b c d e
9. ■ b c d e
10. a ■ c d e
11. a b c ■ e
12. a ■ c d e
13. a b c ■ e
14. a ■ c d e
15. ■ b c d e
16. ■ b c d e
17. a ■ c d e
18. a ■ c d e
19. a b c d ■
20. ■ b c d e

For grading use:

Number Correct	
	(out of 20 problems)

Total	
	(out of 100 points)

Please make sure to list the correct section number on the front page of your exam. In case you forgot your section number, consult the following table. If you are enrolled in a lecture with recitation, then your time and location is based on your recitation, not your lecture.

Section #	Instructor	Lectures
001	Koester/Hamilton	T 8:00 - 9:15 am, CP 243
002	Koester/Hamilton	R 8:00 - 9:15 am, CP 243
003	Koester/Hamilton	T 9:30 - 10:45 am, MMRB 243
004	Koester/May	R 9:30 - 10:45 am, CB 342
005	Koester/May	T 11:00 - 12:15 pm, CP 220
006	Koester/May	R 11:30 - 12:15 pm, CP 220
007	Koester/Kyriopoulos	T 9:30 - 10:45 am, CP 367
008	Koester/Kyriopoulos	R 9:30 - 10:45 am, DH 323
009	Koester/Kyriopoulos	T 11:00 - 12:15 pm, FB 307A
010	Koester/Robinson	R 11:00 - 12:15 pm, CP 345
011	Koester/Robinson	T 12:30 - 1:45pm, CB 219
012	Koester/Robinson	R 12:30 - 1:45pm, CB 219
013	Shaw/Taylor	T 11:00 - 12:15 pm, CB 345
014	Shaw/Taylor	R 11:00 - 12:15 pm, MMRB 243
015	Shaw/Taylor	T 12:30 - 1:45 pm, Nurs 201
016	Shaw/Tarr	R 12:30 - 1:45 pm, Nurs 502A
017	Shaw/Tarr	T 2:00 - 3:15 pm, CB 233
018	Shaw/Tarr	R 2:00 - 3:15 pm, CB 245
019	Shaw/Ozbek	T 3:30 - 4:45pm, CP 208
020	Shaw/Ozbek	R 3:30 - 4:45 pm, CP 208
021	Shaw/Ozbek	T 2:00 - 3:15 pm, FB B2
022	Shaw/Zhi	R 2:00 - 3:15 pm, CP 233
023	Shaw/Zhi	T 9:30 - 10:45 am, CP 211
024	Shaw/Zhi	R 9:30 - 10:45 am, CB 341
025	Beth Kelly	MWF 12:00 - 12:50 pm, CP 153
026	John Maki	MWF 2:00 - 2:50 pm, KAS 213

Multiple Choice Questions

Show all your work on the page where the question appears.
Clearly mark your answer both on the cover page on this exam
and in the corresponding questions that follow.

1. Suppose $g(t) = t^3 + 21t^2 + 99t - 7$. Find the largest interval(s) on which $g(t)$ is decreasing.

Possibilities:

(a) $(-11, 11)$
(b) $(-7, \infty)$
(c) $(-\infty, -7)$
(d) $(-11, -3)$
(e) $(-\infty, -11)$ and $(11, \infty)$

Need $g'(t) < 0$

$g'(t) = 3t^2 + 42t + 99$
$= 3(t^2 + 14t + 33)$
$= 3(t + 11)(t + 3)$

	-11		-3	
$g'(t)$	+ ++		− − −	+ ++++
$g(t)$	Increase		Decrease	Increase

2. Suppose the derivative of $g(t)$ is $g'(t) = (t+2)(t-6)$. Determine the largest interval(s) on which $g(t)$ is increasing.

Possibilities:

(a) $(6, \infty)$
(b) $(-2, 6)$
(c) $(-\infty, -2)$ and $(6, \infty)$
(d) $(-2, \infty)$
(e) $(-\infty, -2)$

Need $g'(t) > 0$.
Notice $g'(t)$ is given.

	-2		6	
	+ ++		− − − −	+ ++++
	Increase		Decrease	Increase

3. Suppose the derivative of $g(t)$ is $g'(t) = (t^2+4)(t-5)$. Find the value of t in the interval $[-80, 80]$ at which $g(t)$ takes on its minimum.

Possibilities:

(a) $t = 80$
(b) $t = -5$
(c) $t = 2$
(d) $t = 4$
(e) $t = 5$

min at endpoint -80 or 80,
OR at critical point, where $g'(t) = 0$.
So could be $t = -80, 80$, OR $t = 5$.

	-80		5		80
$g'(t)$		− − − −		+ ++++	
$g(t)$		Decrease		Increase	

$g(t)$ decreases until $t = 5$ & increases for t to the right of 5.

4. Suppose $f(x) = x^4 - 54x^2 - 7x - 4$. Find the largest interval(s) on which $f(x)$ is concave down.

Need $g''(x) < 0$

$g'(x) = 4x^3 - 108x^1 \to$

$g''(x) = 12x^2 - 108 = 0$

So $x^2 = \frac{108}{12} = 9$, so $x = \pm 3$.

Possibilities:

(a) $(-3, \infty)$

(b) $(-\infty, -3)$ and $(3, \infty)$

(c) $(3, \infty)$

(d) $(-\infty, -3)$

(e) $(-3, 3)$

	-3		3	
$g''(x)$	$+++$	$----$		$++++$
$g(x)$	c. up	c. down		c. up

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5. Suppose $f(x) = 10(x-9)^3 + 6$. Find the x-coordinate of the inflection point of $f(x)$.

For an inflection pt, need $f''(x) = 0$.

Now $f'(x) = 10 \cdot 3(x-9)^2 = 30(x-9)^2$

$f''(x) = 30 \cdot 2(x-9) = 60(x-9)$

$f''(x) = 0 \Rightarrow x = 9$

Possibilities:

(a) 9

(b) 10

(c) 11

(d) 12

(e) 13

Furthermore

		9	
$g''(x)$	$----$		$++++$
$g(x)$	c. down		c. up

} So the concavity changes at $x = 9$.

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6. Find the largest interval on which $f(x) = (x+1)\ln(x-4)$ is concave up.

HINT: You may use $f'(x) = \ln(x-4) + \frac{x+1}{x-4}$ and $f''(x) = \frac{x-9}{(x-4)^2}$ and that $f(x)$ is only defined for $x > 4$.

First, find where $f''(x) = 0$:

$\frac{x-9}{(x-4)^2} = 0 \Rightarrow x = 9$

Possibilities:

(a) $(1, \infty)$

(b) $(4, \infty)$

(c) $f(x)$ is never concave up.

(d) $(4, 9)$

(e) $(9, \infty)$

	4		9	
$f''(x)$		$----$		$++++++$
$f(x)$		c. down		c. up

7. Two positive real numbers, x and y, satisfy $x + y = 3$. What is the maximum value of the expression x^2y?

Possibilities:

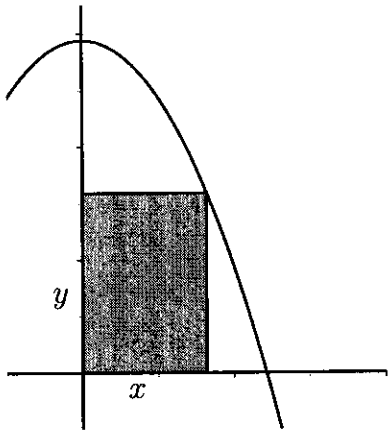
(a) 1
(b) 2
(c) 3
(d) 4
(e) 5

Maximize x^2y,
But $y = 3 - x$, so
Maximize $x^2(3-x) = 3x^2 - x^3$
Now $(3x^2 - x^3)' = 6x - 3x^2 = 3x(2-x)$
so Maximum occurs at $x = 0$ or $x = 2$.
Now $2^2(3-2) = 4$ and $0^2(3-0) = 0$
Max

8. Find the area of the largest rectangle with one corner at the origin, the opposite corner in the first quadrant on the graph of the curve $y = 27 - x^2$. (See the graph, but the graph is not to scale.)

Possibilities:

(a) 54
(b) 55
(c) 56
(d) 57
(e) 58



Area $= xy$
$= x(27 - x^2)$
Now, $A' = 27 - 3x^2$
$A' = 0 \Rightarrow 27 = 3x^2$
$x^2 = 9$
so $x = 3$
Area $= 3 \cdot (27 - 9) = 54$

9. A cylindrical tank has a circular base with radius $r = 5$ inches. The tank is being filled with water at the rate of 3 cubic inches per second. How fast is the height of the water in the tank increasing?

Possibilities:

(a) $\frac{3}{25\pi}$ inches per second
(b) $\frac{28}{25\pi}$ inches per second
(c) $\frac{53}{25\pi}$ inches per second
(d) $\frac{78}{25\pi}$ inches per second
(e) $\frac{103}{25\pi}$ inches per second

$V = \pi r^2 h = \pi \cdot 5^2 h$
$\frac{dV}{dt} = 3$ (Rate of Change Volume)
So $\frac{dV}{dt} = \pi r^2 \cdot \frac{dh}{dt}$
$3 = \pi \cdot 5^2 \cdot \frac{dh}{dt}$
So $\frac{dh}{dt} = \frac{3}{25\pi}$

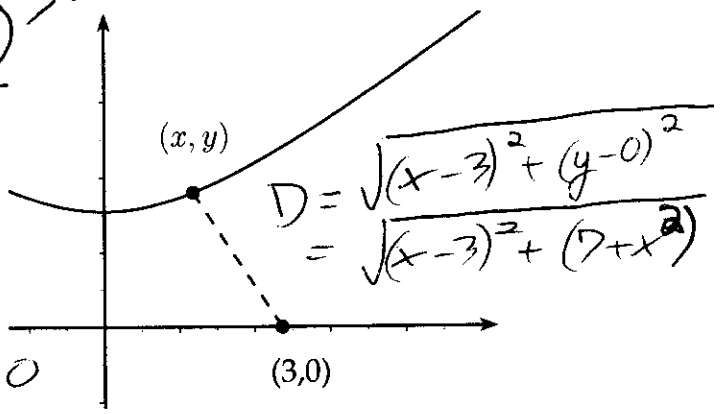
10. Let (x, y) be the point on the hyperbola $y^2 - x^2 = 7$ in the first quadrant that is closest to the point $(3, 0)$. Determine x. (HINT: x and y satisfy $y = \sqrt{7 + x^2}$)

Possibilities:

(a) 1
(b) 3/2
(c) 2
(d) 5/2
(e) 3

$D' = \dfrac{\left((x-3)^2 + 7 + x^2\right)'}{2\sqrt{(x-3)^2 + 7 + x^2}}$

$= \dfrac{2(x-3) + 2x}{2\sqrt{(x-3)^2 + 7 + x^2}}$



$D = \sqrt{(x-3)^2 + (y-0)^2}$

$= \sqrt{(x-3)^2 + (7+x^2)}$

$D' = 0 \Rightarrow 2(x-3) + 2x = 0$

So $4x = 6$

So $x = \frac{6}{4} = \frac{3}{2}$

11. Boyle's Law states that when a sample gas is compressed at a constant temperature, the pressure P and volume V satisfy the equation $PV = c$, where c is a constant. Suppose that a gas is being compressed and at a certain instant the volume is 69 cubic centimeters, the pressure is 4 kPa, and the pressure is increasing at a rate of 2 kPa/min. At what rate is the volume decreasing at this instant?

Possibilities:

(a) 33 cubic centimeters per minute
(b) 67/2 cubic centimeters per minute
(c) 34 cubic centimeters per minute
(d) 69/2 cubic centimeters per minute
(e) 35 cubic centimeters per minute

Given: $\frac{dP}{dt} = 2$ (Rate of change Pressure)

PV = constant, Take derivatives

$\Rightarrow \frac{dP}{dt} \cdot V + P \cdot \frac{dV}{dt} = 0$

Now $2 \cdot 69 + 4 \cdot \frac{dV}{dt} = 0$, So $\frac{dV}{dt} = \frac{-2 \cdot 69}{4}$

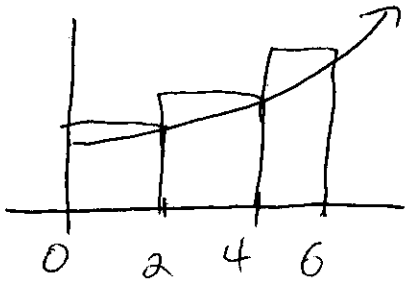
12. A train is traveling over a bridge at 32 miles per hour. A man on the train is walking toward the back of the train at 4 miles per hour. How fast is the man traveling across the bridge in miles per hour?

Possibilities:

(a) 32 miles per hour.
(b) 28 miles per hour.
(c) 40 miles per hour.
(d) 36 miles per hour.
(e) 128 miles per hour.

Motion in opposite directions, Subtract the speeds

13. Estimate the area under the graph of $f(x) = x^2 + 4$ for x between 0 and 6. Use a partition that consists of 3 equal subintervals of $[0, 6]$ and use the right endpoint of each subinterval as the sample point.



$\Delta x = \frac{6-0}{3} = 2$

Heights are

$f(2) = 2^2 + 4 = 8$

$f(4) = 4^2 + 4 = 20$

$f(6) = 6^2 + 4 = 40$.

Possibilities:

(a) 51

(b) 159

(c) 64

(d) 136

(e) 240

Area $= 8 \cdot 2 + 20 \cdot 2 + 40 \cdot 2$

$= 136.$

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14. Suppose you are given the data points for a function $g(t)$:

t	0	1	2
$g(t)$	10	18	22

If $g(t)$ is a linear function on each interval between the given points, find

$\int_0^2 g(t)\,dt$ = Area of the two trapezoids

$= \left(\frac{10+18}{2}\right)\cdot 1 + \left(\frac{18+22}{2}\right)\cdot 1$

$= 34$

Possibilities:

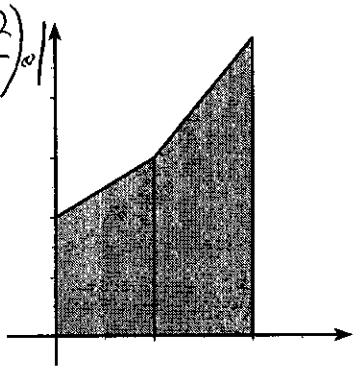
(a) 28

(b) 34

(c) 68

(d) 50

(e) 16



(Not drawn to scale)

15. Suppose that the integral $\int_8^{22} x^3\,dx$ is estimated by the sum $\sum_{k=1}^{N}(8+k\,\Delta x)^3\cdot\Delta x$. The terms in the sum equal areas of rectangles obtained using right endpoints of the subintervals of length Δx as sample points. If $N = 28$ equal subintervals are used, what is area of the second rectangle?

Possibilities:

(a) 729/2
(b) 4913/16
(c) 4913/8
(d) 729
(e) 256

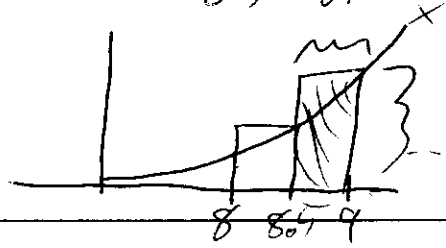
2nd Rectangle $\Rightarrow k=2$ in summand

$(8+2\cdot\frac{1}{2})^3\cdot\frac{1}{2} = \frac{1}{2}\cdot 9^3$

Alternatively, Width $= \frac{1}{2}$

Height $= 9^3$

So area $= \frac{1}{2}\cdot 9^3$



16. Suppose that the integral $\int_{47}^{87} g(x)\,dx$ is estimated by the sum $\sum_{k=1}^{N} g(47+k\,\Delta x)\cdot\Delta x$. The terms in the sum equal areas of rectangles obtained using right endpoints of the subintervals of length Δx as sample points. If $N = 800$ equal subintervals are used, what is the value of Δx ?

Possibilities:

(a) $\Delta x = 0.05$
(b) $\Delta x = 0.06$
(c) $\Delta x = 0.07$
(d) $\Delta x = 0.08$
(e) $\Delta x = 0.09$

$\Delta x = \frac{87-47}{800} = \frac{40}{800} = \frac{1}{20}$

$= .05$

17. Evaluate the sum $\sum_{k=5}^{7}(k^2+4k)$

Only 3 terms, Easiest to just evaluate sum

$k=5 \quad k=6 \quad k=7$

$= (5^2+4\cdot5)+(6^2+4\cdot6)+(7^2+4\cdot7)$

$= 45 + 60 + 77$

$= 182$

Possibilities:

(a) 181
(b) 182
(c) 183
(d) 184
(e) 185

18. Evaluate the sum

$$\sum_{k=1}^{34} (k^2 + k)$$

Possibilities:

(a) 14270
(b) 14280
(c) 14290
(d) 14300
(e) 14310

$$= \left(\sum_{k=1}^{34} k^2\right) + \left(\sum_{k=1}^{34} k\right)$$

$$= \frac{34 \cdot (34+1)(2 \cdot 34+1)}{6} + \frac{34 \cdot (34+1)}{2}$$

$$= 14280$$

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19. Evaluate the sum

$$8 + 16 + 24 + 32 + \ldots + 184 + 192$$

Possibilities:

(a) 2368
(b) 2376
(c) 2384
(d) 2392
(e) 2400

$$= 8(1 + 2 + 3 + 4 + \cdots 23 + 24)$$

$$= 8 \cdot \frac{24 \cdot 25}{2}$$

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20. Evaluate the sum

$$\sum_{k=6}^{42} k$$

Possibilities:

(a) 888
(b) 893
(c) 882
(d) 903
(e) 25530

$$= \sum_{k=1}^{42} k - \sum_{k=1}^{5} k$$

$$= \frac{42 \cdot 43}{2} - \frac{5 \cdot 6}{2}$$

$$= 903 - 15$$

Some Formulas

1. **Summation formulas:**

$$\sum_{k=1}^{n} k = \frac{n(n+1)}{2}$$

$$\sum_{k=1}^{n} k^2 = \frac{n(n+1)(2n+1)}{6}$$

2. **Areas:**

   (a) Triangle $A = \frac{bh}{2}$

   (b) Circle $A = \pi r^2$

   (c) Rectangle $A = lw$

   (d) Trapezoid $A = \frac{b_1 + b_2}{2} h$

3. **Volumes:**

   (a) Rectangular Solid $V = lwh$

   (b) Sphere $V = \frac{4}{3}\pi r^3$

   (c) Cylinder $V = \pi r^2 h$

   (d) Cone $V = \frac{1}{3}\pi r^2 h$

4. **Distance:**

   (a) Distance between (x_1, y_1) and (x_2, y_2)

$$D = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$