MA 123 — Elem. Calculus EXAM 3	Spring 2012 4/11/2012	**Name:** Solutions **Sec.:** ______

Do not remove this answer page — you will turn in the entire exam. You have two hours to do this exam. No books or notes may be used. You may use a graphing calculator during the exam, but NO calculator with a Computer Algebra System (CAS) or a QWERTY keyboard is permitted. Absolutely no cell phone use during the exam is allowed.

The exam consists of multiple choice questions. Record your answers on this page. For each multiple choice question, you will need to fill in the box corresponding to the correct answer. For example, if (b) is correct, you must write

[a] ■ [c] [d] [e]

Do not circle answers on this page, but please circle the letter of each correct response in the body of the exam. It is your responsibility to make it CLEAR which response has been chosen. You will not get credit unless the correct answer has been marked on both this page and in the body of the exam.

GOOD LUCK!

1.	a	**b**	c	d	e	**11.**	a	b	**c**	d	e
2.	**a**	b	c	d	e	**12.**	**a**	b	c	d	e
3.	a	b	c	**d**	e	**13.**	a	**b**	c	d	e
4.	**a**	b	c	d	e	**14.**	a	b	c	d	**e**
5.	a	**b**	c	d	e	**15.**	a	b	c	**d**	e
6.	a	b	**c**	d	e	**16.**	a	b	c	d	**e**
7.	a	**b**	c	d	e	**17.**	**a**	b	c	d	e
8.	a	b	c	d	**e**	**18.**	**a**	b	c	d	e
9.	a	b	**c**	d	e	**19.**	a	b	**c**	d	e
10.	a	**b**	c	d	e	**20.**	a	b	**c**	d	e

For grading use:

Number Correct	
	(out of 20 problems)

Total	
	(out of 100 points)

Please make sure to list the correct section number on the front page of your exam. In case you forgot your section number, consult the following table. Your section number is determined by your recitation time and location.

Section #	**Instructor**	**Recitation**
001	D. Akers	T 8:00 am - 9:15 am, CB 243
002	D. Akers	R 8:00 am - 9:15 am, CB 243
003	D. Akers	T 12:30 pm - 1:45 pm, TEB 231
004	Q. Liang	R 9:30 am - 10:45 am, NURS 502A
005	Q. Liang	T 11:00 am - 12:15 pm, CB 243
006	Q. Liang	R 11:00 am - 12:15 pm, CB 243
007	D. Corral	T 2:00 pm - 3:15 pm, DH 301
008	D. Corral	R 2:00 pm - 3:15 pm, DH 301
009	D. Corral	T 11:00 am - 12:15 pm, DH 353
010	A. Barra	R 11:00 am - 12:15 pm, DH 353
011	A. Barra	T 12:30 pm - 1:45 pm, MMRB 243
012	A. Barra	R 12:30 pm - 1:45 pm, MMRB 243
013	J. Jung	T 11:00 am - 12:15 pm, TPC 113
014	J. Jung	R 11:00 am - 12:15 pm, TPC 113
015	F. Camacho	T 12:30 pm - 1:45 pm, CB 219
016	J. Jung	R 12:30 pm - 1:45 pm, CB 219
017	F. Camacho	T 2:00 pm - 3:15 pm, FB B8
018	F. Camacho	R 2:00 pm - 3:15 pm, TPC 212
019	S. Hamilton	T 3:30 pm - 4:45 pm, CP 345
020	S. Hamilton	R 3:30 pm - 4:45 pm, BE 301
021	S. Hamilton	T 2:00 pm - 3:15 pm, CB 340
022	J. Constable	R 2:00 pm - 3:15 pm, CB 345
023	J. Constable	T 9:30 am - 10:45 am, L 201
024	J. Constable	R 9:30 am - 10:45 am, L 201
025	M. Shaw	MWF 9:00 am - 9:50 am, CB 110

Multiple Choice Questions

Show all your work on the page where the question appears.
Clearly mark your answer both on the cover page on this exam
and in the corresponding questions that follow.

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1. Suppose $f(x) = x^4 - 8x^3 - 30x^2 + 2x - 3$. Find the largest interval or collection of intervals on which $f(x)$ is concave down.

Need $f''(x) < 0$.

Possibilities:

$f'(x) = 4x^3 - 24x^2 - 60x + 2$

$f''(x) = 12x^2 - 48x - 60 = 12(x^2 - 4x - 5)$

$f''(x) = 12(x-5)(x+1)$

so $f''(x) = 0 \Rightarrow x = 5$ or $x = -1$.

-1 concave down 5

$f''(x)$ + + + + | − − − − − | + + + +

(a) ~~$(-\infty, -1)$~~ and $(5, \infty)$
(b) $(-1, 5)$
(c) $(-\infty, -1)$
(d) $(-1, \infty)$
(e) $(5, \infty)$

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2. Suppose the derivative of $g(t)$ is $g'(t) = (t+9)(t-3)(t-1)$. Determine the largest interval or collection of intervals on which $g(t)$ is decreasing. Need $g'(t) < 0$.

Possibilities:

-9 1 3

$g'(t)$ − − − | + + + + + | − − − − | + + + +

$g(t)$ ↘ ↗ ↘ ↗

(a) $(-\infty, -9)$ and $(1, 3)$
(b) $(3, \infty)$
(c) $(-9, 1)$ and $(3, \infty)$
(d) $(-\infty, -9)$ and $(3, \infty)$
(e) $(-9, 3)$

Decreasing $(-\infty, -9) \cup (1, 3)$

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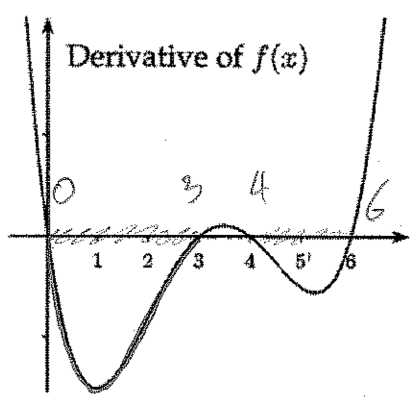
3. Determine the interval or collection of intervals on which $y = f(x)$ is decreasing. Please note the graph is of the derivative of $y = f(x)$.

Need $f'(x) < 0$

i.e. $y = f'(x)$ below x-axis



Possibilities:

(a) $f(x)$ is never decreasing
(b) $(-\infty, 0)$, $(3, 4)$, and $(6, \infty)$
(c) ~~$(-\infty, 1)$ and~~ $(3.5, 5)$
(d) $(0, 3)$ and $(4, 6)$
(e) $(1, 3.5)$ and $(5, \infty)$

4. Suppose that $y = f(x)$ is continuous and differentiable for all real numbers, and $f'(x) < 0$ for all x. Which of the following must be true about $y = f(x)$?

Possibilities:

(a) $y = f(x)$ is always decreasing.
(b) $y = f(x)$ is always concave down.
(c) $y = f(x)$ is always below the x-axis.
(d) $y = f(x)$ is always increasing.
(e) $y = f(x)$ is always concave up.

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5. Suppose that $f(x) = x^3 - 9x^2 + 48x + 9$. Determine the x coordinate of the inflection point of $y = f(x)$.

concavity changes.

Possibilities:

$f'(x) = 3x^2 - 18x + 48$, $f''(x) = 6x - 18 = 6(x-3)$

(a) $x = 2$
(b) $x = 3$
(c) $x = 4$
(d) $x = 5$
(e) $x = 6$

	3	
$f''(x)$	$- - -$	$+ + + + +$
$f(x)$	C.D.	C.U.

---

6. The product of two postive numbers, x and y is 19. Determine the maximum value of the expression $x + 2y$.

Possibilities:

(a) $\sqrt{38}$
(b) 19
(c) $2\sqrt{38}$
(d) $3\sqrt{38}$
(e) 38

As stated, this problem has no solution. It should have read "Determine the minimum value..." Legitimate attempts to find either the max. OR min were granted credit.

Revised #6.

The product of two positive numbers, x and y, is 19. Determine the $\boxed{\text{minimum}}$ value of the expression $x+2y$.

Solution.

$xy = 19 \Rightarrow y = 19/x = 19x^{-1}$

Want $x + 2y = x + 2 \cdot 19x^{-1} = x + 38x^{-1}$
minimal

But $(x + 38x^{-1})' = 1 - 38x^{-2} = 0$

$\Rightarrow 1 = 38x^{-2}$

$\Rightarrow x^2 = 38$

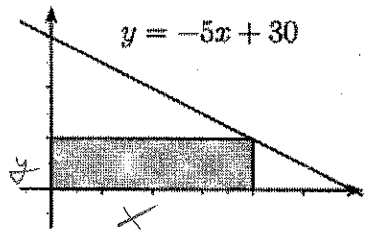
$\Rightarrow x = \pm\sqrt{38}$

(But x should be positive, so $x = \sqrt{38}$)

So $x + 2y = x + \frac{38}{x} = \sqrt{38} + \frac{38}{\sqrt{38}} = 2\sqrt{38}$.

7. Determine the area of the largest rectangle which has one corner at the origin (0,0) and opposite corner in the first quadrant on the line $y = -5x + 30$.

Want $A = xy$ maximal.



Possibilities:

But $y = -5x + 30$ so

(a) 90

(b) 45

$A = (-5x+30)x = -5x^2 + 30x,$

(c) 450

Now, $A' = -10x + 30 = 0$

(d) 180

$\Rightarrow x = 3$

(e) 3/2

So $A(3) = (-5\cdot 3 + 30)\cdot 3 = 45$

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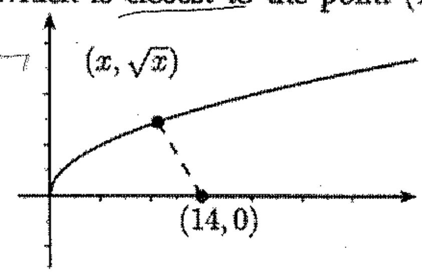
8. Find the x-coordinate of the point on the curve $y = \sqrt{x}$ which is closest to the point $(14, 0)$

Want distance minimized.



Possibilities: $D = \sqrt{(x-14)^2 + (\sqrt{x} - 0)^2}$

(a) $\sqrt{7}$

$= \sqrt{(x-14)^2 + x}$

(b) 14

(c) $\sqrt{14}$

Now,

(d) 28

(e) 27/2

$D' = \dfrac{2(x-14)+1}{2\sqrt{(x-14)^2 + x}} = 0 \Rightarrow 2(x-14)+1 = 0$

$\Rightarrow 2x - 28 + 1 = 0$

$\Rightarrow x = \dfrac{27}{2}.$

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9. Determine the rate of increase of the area of a circle when the radius of the circle is 14 feet and the radius is increasing at the rate of 7 feet per minute.

$\frac{dr}{dt} = 7$

$A = \pi r^2,$

Possibilities:

(a) 140π square feet per minute

$\frac{dA}{dt} = \pi\, 2r \cdot \frac{dr}{dt}$

(b) 98π square feet per minute

(c) 196π square feet per minute

$= \pi \cdot 2 \cdot 14 \cdot 7 = 196\pi$

(d) 49π square feet per minute

(e) 28π square feet per minute

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10. The price of a share of stock is increasing at a rate of 19 dollars per share per year. An investor is buying stock at a rate of 11 shares per year. How fast is the value of the investor's stock growing when the price of the stock is 63 dollars per share and the investor owns 50 shares of the stock? (Hint: Write down an expression for the total value, V, of the stock owned by the investor.)

$V = pn$, p = price/share, n = # shares

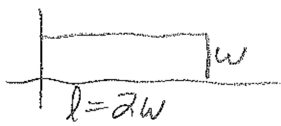
Possibilities:

(a) \$950 per year.
(b) \$1643 per year.
(c) \$1747 per year.
(d) \$3150 per year.
(e) \$209 per year.

$$\frac{dV}{dt} = \frac{dp}{dt}\cdot n + p\cdot\frac{dn}{dt}$$

$$= 19\cdot 50 + 63\cdot 11 = \$1643.$$

11. An expanding rectangle has its length always equal to twice its width. The area is increasing at a rate of 72 square feet per minute. At what rate is the width increasing when the width is 9 feet?



$A = lw = (2w)w = 2w^2$

Possibilities:

(a) 36 feet per minute.
(b) 18 feet per minute.
(c) 2 feet per minute.
(d) 8 feet per minute.
(e) 162 feet per minute.

$$\frac{dA}{dt} = 2\cdot 2w\,\frac{dw}{dt} = 4w\frac{dw}{dt}$$

This 2 comes from power rule.

But $\frac{dA}{dt} = 72$

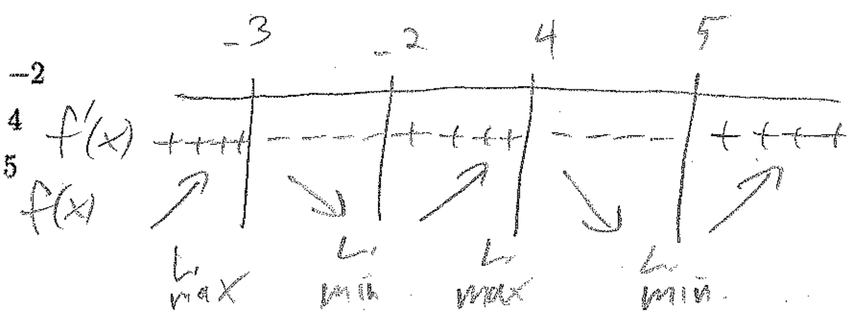
So $72 = 4\cdot 9\cdot\frac{dw}{dt} \Rightarrow \frac{dw}{dt} = \frac{72}{4\cdot 9} = 2$

12. The derivative of $f(x)$ is

$$f'(x) = (x+3)(x+2)(x-4)(x-5).$$

Which of the following are true?

~~(I)~~ $f(x)$ has a local maximum at $x = -2$
(II) $f(x)$ has a local maximum at $x = 4$
(III) $f(x)$ has a local minimum at $x = 5$



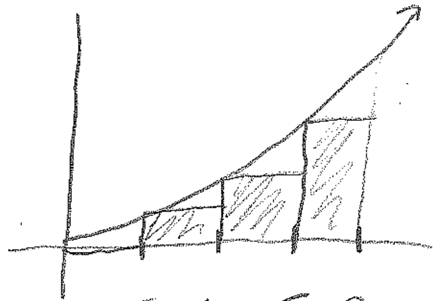
Possibilities:

(a) (II) and (III) true
~~(b) Only (III) is true.~~
~~(c) Only (I) is true.~~
(d) Only (II) is true.
~~(e) (I) and (II) true.~~

13. Estimate the area under the graph of $f(x) = x^2 + 5x$ for x between 0 and 2. Use a partition that consists of 4 equal subintervals of $[0, 2]$ and use the left endpoint of each subinterval as the sample point.

Possibilities:

(a) 6
(b) 37/4
(c) 65/4
(d) 37/2
(e) 20



$f(0) = 0$
$f(.5) = (.5)^2 + 5(.5) = 2.75$
$f(1) = 1^2 + 5 \cdot 1 = 6$
$f(1.5) = 1.5^2 + 5 \cdot (1.5) = 9.75$

0 .5 1 1.5 2

Area $= \frac{1}{2}(0 + 2.75 + 6 + 9.75) = 9.25 = \frac{37}{4}$

14. Suppose you are given the data points for a function $g(t)$:

t	0	1	2
$g(t)$	9	12	16

If $g(t)$ is a linear function on each interval between the given points, find

$$\int_0^2 g(t)\,dt$$

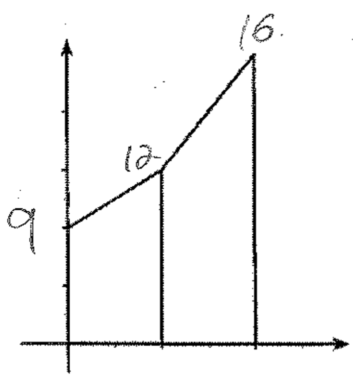
Widths = 1

Possibilities:

(a) 25/2
(b) 21
(c) 37
(d) 49
(e) 49/2

Area $= \left[\frac{9+12}{2} + \frac{12+16}{2} \right]$

$= \frac{49}{2}$



(Not drawn to scale)

15. Evaluate the sum

$$\rightarrow \sum_{k=6}^{8} k^3 = 6^3 + 7^3 + 8^3 = 1071$$

Only 3 terms,
simplest to just expand
the sum

Possibilities:

(a) 1068

(b) 1069

(c) ~~1070~~

(d) 1071

(e) 1072

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16. Evaluate the sum

$$\rightarrow \sum_{k=1}^{45} (k^2 + k) = \sum_{k=1}^{45} k^2 + \sum_{k=1}^{45} k$$

Use sum formulas!

Possibilities:

(a) 32390

(b) 32400

(c) 32410

(d) 32420

(e) 32430

$$= \frac{45(45+1)(45 \cdot 2 + 1)}{6} + \frac{45(45+1)}{2}$$

$$= 32430$$

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17. Evaluate the sum

$$15 + 20 + 25 + 30 + \ldots + 250 + 255$$

$$= 5(3 + 4 + 5 + \cdots + 50 + 51)$$

Possibilities:

(a) 6615

(b) ~~6630~~

(c) 1323

(d) 6625

(e) 6600

$$= 5\left[\overbrace{1 + 2 + 3 + 4 + \cdots + 50 + 51}^{51 \cdot 52/2} - 1 - 2\right]$$

$$= 5 \cdot \left[\frac{51 \cdot 50}{2} - 3\right]$$

18. Suppose that the integral $\int_{32}^{42} f(x)\,dx$ is estimated by the sum $\sum_{k=1}^{N} f(32 + k\,\Delta x)\cdot\Delta x$. The terms in the sum equal areas of rectangles obtained using right endpoints of the subintervals of length Δx as sample points. If $N = 500$ equal subintervals are used, what is the value of Δx ?

Possibilities:

(a) $\Delta x = 0.02$
(b) $\Delta x = 0.03$
(c) $\Delta x = 0.04$
(d) $\Delta x = 0.05$
(e) $\Delta x = 0.06$

$\Delta x = \frac{b-a}{N} = \frac{42-32}{500} = \frac{10}{500}$

19. Suppose that the integral $\int_{9}^{30} x^2\,dx$ is estimated by the sum $\sum_{k=1}^{N} (9 + k\,\Delta x)^2\cdot\Delta x$. The terms in the sum equal areas of rectangles obtained using right endpoints of the subintervals of length Δx as sample points. If $N = 42$ equal subintervals are used, what is area of the second rectangle?

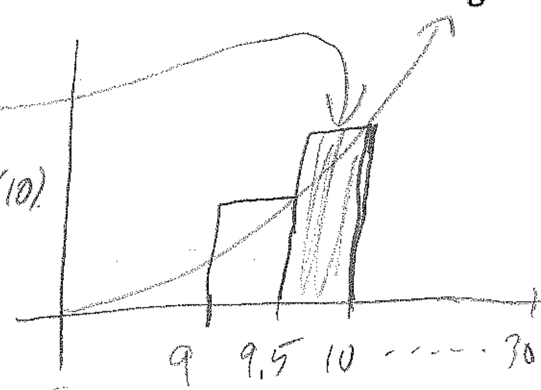
Possibilities:

(a) 100
(b) 81/2
(c) 50
(d) 361/8
(e) 361/4

$\Delta x = \frac{30-9}{42} = \frac{1}{2}$

Second rectangle:
width $= \frac{1}{2}$, Height $= f(10)$

$A = \frac{1}{2}\cdot 10^2 = 50$



20. Estimate the area under the graph of $y = 1/x$ for x between 1 and 50 by dividing the interval $[1, 50]$ into 49 equal subintervals and using the left endpoint of each subinterval as sample point. Next, estimate the area using the right endpoint as sample point. Find the difference between the two estimates (left endpoint estimate minus right endpoint estimate).

Possibilities:

(a) 47/48
(b) 48/49
(c) 49/50
(d) 50/51
(e) 51/52

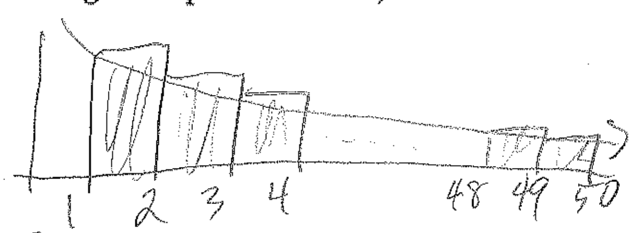
Left: $1 + \frac{1}{2} + \dots \frac{1}{49}$

Right: $\frac{1}{2} + \frac{1}{3} + \dots + \frac{1}{50}$

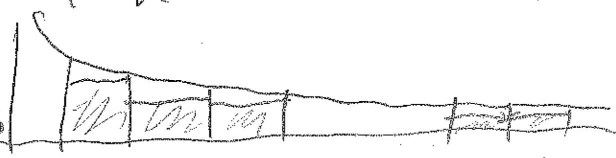
Left − Right

$= 1 + \frac{1}{2} + \frac{1}{3} + \dots + \frac{1}{49} - \frac{1}{2} - \frac{1}{3} - \dots - \frac{1}{49} - \frac{1}{50}$

$= 1 - \frac{1}{50} = \frac{49}{50}$



Left



Right

Some Formulas

1. Summation formulas:

$$\sum_{k=1}^{n} k = \frac{n(n+1)}{2}$$

$$\sum_{k=1}^{n} k^2 = \frac{n(n+1)(2n+1)}{6}$$

2. Areas:

(a) Triangle $A = \frac{bh}{2}$

(b) Circle $A = \pi r^2$

(c) Rectangle $A = lw$

(d) Trapezoid $A = \frac{b_1 + b_2}{2} h$

3. Volumes:

(a) Rectangular Solid $V = lwh$

(b) Sphere $V = \frac{4}{3}\pi r^3$

(c) Cylinder $V = \pi r^2 h$

(d) Cone $V = \frac{1}{3}\pi r^2 h$

4. Distance:

(a) Distance between (x_1, y_1) and (x_2, y_2)

$$D = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$