MA 123 — Elem. Calculus EXAM 3	Spring 2013 4/10/2013	**Name:** Solutions	**Sec.:**

Do not remove this answer page — you will turn in the entire exam. You have two hours to do this exam. No books or notes may be used. You may use a graphing calculator during the exam, but NO calculator with a Computer Algebra System (CAS) or a QWERTY keyboard is permitted. Absolutely no cell phone use during the exam is allowed.

The exam consists of multiple choice questions. Record your answers on this page. For each multiple choice question, you will need to fill in the box corresponding to the correct answer. For example, if (b) is correct, you must write

a **b** c d e

Do not circle answers on this page, but please circle the letter of each correct response in the body of the exam. It is your responsibility to make it CLEAR which response has been chosen. You will not get credit unless the correct answer has been marked on both this page and in the body of the exam.

GOOD LUCK!

Problem	Answer	Problem	Answer
1.	a	11.	c
2.	c	12.	d
3.	a	13.	c
4.	a	14.	d
5.	d	15.	a
6.	b	16.	c
7.	c	17.	e
8.	e	18.	b
9.	d	19.	e
10.	d	20.	d

For grading use:

Number Correct	
	(out of 20 problems)

Total	
	(out of 100 points)

Please make sure to list the correct section number on the front page of your exam. In case you forgot your section number, consult the following table. Your section number is determined by your recitation time and location.

Section #	Instructor	Day and Time	Room
001	F. Smith	T, 8:00 am - 9:15 am	CB 213
002	W. Hough	R, 8:00 am - 9:15 am	CB 213
003	D. Akers	T, 12:30 pm - 1:45 pm	CB 342
004	W. Hough	R, 9:30 am - 10:45 am	CP 397
005	D. Akers	T, 11:00 am - 12:15 pm	TPC 212
006	W. Hough	R, 11:00 am - 12:15 pm	TPC 113
007	A. Happ	T, 2:00 pm - 3:15 pm	TPC 109
008	A. Hubbard	R, 2:00 pm - 3:15 pm	L 108
009	A. Happ	T, 11:00 am - 12:15 pm	TPC 113
010	A. Hubbard	R, 11:00 am - 12:15 pm	CB 340
011	A. Happ	T, 12:30 pm - 1:45 pm	TEB 231
012	A. Hubbard	R, 12:30 pm - 1:45 pm	EH 307
013	L. Solus	T, 11:00 am - 12:15 pm	CB 340
014	D. Akers	R, 11:00 am - 12:15 pm	TPC 101
015	L. Solus	T, 12:30 pm - 1:45 pm	OT 0B7
016	F. Smith	R, 12:30 pm - 1:45 pm	FB B4
017	L. Solus	T, 2:00 pm - 3:15 pm	FB B4
018	F. Smith	R, 2:00 pm - 3:15 pm	CB 245
019	X. Kong	T, 3:30 pm - 4:45 pm	BH 303
020	Q. Liang	R, 3:30 pm - 4:45 pm	EGJ 115
021	X. Kong	T, 12:30 pm - 1:45 pm	CB 205
022	X. Kong	R, 2:00 pm - 3:15 pm	CB 233
023	L. Davidson	T, 9:30 am - 10:45 am	OT 0B7
024	L. Davidson	R, 9:30 am - 10:45 am	OT 0B7
026	L. Davidson	R, 8:00 am - 9:15 am	CB 243
027	Q. Liang	T, 9:30 am - 10:45 am	DH 131

Multiple Choice Questions

Show all your work on the page where the question appears.
Clearly mark your answer both on the cover page on this exam
and in the corresponding questions that follow.

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1. Consider the function $f(x) = x^4 - 18x^2 + 10$. Determine the largest interval or collection of intervals on which $f(x)$ is decreasing.

Possibilities:

Need $f'(x) < 0$

Now $f'(x) = 4x^3 - 36x = 4x(x^2-9)$

$f'(x) = 0 \Rightarrow x = 0$ or $x = 3$ or $x = -3$

(a) $(-\infty, -3)$ and $(0, 3)$
(b) $(-\infty, -3)$
(c) $(0, 3)$
(d) $(-\infty, -3)$ and $(3, \infty)$
(e) $(-3, 0)$ and $(3, \infty)$

$-\infty$ -4 -3	-1 0	1 3	4 ∞	
$f'(x)$: $- - -$	$+ + + + +$	$- - -$	$+ + + + +$
$f(x)$: $\searrow$	$\nearrow$	$\searrow$	$\nearrow$

---

2. The derivative of $g(t)$ is given by $g'(t) = (t-1)(t-5)(t-8)$. Determine the largest interval or collection of intervals on which $g(t)$ is increasing.

Possibilities:

Need $g'(t) > 0$

But $g'(t) = 0 \Rightarrow t = 1, 5, 8$.

(a) $(-\infty, 1)$
(b) $(-8, -5)$
(c) $(1, 5)$ and $(8, \infty)$
(d) $(5, 8)$
(e) $(-\infty, -8)$ and $(-5, -1)$

	1	5	8	
$g'(t)$	$- -$	$+ + +$	$- - -$	$+ + +$
$g(t)$	$\searrow$	$\nearrow$	$\searrow$	$\nearrow$

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3. The derivative of $H(s)$ is $H'(s) = s^2(s^2+3)(s+4)$. Determine the value of s in the interval $[-100, 100]$ at which $H(s)$ takes its minimum value.

Possibilities: Min $\Rightarrow$ function goes from $\searrow$ to $\nearrow$

$H'(s) = 0 \Rightarrow s^2 = 0$, $s^2+3=0$, OR $s+4=0$

So $s = 0$, No, $s = -4$.

(a) -4
(b) 0
(c) -100
(d) 100
(e) -3

	-100	-4	0	100
$H'(s)$	$- - - -$	$+ + + + +$	$+ + + + + +$	
$H(s)$	$\searrow$	$\nearrow$	$\nearrow$	

Min @ $s = -4$.

4. Consider the function $g(x) = x^4 - 54x^2 + 5$. Determine the largest interval or collection of intervals on which $g(x)$ is concave up.

Need $g''(x) \geq 0$

$g'(x) = 4x^3 - 108x$, $g''(x) = 12x^2 - 108$

Now $g''(x) = 0 \Rightarrow 12x^2 = 108 \Rightarrow x^2 = \frac{108}{12} = 9$

So $x = \pm 3$

Possibilities:

(a) $(-\infty, -3)$ and $(3, \infty)$

(b) $(-3, 3)$

(c) $(-\infty, 3)$

(d) $(3, \infty)$

(e) $(-\infty, -3)$

		-3		3	
$g''(x)$	$++++$		$- - - -$		$+++$
$g(x)$	C. up		C. down		C. up

5. The derivative of $g(x)$ is given by $g'(x) = x^2 + 4x + 6$. Determine the largest interval or collection of intervals on which $g(x)$ is concave down.

Need $g''(x) \leq 0$

Note: $g'(x)$ given.

So $g''(x) = 2x + 4$, Set $g''(x) = 0 \Rightarrow x = -2$.

Possibilities:

(a) $(-\infty, 2)$

(b) $(-\infty, 0)$

(c) $(0, \infty)$

(d) $(-\infty, -2)$

(e) $(-2, \infty)$

		-2	
$g''(x)$	$- - - -$		$+++++$
$g(x)$	C. Down		C. up

6. Consider the function $f(x) = x^4 - 32x + 5$. Determine the x-coordinate of the inflection point of $f(x)$.

Concavity switches up to down ~~or~~ down to up at Inf. Point

Possibilities:

(a) 0

(b) $f(x)$ does not have any inflection points.

(c) 5

(d) 32

(e) 8

$f'(x) = 4x^3 - 32$, $f''(x) = 12x^2$

$f''(x) = 0 \Rightarrow x = 0$.

		-1		0		1	
$f''(x)$	$++++$				$++++$		
$f(x)$	C. up				C. up		

Concavity did not change

7. Two positive real numbers, x and y, satisfy $xy = 48$. What is the minimum value of the expression $2x + 6y$?

Constraint: $xy = 48 \Rightarrow y = 48x^{-1}$

Possibilities:

(a) 46

(b) 47

(c) 48

(d) 49

(e) 50

Objective: $2x + 6\cdot(48x^{-1}) = 2x + 288x^{-1}$

(Objective)$' = 2 - 288x^{-2} = 0$

$2 = 288x^{-2} \Rightarrow 2x^2 = 288 \Rightarrow x^2 = 144 \Rightarrow x = 12$

Plug back in Objective

$2\cdot 12 + 6\cdot\left(\frac{48}{12}\right) = 48$

8. Find the area of the largest rectangle with one corner at the origin, the opposite corner in the first quadrant on the graph of the curve $f(x) = -3x + 4$. (See the graph, but the graph is not to scale.)

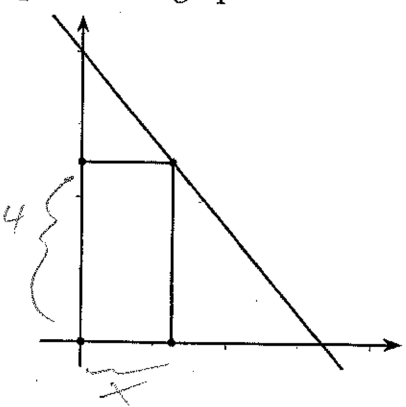
Area $= x(-3x+4) = -3x^2 + 4x$

$A' = -6x + 4$

$A' = 0 \Rightarrow 6x = 4$

$x = 4/6 = \frac{2}{3}$

Want $\underline{\text{Area}}$ so plug back into A:



Possibilities:

(a) $\frac{1}{3}$

(b) $\frac{1}{2}$

(c) $\frac{2}{3}$

(d) 1

(e) $\frac{4}{3}$

$A = \frac{2}{3}\left(-3\cdot\frac{2}{3} + 4\right) = \frac{4}{3}$

9. A cylindrical tank has a circular base with radius $r = 4$ feet. The tank is being filled with water at the rate of 6 cubic feet per minute. How fast is the height of the water in the tank increasing?

Possibilities: feet

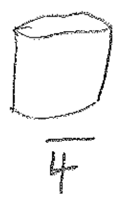
(a) $3/(16\pi)$ ~~inches~~ per minute

(b) $1/(4\pi)$ inches per minute

(c) $5/(16\pi)$ ~~inches~~ per minute

(d) $3/(8\pi)$ inches per minute

(e) $7/(16\pi)$ ~~inches~~ per minute



$\frac{dV}{dt} = 6$ ft^3/min

$V = \pi r^2 h = \pi \cdot 4^2 h$

$V = 16\pi h$

so $\frac{dV}{dt} = 16\pi \frac{dh}{dt}$ $\quad \frac{dh}{dt} = \frac{6}{16\pi} = \frac{3}{8\pi}$

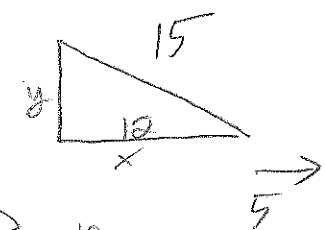
$6 = 16\pi \frac{dh}{dt} \Rightarrow$

10. A ladder of length 15 feet rests against a wall. The bottom of the ladder is being pulled away from the wall at a rate of 5 feet per second. How fast is the top of the ladder sliding down the wall when the bottom of the ladder is 12 feet from the wall? (Just give the numeric value of the answer. Do not worry about the plus or minus sign.)

$x^2 + y^2 = 15^2$

Possibilities:

Take derivative both sides



(a) 17/3 feet per second

(b) 6 feet per second

(c) 19/3 feet per second

(d) 20/3 feet per second

(e) 7 feet per second

$2x\frac{dx}{dt} + 2y\frac{dy}{dt} = 0$

$x = 12 \Rightarrow y = \sqrt{15^2 - 12^2} = 9$

$2 \cdot 12 \cdot 5 + 2 \cdot 9 \cdot \frac{dy}{dt} = 0$

$60 + 9\frac{dy}{dt} = 0$

$\frac{dy}{dt} = \frac{-60}{9} = \frac{-20}{3}$

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11. The price of a share of stock is increasing at a rate of 6 dollars per share per year. An investor is buying stock at a rate of 19 shares per year. How fast is the value of the investor's stock growing when the price of the stock is 58 dollars per share and the investor owns 55 shares of the stock? (Hint: Write down an expression for the total value, V, of the stock owned by the investor.)

$\frac{dp}{dt}$ $\frac{dn}{dt}$ p n

Possibilities: $n = $ # shares, $p = $ price per share

(a) \$3190 per year.

(b) \$330 per year.

(c) \$1432 per year.

(d) \$1393 per year.

(e) \$114 per year.

$V = np$

Want $\frac{dV}{dt}$.

$\frac{dV}{dt} = \frac{dn}{dt} \cdot p + n \cdot \frac{dp}{dt} = 19 \cdot 58 + 55 \cdot 6$

$= \$1432/\text{year}$

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12. Estimate the area under the graph of $f(x) = x^2 + 4x$ for x between 0 and 2. Use a partition that consists of 4 equal subintervals of $[0, 2]$ and use the right endpoint of each subinterval as the sample point.

$\Delta x = \frac{2-0}{4} = \frac{1}{2}$.

Possibilities:

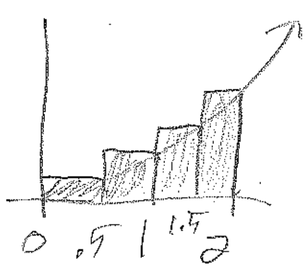
(a) 24

(b) 5

(c) 17

(d) 55/4

(e) 31/4



Area

$= f(.5) \cdot \frac{1}{2} + f(1) \cdot \frac{1}{2} + f(1.5)\frac{1}{2} + f(2) \cdot \frac{1}{2}$

$= 13.75 = \frac{55}{4}$

13. Suppose that the integral $\int_7^{39} x^2\,dx$ is estimated by the sum $\sum_{k=1}^{N}(7+k\,\Delta x)^2 \cdot \Delta x$. The terms in the sum equal areas of rectangles obtained using right endpoints of the subintervals of length Δx as sample points. If $N = 64$ equal subintervals are used, what is area of the second rectangle?

Possibilities:

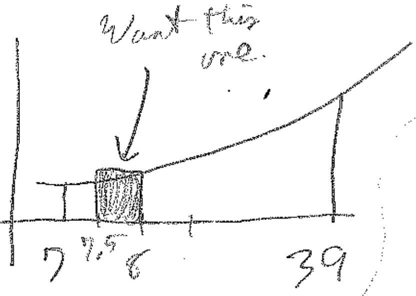
(a) 49/2

(b) 225/8

(c) 32

(d) 64

(e) 225/4



$\Delta x = \frac{39-7}{64} = \frac{32}{64} = \frac{1}{2}$

Width $= \frac{1}{2}$

Height $= f(8) = 8^2 = 64$

Area of 2^{nd} $= 64 \cdot \frac{1}{2}$

$= 32$

14. Suppose you estimate the integral

$$\int_{-8}^{0} x^2\,dx = \lim_{n\to\infty} \sum_{k=1}^{n} \Delta x\, f(x_k)$$

by the sum

$$\sum_{k=1}^{N} \Delta x\,\left(A(k\Delta x)^2 + B(k\Delta x) + C\right)\,dx.$$

$x_k = -8 + k\Delta x$

$f(x) = x^2$

The terms in the sum equal areas of rectangles obtained by using right endpoints of the subintervals of length Δx as sample points. What is the value of C?

Possibilities:

(a) 61

(b) 62

(c) 63

(d) 64

(e) 65

$\sum_{k=1}^{n} (-8+k\Delta x)^2 \Delta x$

C is the constant term.

$= \sum_{k=1}^{n} \left(64 - 16k\Delta x + (k\Delta x)^2\right)\Delta x$

15. Suppose that the integral $\int_{27}^{34} f(x)\,dx$ is estimated by the sum $\sum_{k=1}^{N} f(27 + k\,\Delta x)\cdot \Delta x$. The terms in the sum equal areas of rectangles obtained using right endpoints of the subintervals of length Δx as sample points. If $\Delta x = 0.01$, what is the value of N, the number of rectangles?

Possibilities:

(a) $N = 700$

(b) $N = 800$

(c) $N = 900$

(d) $N = 1000$

(e) $N = 1100$

$\Delta x = \frac{b-a}{n} = \frac{34-27}{n} = \frac{7}{n}$

So $0.01 = \frac{7}{n} \Rightarrow n = \frac{7}{0.01} = 700$

16. Evaluate the sum

$$\sum_{k=2}^{4}(3k^3+5)$$

Possibilities:

(a) 302
(b) 307
(c) 312
(d) 317
(e) 322

$k=2$ $\quad$ $k=3$ $\quad$ $k=4$

$$= (3\cdot 2^3+5) + (3\cdot 3^3+5) + (3\cdot 4^3+5)$$

$$= 312$$

17. Evaluate the sum

$$\sum_{k=3}^{4274} 2 = 2\cdot \#\text{terms in sum}$$

Possibilities:

(a) 8536
(b) 8538
(c) 8540
(d) 8542
(e) 8544

$\#\text{terms} = 4274 - 3 + 1 = 4272$

so $\text{sum} = 4272\cdot 2 = 8544$

18. Evaluate the sum

$$-7-6-5-\cdots+2640+2641+2642$$

Possibilities:

(a) 3494018
(b) 3491375
(c) 3491367
(d) 3488733
(e) 3491382

$$= -(7+6+5+4+3+2+1)+0+(1+2+\cdots+2642)$$

$$= -\frac{7\cdot 8}{2} + \frac{2642\cdot 2643}{2}$$

$$= 3,491,375$$

19. Suppose you are given the data points for a function $f(x)$:

x	0	1	2
$f(x)$	19	22	27

If $f(x)$ is a linear function on each interval between the given points, find

$$\int_0^2 f(x)\,dx$$ = Area under curve, But curve define two trapezoids

Widths are each 1.

Possibilities:

(a) 68

(b) 90

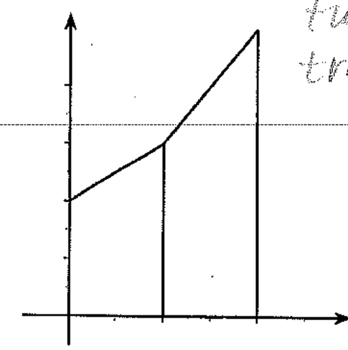
(c) 41

(d) 23

(e) 45

Area $\frac{19+22}{2}\cdot 1 + \frac{22+27}{2}\cdot 1$

$= 45.$



(Not drawn to scale)

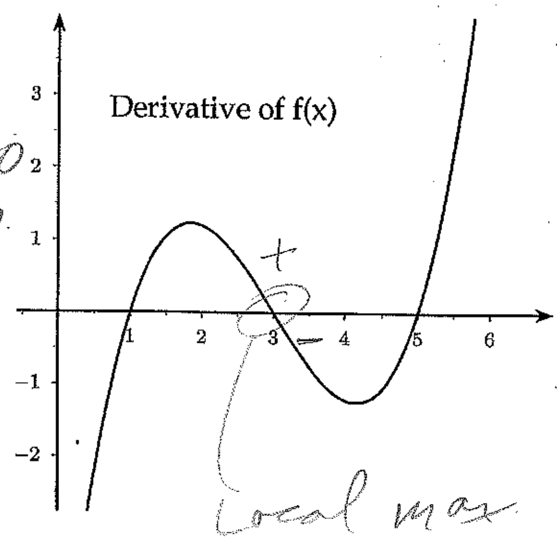
20. At what value of x does $f(x)$ have a local maximum? Please note the graph is of the derivative of $y = f(x)$.

At local maximum, $f(x)$ changes from ↗ to ↘

Now, $f(x)$ ↘ when $f'(x) < 0$

& $f(x)$ ↗ when $f'(x) > 0$.

So $f(x)$ has local maximum where $f'(x)$ changes from positive to negative



Possibilities:

(a) $x = 5$

(b) $x = 1$

(c) ~~$x = 2$~~

(d) $x = 3$

(e) $f(x)$ has no local maximum

Some Formulas

1. Summation formulas:

$$\sum_{k=1}^{n} k = \frac{n(n+1)}{2}$$

$$\sum_{k=1}^{n} k^2 = \frac{n(n+1)(2n+1)}{6}$$

2. Areas:

(a) Triangle $A = \dfrac{bh}{2}$

(b) Circle $A = \pi r^2$

(c) Rectangle $A = lw$

(d) Trapezoid $A = \dfrac{h_1 + h_2}{2} b$

3. Volumes:

(a) Rectangular Solid $V = lwh$

(b) Sphere $V = \dfrac{4}{3}\pi r^3$

(c) Cylinder $V = \pi r^2 h$

(d) Cone $V = \dfrac{1}{3}\pi r^2 h$

4. Distance:

(a) Distance between (x_1, y_1) and (x_2, y_2)

$$D = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$