MA123 — Elem. Calculus Spring 2014
Exam 3 2014-04-17

Name: KEY Sec.: _____

Do not remove this answer page — you will turn in the entire exam. You have two hours to do this exam. No books or notes may be used. You may use a graphing calculator during the exam, but NO calculator with a Computer Algebra System (CAS) or a QWERTY keyboard is permitted. Absolutely no cell phone use during the exam is allowed.

The exam consists of multiple choice questions. Record your answers on this page. For each multiple choice question, you will need to fill in the box corresponding to the correct answer. For example, if (a) is correct, you must write

(a) (b) (c) (d) (e)

Do not circle answers on this page, but please circle the letter of each correct response in the body of the exam. It is your responsibility to make it CLEAR which response has been chosen. You will not get credit unless the correct answer has been marked on both this page and in the body of the exam.

GOOD LUCK!

1.	**a**	b	c	d	e	11.	a	**b**	c	d	e
2.	a	b	c	**d**	e	12.	**a**	b	c	d	e
3.	a	b	c	d	**e**	13.	**a**	b	c	d	e
4.	**a**	b	c	d	e	14.	a	b	**c**	d	e
5.	a	b	c	**d**	e	15.	a	b	**c**	d	e
6.	a	b	c	d	**e**	16.	a	b	c	d	**e**
7.	a	b	**c**	d	e	17.	a	b	c	d	**e**
8.	a	b	**c**	d	e	18.	a	**b**	c	d	e
9.	**a**	b	c	d	e	19.	a	b	c	**d**	e
10.	a	b	c	**d**	e	20.	a	**b**	c	d	e

For grading use:

Number Correct	
	(out of 20 problems)

Total	
	(out of 100 points)

Please make sure to list the correct section number on the front page of your exam.
In case you forgot your section number, consult the following table.

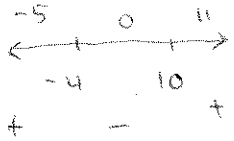
Section	Instructor	Day	Time	Room
	Jack Schmidt	MWF	10:00 am	CB 106
001	Wenwen Du	Tu	8:00 am	CB 349
002	Wenwen Du	Th	8:00 am	CB 349
003	Jinping Zhuge	Tu	12:30 pm	CP 201
004	Wenwen Du	Th	9:30 am	CP 211
005	Jinping Zhuge	Tu	11:00 am	TPC 113
006	Jinping Zhuge	Th	11:00 am	CP 103
	Jack Schmidt	MWF	12:00 pm	CB 118
007	Stephen Sturgeon	Tu	2:00 pm	FB 313
008	John Mosley	Th	2:00 pm	FB 313
009	Stephen Sturgeon	Tu	11:00 am	CB 335
010	John Mosley	Th	11:00 am	CB 335
011	Stephen Sturgeon	Tu	12:30 pm	CP 111
012	John Mosley	Th	12:30 pm	CB 233
013	Sarah Orchard	Tu	11:00 am	CP 111
014	Sarah Orchard	Th	11:00 am	CB 334
015	Sarah Orchard	Tu	12:30 pm	CP 103
	Nicholas Nguyen	MWF	2:00 pm	KAS 213
016	Jiaqi Liu	Th	12:30 pm	CB 201
017	Jiaqi Liu	Tu	2:00 pm	CP 345
018	Jiaqi Liu	Th	2:00 pm	CP 345
019	Hao Wang	Tu	3:30 pm	FB B9
020	Hao Wang	Th	3:30 pm	CP 297
021	Fernando Camacho	Tu	12:30 pm	TPC 212
	Drew Butcher	MWF	3:00 pm	BS 107
022	Hao Wang	Th	2:00 pm	BS 109
023	Fernando Camacho	Tu	9:30 am	CB 349
024	Fernando Camacho	Th	9:30 am	CB 349
025	Isaiah Harney	Tu	3:30 pm	CB 345
026	Isaiah Harney	Th	3:30 pm	CB 345
027	Luis Sordo Vieira	Tu	12:30 pm	CP 220
028	Isaiah Harney	Th	2:00 pm	TPC 212

Multiple Choice Questions

Show all your work on the page where the question appears.
Clearly mark your answer both on the cover page on this exam
and in the corresponding questions that follow.

1. Find the largest value of A such that the function $f(t) = t^3 - 9t^2 - 120t + 4$ is decreasing for all t in the interval $(0, A)$.

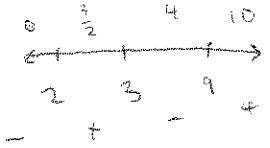
$f'(t) = 3t^2 - 18t - 120 = 3(t^2 - 6t - 40) = 3(t-10)(t+4)$



Possibilities:

(a) 10
(b) ∞
(c) 3
(d) -4
(e) 4

2. Suppose $g'(t) = (t-2)(t-3)(t-9)$. Find the largest value of A such that the function $g(t)$ is increasing for all t in the interval $(2, A)$.



Possibilities:

(a) 2
(b) 54
(c) ∞
(d) 3
(e) 9

3. Suppose the derivative of $H(s)$ is given by $H'(s) = (s^2+3)(s^2+7)$. Find the value of s in the interval $[-10, 10]$ where $H(s)$ takes on its maximum.

$H'(s) > 0$ for $s \in [-10, 10]$, so function is increasing. Thus max occurs at 10

Possibilities:

(a) 7
(b) 3
(c) -7
(d) -10
(e) 10

4. Suppose the derivative of $g(t)$ is $g'(t) = -9(t-4)(t-8)$. For t in which interval(s) is g concave up?

Possibilities:

(a) $(-\infty, 6)$

(b) $(6, \infty)$

(c) $(-9, 4) \cup (6, 8)$

(d) $(-\infty, 4) \cup (8, \infty)$

(e) $(4, 8)$

$g''(t) = -9(t-8) + -9(t-4)$

$= -18t + 108$

$-18t + 108 = 0$

$t = 6$

5. Suppose the derivative of $h(x)$ is given by $h'(x) = (x-3)(x-7)$. If $h(x)$ is concave upward on the interval (a, ∞), what is a?

Possibilities:

(a) 3

(b) 7

(c) $-\infty$

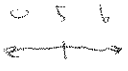
(d) 5

(e) 10

$h''(x) = x-3 + x-7 = 2x-10$

$2x-10 = 0$

$x = 5$



6. The following is the graph of the derivative, $f'(x)$, of the function $f(x)$. The zeroes, local extrema, and points of inflection of $f'(x)$ are marked. Where is $f(x)$ increasing?

Possibilities:

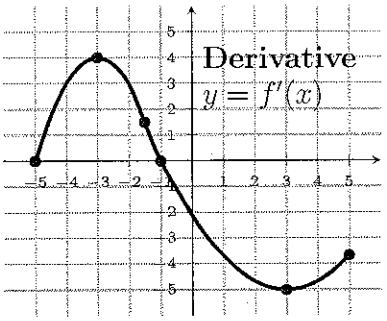
(a) between -1 and 5

(b) between -5 and -3, also between 3 and 5

(c) between -3 and 3

(d) between -5 and -1.5

(e) between -5 and -1



7. Find the area of the largest rectangle whose sides are parallel to the coordinate axes, whose bottom-left corner is at $(0,0)$ and whose top-right corner is on the graph of $y = 6x - x^2$.

Possibilities:

(a) 0

(b) 30

(c) 32

(d) 3

(e) 27

$A = xy = x(6x - x^2) = 6x^2 - x^3$

$A' = 12x - 3x^2 = 3x(4 - x)$

$3x(4-x) = 0$

$x = 0, 4$

$A = 6(4)^2 - (4)^3$

$96 - 64 = 32$

---

8. Find the point in the first quadrant that lies on the hyperbola $y^2 - x^2 = 4$ and is closest to the point $(8,0)$.

Possibilities:

(a) $(8, 2\sqrt{17})$

(b) $(3, \sqrt{13})$

(c) $(4, 2\sqrt{5})$

(d) $(0, 2)$

(e) $(5, \sqrt{29})$

$d^2 = (x-8)^2 + (y-0)^2$

$= x^2 - 16x + 64 + y^2$

$= x^2 - 16x + 64 + 4 + x^2$

$= 2x^2 - 16x + 68$

$y^2 - x^2 = 4$

$y^2 = 4 + x^2$

$(d^2)' = 4x - 16$

$4x - 16 = 0$

$x = 4$

---

9. A farmer builds a rectangular pen with 4 vertical partitions (5 vertical sides) using 800 feet of fencing. What is the maximum possible total area of the pen?

Possibilities:

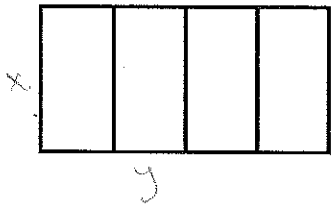
(a) 16000

(b) 400

(c) 800

(d) 40000

(e) $\frac{40000}{3}$



$A = xy = -\frac{5}{2}x^2 + 400x$

$800 = 5x + 2y$

$y = -\frac{5}{2}x + 400$

$A' = -5x + 400$

$-5x + 400 = 0$

$x = 80$

$A = -\frac{5}{2}(80)^2 + 400(80) = 16000$

---

10. Boyle's Law states that when a sample gas is compressed at a constant temperature, the pressure P and volume V satisfy the equation $PV = c$, where c is a constant. Suppose that at a certain instant the volume is 61 cubic centimeters, the pressure is 7 kPa, and the pressure is increasing at a rate of 2 kPa/min. At what rate is the volume decreasing at this instant?

Possibilities:

(a) $\frac{125}{7}$ cubic centimeters per minute

(b) $\frac{123}{7}$ cubic centimeters per minute

(c) $\frac{124}{7}$ cubic centimeters per minute

(d) $\frac{122}{7}$ cubic centimeters per minute

(e) 18 cubic centimeters per minute

$PV = C$

$PV' + PV' = 0$

$(2)(61) + (7)V' = 0$

$V' =$

---

11. A ladder 10 feet long rests against a vertical wall. If the bottom of the ladder slides away from the wall at a rate of 2 feet per second, how fast is the top of the ladder sliding down the wall (in feet per second) when the bottom of the ladder is 8 feet from the wall? (answer should be positive)

Possibilities:

(a) $\frac{6}{5}$ feet per second

(b) $\frac{8}{3}$ feet per second

(c) $\frac{8}{5}$ feet per second

(d) $\frac{4}{3}$ feet per second

(e) $\frac{10}{3}$ feet per second

$10^2 = x^2 + y^2$ $\quad y = \sqrt{100 - x^2} = \sqrt{36} = 6$

$0 = 2xx' + 2yy'$

$0 = xx' + yy'$

$0 = (8)(2) + (6)y'$

$y' = \frac{8}{3}$

---

12. Estimate the area under the graph of $x^2 - 6x$ for x between 1 and 9, by using a partition that consists of 4 equal subintervals of $[1, 9]$ and use the right endpoint of each subinterval as a sample point.

Possibilities:

(a) 40

(b) 20

(c) -24

(d) $\frac{8}{3}$

(e) 30

$A \approx \sum_{k=1}^{4} f(a + k\Delta x)\Delta x = 2[f(3) + f(5) + f(7) + f(9)]$

$= 2[-9 - 5 + 7 + 27] = 40$

13. A train travels in a straight westward direction along a track. The speed of the train varies, but it is measured at regular time intervals of 1/10 hour. The measurements for the first half hour are:

time	0	.1	.2	.3	.4	.5
speed	0	5	12	16	22	25

Estimate the total distance (in miles) traveled by the train during the first half hour by assuming the speed is a linear function of t on the subintervals. The speed in the table is given in miles per hour. Use all six speed measurements in your estimate.

Possibilities:

(a) 6.75 miles

(b) 2.5 miles

(c) 8.0 miles

(d) 6 miles

(e) 12.5 miles

$\frac{1}{10}\cdot\frac{1}{2}f(t_0) + \frac{1}{10}(5+12+16+22+25)$

$\frac{\Delta x}{2}\left(f(t_0)+2f(t_1)+2f(t_2)+2f(t_3)+2f(t_4)+f(t_5)\right)$

$= \frac{1}{20}(0+10+24+32+44+25) = 6.75$

---

14. One way to approximate $\int_8^{33} e^{17-2x}\,dx$ is with the sum $\sum_{k=1}^{100}\left((\Delta x)\cdot\left(e^{17-2(8+k\Delta x)}\right)\right)$. What is the best value of Δx to use?

Possibilities:

(a) 100

(b) 1.359140914

(c) $\frac{1}{4}$

(d) 8

(e) 33

---

15. Suppose you estimate the area under the graph of $f(x) = x^3$ from $x = 4$ to $x = 24$ by adding the areas of the rectangles as follows: partition the interval into 20 equal subintervals and use the right endpoint of each interval to determine the height of the rectangle. What is the area of the 14th rectangle?

Possibilities:

(a) 4913

(b) $\frac{21455}{4}$

(c) 5832

(d) 89900

(e) 24

$A_{14} = f(4+14\Delta x)\,\Delta x = f(18)\cdot 1 = 18^3 = 5832$

---

16. Evaluate the sum

$$\sum_{k=5}^{7}(4k^3+7) = [4(5)^3+7] + [4(6)^3+7] + [4(7)^3+7]$$

$$= 2757$$

Possibilities:

(a) 507

(b) 28

(c) 1886

(d) 1379

(e) 2757

17. Evaluate the sum

$$\sum_{k=1}^{140}(3k^2) = 3\sum_{k=1}^{140} k^2 = 3\left(\frac{140(141)(281)}{6}\right) = 2\,773\,470$$

Possibilities:

(a) 58800

(b) 29610

(c) 924490

(d) 58803

(e) 2773470

18. Evaluate the sum $4+8+12+16+20+24+\cdots+76+80$.

$$= \sum_{n=1}^{20} 4n = 4\sum_{n=1}^{20} n = 4\left(\frac{20\cdot 21}{2}\right)$$

$$= 840$$

Possibilities:

(a) 5

(b) 840

(c) 4

(d) 120

(e) 3240

19. Evaluate the sum $\sum_{k=3}^{200}(6+5k)$.

$= \sum_{k=1}^{200}(6+5k) - 16 - 11$

$= 200 \cdot 6 + 5\sum_{k=1}^{200} k - 27$

$= 1200 + 5\frac{(200)(201)}{2} - 27 = 101673$

Possibilities:

(a) 101700

(b) 1006

(c) 100506

(d) 101673

(e) 21

20. Evaluate the sum $\sum_{k=3}^{n}(9k)$.

$= \sum_{k=1}^{n} 9k - 18 - 9$

$= 9\left(\frac{n(n+1)}{2}\right) - 27$

Possibilities:

(a) $\frac{27}{2} + \frac{9}{2}n$

(b) $\frac{9}{2}n(n+1) - 27$

(c) $\frac{9}{2}n(n+1) - 54$

(d) $9n$

(e) $\frac{9}{2}n(n+1)$

Some Formulas

1. **Summation formulas:**

$$\sum_{k=1}^{n} k = \frac{n(n+1)}{2}$$

$$\sum_{k=1}^{n} k^2 = \frac{n(n+1)(2n+1)}{6}$$

2. **Areas:**

   (a) Triangle $A = \frac{bh}{2}$

   (b) Circle $A = \pi r^2$

   (c) Rectangle $A = lw$

   (d) Trapezoid $A = \frac{h_1 + h_2}{2} b$

3. **Volumes:**

   (a) Rectangular Solid $V = lwh$

   (b) Sphere $V = \frac{4}{3}\pi r^3$

   (c) Cylinder $V = \pi r^2 h$

   (d) Cone $V = \frac{1}{3}\pi r^2 h$

4. **Distance:**

   (a) Distance between (x_1, y_1) and (x_2, y_2)

$$D = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$