MA123 — Elem. Calculus	Spring 2015	Name: SOLUTIONS Sec.: ________
Exam 3	2015-04-16	

Do not remove this answer page — you will turn in the entire exam. You have two hours to do this exam. No books or notes may be used. You may use an ACT-approved calculator during the exam, but NO calculator with a Computer Algebra System (CAS), networking, or camera is permitted. Absolutely no cell phone use during the exam is allowed.

The exam consists of two short answer questions and eighteen multiple choice questions. Answer the short answer questions on the back of this page, and record your answers to the multiple choice questions on this page. For each multiple choice question, you will need to fill in the circle corresponding to the correct answer. For example, if (a) is correct, you must write

(a) (b) (c) (d) (e)

Do not circle answers on this page, but please circle the letter of each correct response in the body of the exam. It is your responsibility to make it CLEAR which response has been chosen. You will not get credit unless the correct answer has been marked on both this page and in the body of the exam.

GOOD LUCK!

3.	**(a)**	(b)	(c)	(d)	(e)	**12.**	**(a)**	(b)	(c)	(d)	(e)
4.	(a)	**(b)**	(c)	(d)	(e)	**13.**	(a)	(b)	**(c)**	(d)	(e)
5.	(a)	(b)	**(c)**	(d)	(e)	**14.**	(a)	(b)	(c)	(d)	**(e)**
6.	**(a)**	(b)	(c)	(d)	(e)	**15.**	**(a)**	(b)	(c)	(d)	(e)
7.	(a)	(b)	(c)	**(d)**	(e)	**16.**	(a)	**(b)**	(c)	(d)	(e)
8.	(a)	(b)	(c)	(d)	**(e)**	**17.**	(a)	(b)	**(c)**	(d)	(e)
9.	(a)	(b)	(c)	**(d)**	(e)	**18.**	(a)	(b)	(c)	(d)	**(e)**
10.	(a)	(b)	(c)	**(d)**	(e)	**19.**	**(a)**	(b)	(c)	(d)	(e)
11.	(a)	(b)	**(c)**	(d)	(e)	**20.**	(a)	(b)	(c)	**(d)**	(e)

For grading use:

Multiple Choice	Short Answer
(number right) (5 points each)	(out of 10 points)

Total	
	(out of 100 points)

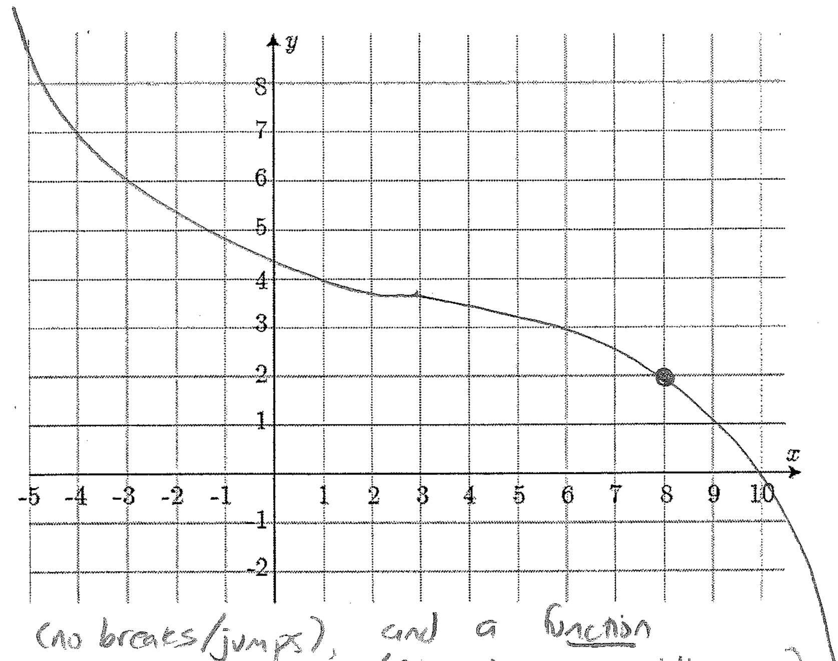
Spring 2015 Exam 3 Short Answer Questions

Write your answers on this page.

1. Sketch the graph of a **continuous** function $y = f(x)$ which satisfies the following:

   $f(8)=2$, $f'(x) \le 0$ for all x; $f''(x) > 0$ for $x<3$; $f''(x)<0$ for $x>3$.

Conditions above require the graph to go through the point (8, 2), to be decreasing (never increasing), to be concave up for $x < 3$, and concave down for $x > 3$.



Also must be continuous (no breaks/jumps), and a function (pass the vertical line test.)

2. The area of a square is increasing at a rate of 140 cm^2/min. At what rate is the length of the side of the square increasing when the area is 25 cm^2? *You must show proper, appropriate, and legible work to be sure you will get full credit.*

$$A = x^2$$

$$\frac{dA}{dt} = 2x \frac{dx}{dt}$$

when the area is 25, $x = 5$.

$$140 = 2(5)\frac{dx}{dt}$$

$$\frac{dx}{dt} = \frac{140}{10} \frac{\text{cm}}{\text{min}}$$

Final answer: 14 cm/min

Name: SOLUTIONS

Multiple Choice Questions

Show all your work on the page where the question appears.
Clearly mark your answer both on the cover page on this exam
and in the corresponding questions that follow.

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3. Where is the function $f(t) = t^3 - 9t^2 - 48t + 1$ decreasing?

Possibilities:

(a) $-2 < t < 8$

(b) $t < 3$

(c) $t < -2$ and $t > 8$

(d) $f(t)$ is always decreasing

(e) $t > 3$

$$f'(t) = 3t^2 - 18t - 48$$
$$= 3(t^2 - 6t - 16)$$
$$= 3(t-8)(t+2)$$

$f'(t) = 0$ when $t = 8$ or $t = -2$.

(Test in the first derivative, caring only about sign)

f

f' $+$ $-$ $+$

test -100 -2 test 0 8 test 100

f is decreasing for t between -2 and 8.

---

4. Where is the function $f(t) = t^3 - 9t^2 - 48t + 1$ concave up?

Possibilities:

(a) $f(t)$ is always concave up

(b) $t > 3$

(c) $-2 < t < 8$

(d) $t < 3$

(e) $t < -2$ and $t > 8$

$$f'(t) = 3t^2 - 18t - 48$$
$$f''(t) = 6t - 18 = 6(t-3)$$

$f''(t) = 0$ when $t = 3$.

(test in the second deriv., caring only about sign)

f $\cap$ $\cup$

f'' $-$ $+$

test 0 3 test 100

f is concave up for $t > 3$.

---

5. If $g'(t) = 4 - t^2$, where is the function $g(t)$ decreasing?

Possibilities:

(a) $t < 0$

(b) $-2 < t < 2$

(c) $t < -2$ and $t > 2$

(d) $f(t)$ is always decreasing

(e) $t > 0$

$g'(t) = 4 - t^2 = (2-t)(2+t)$

$g'(t) = 0$ when $t = 2$ or $t = -2$

g

g' $-$ $+$ $-$

test -100 -2 test 0 2 test 100

(g is decreasing if $g' < 0$)

g decreases for $t < -2$ and $t > 2$.

---

6. If $g'(t) = 4 - t^2$, where is the function $g(t)$ concave up?

Possibilities:

(a) $t < 0$

(b) $-2 < t < 2$

(c) $t < -2$ and $t > 2$

(d) $f(t)$ is always concave up

(e) $t > 0$

$g'(t) = 4 - t^2$

$g''(t) = 0 - 2t = -2t$

$g''(t) = 0$ when $t = 0$.

(g is concave up if $g'' > 0$.)

g

g'' $+$ $-$

test -1 0 test 1

g is concave up for $t < 0$.

7. The following is the graph of the derivative, $f'(x)$, of the function $f(x)$. Where is the regular function $f(x)$ decreasing?

Possibilities:

(a) $(-2, \infty)$

(b) $(-\infty, -1)$

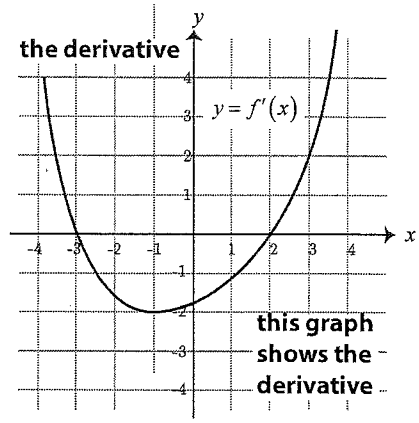
(c) $(-1, \infty)$

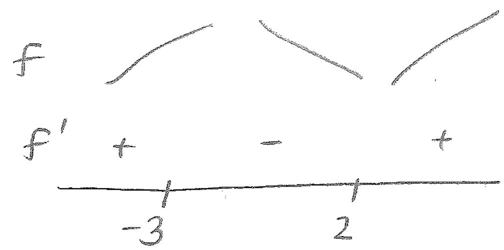
(d) $(-3, 2)$

(e) $(-\infty, -3)$ and $(2, \infty)$

f is decreasing where $f'(x) < 0$,

So we notice where this graph is above/below the axis.





f is decreasing for x between −3 and 2.

8. The following is the graph of the derivative, $f'(x)$, of the function $f(x)$. Where is the regular function $f(x)$ concave up?

Possibilities:

(a) $(-2, \infty)$

(b) $(-\infty, -3)$ and $(2, \infty)$

(c) $(-3, 2)$

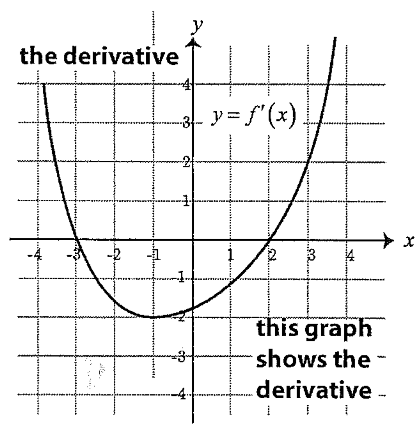
(d) $(-\infty, -1)$

(e) $(-1, \infty)$

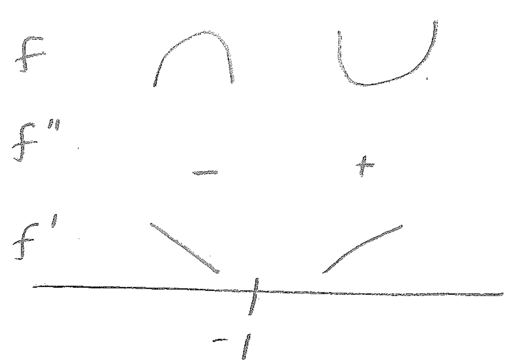
If $f' > 0$, then f is increas.

$\Rightarrow$ If $f'' > 0$, then f' is increas.

So, we notice where this graph is increasing and decreasing.



read this from graph $\longrightarrow$



f is concave up for $x > -1$.

9. Two trains leave the same station at different times, one traveling due East, and the other traveling due North. At 2pm the eastbound train is traveling at 50 mph and is 400 miles from the station, while the northbound train is traveling at 60 mph and is 300 miles from the station. At what rate is the distance between the trains increasing?

Possibilities:

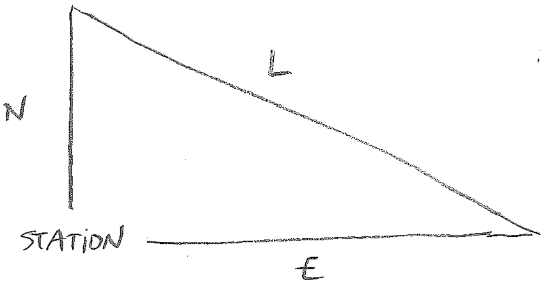
(a) 76000 mph

(b) $10\sqrt{61}$ mph

(c) 500 mph

(d) 76 mph

(e) 110 mph



Let N = distance between northbound train and station, E the distance between the eastbound train and station, and L the distance between the two trains.

At 2 pm, if E = 400 and N = 300, then $L^2 = 400^2 + 300^2$ $\Rightarrow L = 500$

$E^2 + N^2 = L^2$

$2E\frac{dE}{dt} + 2N\frac{dN}{dt} = 2L\frac{dL}{dt}$

$2(400)(50) + 2(300)(60) = 2(500)\frac{dL}{dt}$

$76000 = 1000\frac{dL}{dt}$

$\frac{dL}{dt} = \frac{76000}{1000} = 76$ miles/hour

10. Find the area of the largest rectangle whose sides are parallel to the coordinate axes, whose bottom-left corner is at $(0,0)$ and whose top-right corner is on the graph of $y = 6x - x^2$.

Possibilities:

(a) 27

(b) 3

(c) 30

(d) 32

(e) 0

$A = xy$

$y = 6x - x^2$

$A = x(6x - x^2)$

$= 6x^2 - x^3$

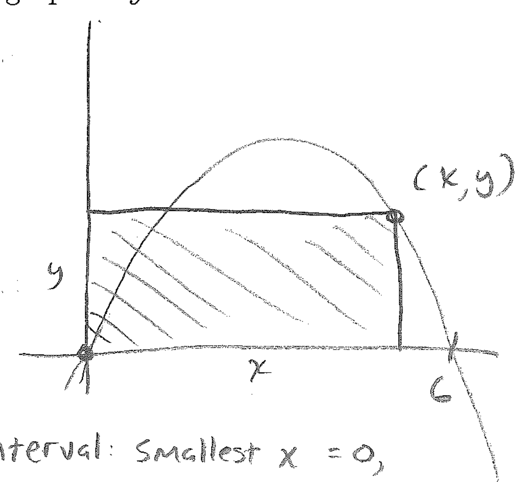
$A' = 12x - 3x^2$

$A' = 0$ when

$12x - 3x^2 = 0$

$3x(4 - x) = 0$

$x = 0$ or $x = 4$.



Interval: Smallest $x = 0$, largest $x = 6$.

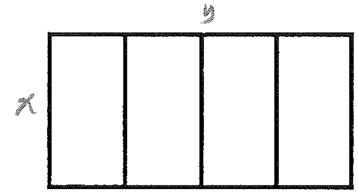
check orig. eqn: $A = 6x^2 - x^3$

$A(0) = 0$

$A(4) = 6 \cdot 4^2 - 4^3 = 32$ ← maximum

$A(6) = 0$

11. A farmer builds a rectangular pen with 4 vertical partitions (5 vertical sides) using 600 feet of fencing. What is the maximum possible total area of the pen?



Possibilities:

(a) 300

(b) 7500

(c) 9000

(d) 600

(e) 22500

$A = xy$

$5x + 2y = 600$

$2y = 600 - 5x$

$y = 300 - \frac{5}{2}x$

$A = x(300 - \frac{5}{2}x) = 300x - \frac{5}{2}x^2$

$A' = 300 - 5x$

$A' = 0$ when $5x = 300 \Rightarrow x = 60$.

Check orig. function: $A(0) = 0$

$A(60) = 300(60) - \frac{5}{2} \cdot 60^2 = \underline{9000}$

$A(120) = 0$.

Interval:
Smallest $x = 0$.
largest x when $y = 0$,
So $5x = 600$
$\Rightarrow x = 120$.

12. A farmer currently has harvested 130 bushels of collard greens that are currently worth \$12.74 per bushel. The way things are going, he expects to be harvesting 3.00 bushels per day, and expects the price to be increasing at \$0.75 per bushel per day. What is the instantaneous rate of change (measured in dollars per day) of the total value of his collard greens?

Possibilities:

(a) \$135.72 per day

(b) \$135.73 per day

(c) \$135.74 per day

(d) \$135.75 per day

(e) \$135.76 per day

Total value = number of bushels · price per bushel

$V = NP$ (now differentiate, using product rule)

$\frac{dV}{dt} = N \cdot \frac{dP}{dt} + P \cdot \frac{dN}{dt}$

$\frac{dV}{dt} = 130 \cdot (.75) + (12.74) \cdot 3$

$= \$135.72/\text{day}$

13. Estimate the area under the graph of $-x^2 + 20x$ for x between 4 and 10, by using a partition tha consists of 3 equal subintervals of $[4, 10]$ and use the right endpoint of each subinterval as a sample point.

$\Delta x = \frac{b-a}{n} = \frac{10-4}{3} = \frac{6}{3} = 2$

Possibilities:

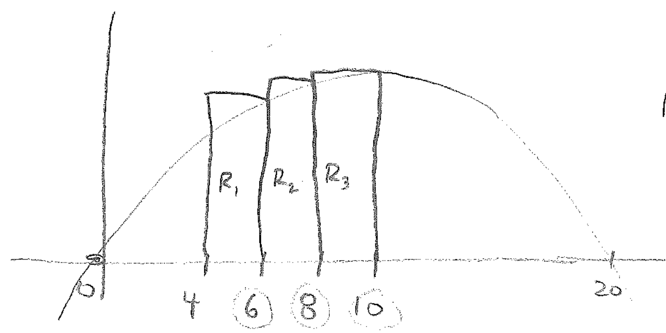
(a) 528

(b) 688

(c) 560

(d) 280

(e) 488



height

$R_1: -6^2 + 20(6) = 84$

$R_2: -8^2 + 20(8) = 96$

$R_3: -10^2 + 20(10) = 100$

Area $\approx 84(2) + 96(2) + 100(2)$

$= 2(84 + 96 + 100) = 560$

14. A train travels in a straight westward direction along a track. The speed of the train varies, but it is measured at regular time intervals of 1/10 hour. The measurements for the first half hour are:

time	0	.1	.2	.3	.4	.5
speed	0	4	10	14	20	23

Estimate the total distance (in miles) traveled by the train during the first half hour by assuming the speed is a linear function of t on the subintervals. The speed in the table is given in miles per hour. Use all six speed measurements in your estimate.

Possibilities:

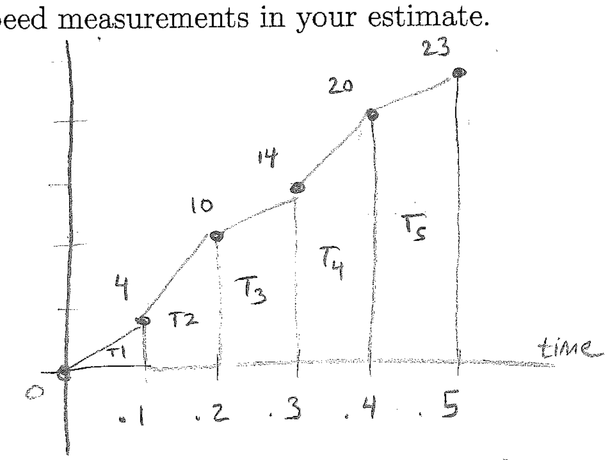
(a) 7.00 miles

(b) 7.10 miles

(c) 2.00 miles

(d) 11.50 miles

(e) 5.95 miles



$T_1: \frac{1}{2}(4)(.1) = .2$

$T_2: \frac{4+10}{2}(.1) = .7$

$T_3: \frac{10+14}{2}(.1) = 1.2$

$T_4: \frac{14+20}{2}(.1) = 1.7$

$T_5: \frac{20+23}{2}(.1) = 2.15$

(use trapezoid area formula on T_2-T_5, triangle area for T_1.)

Total area $= 5.95$

$=$ total distance.

15. One way to approximate $\int_A^{59} e^{19-2x}\,dx$ is with the sum $\sum_{k=1}^{n} ((\Delta x)\cdot(e^{19-2(9+k\Delta x)}))$ where $\Delta x = \frac{1}{4}$. What is the best value of A to use?

200 n

Possibilities:

(a) 9

(b) $\frac{1}{4}$

(c) 1.359140914

(d) 0.01

(e) 200

$\Delta X = \frac{b-a}{n}$ ← a and b are the limits of integration

← n is the # of rectangles we add

$\frac{1}{4} = \frac{59-A}{200}$

cross-multiply: $4(59-A) = 200$

divide by 4 $59 - A = 50$

$A = 9.$

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16. Suppose you estimate the area under the graph of $f(x) = x^3$ from $x = 5$ to $x = 25$ by adding the areas of the rectangles as follows: partition the interval into 20 equal subintervals and use the right endpoint of each interval to determine the height of the rectangle. What is the area of the 7th rectangle?

$\Delta X = \frac{b-a}{n} = \frac{25-5}{20} = 1 =$ width of each rectangle

Possibilities:

(a) 25

(b) 1728

(c) 105400

(d) $\frac{6095}{4}$

(e) 1331

To find x-values used, start at 5 and add ΔX each time.

1st rectangle: $5 + \Delta X$
$= 5 + 1 = 6.$

2nd rectangle: $5 + 2\Delta X$
$= 5 + 2(1) = 7$

⋮

7th rectangle: $5 + 7\Delta X$
$= 5 + 7(1) = 12$

→ height of 7th rectangle
$f(12) = 12^3$
$= 1728$

width $= 1$

Area $= 1728(1)$
$= 1728.$

17. Evaluate the difference of sums

$$\left(\sum_{k=1}^{40000}(6k^3+5)\right)-\left(\sum_{k=3}^{40000}(6k^3+5)\right)$$

$\downarrow$ we add 40000 things,

$\downarrow$ then take away all but the 1st two of them.

All that remains are $k=1$ and $k=2$:

$k=1$ $k=2$

$$6\cdot 1^3+5 \quad + \quad 6\,2^3+5$$

$$= 6+5 \quad + \quad 48+5$$

$$= 64$$

Possibilities:

(a) 800020000

(b) 384000000000005

(c) 64

(d) 0

(e) ∞

18. Evaluate the sum

$$\sum_{k=1}^{N}(11k^2)$$

$$= 11\sum_{k=1}^{N}k^2$$

$$= 11\cdot\frac{N(N+1)(2N+1)}{6}$$

Possibilities:

(a) $11N^2$

(b) $11N^2-11$

(c) $11\dfrac{N(N+1)}{2}$

(d) $11N^2+11$

(e) $11\dfrac{N(N+1)(2N+1)}{6}$

19. Evaluate the sum $5 + 10 + 15 + 20 + 25 + 30 + 35 + 40 + 45 + 50 + \cdots + 370 + 375$.

Possibilities:

(a) 14250

(b) 70500

(c) 717250

(d) 140625

(e) 1020

$$= 5(1+2+3+4+\cdots+74+75)$$

$$= 5\cdot\sum_{k=1}^{75} k$$

$$= 5\cdot\frac{75(76)}{2}$$

$$= 5(2850)$$

$$= 14250$$

20. Evaluate the sum $\frac{1}{13} + \frac{4}{13} + \frac{9}{13} + \frac{16}{13} + \frac{25}{13} + \frac{36}{13} + \frac{49}{13} + \frac{64}{13} + \frac{81}{13} + \frac{100}{13} + \cdots + \frac{841}{13} + \frac{900}{13}$.

Possibilities:

(a) $\frac{13515}{13}$

(b) $\frac{810000}{169}$

(c) $\frac{2126}{13}$

(d) $\frac{9455}{13}$

(e) $\frac{410850}{169}$

$$= \frac{1}{13}(1+4+9+16+25+\cdots+900)$$

$$= \frac{1}{13}(1^2+2^2+3^2+4^2+5^2+\cdots+30^2)$$

$$= \frac{1}{13}\sum_{k=1}^{30} k^2$$

$$= \frac{1}{13}\cdot\frac{30(31)(61)}{6}$$

$$= \frac{56730}{13(6)} = \frac{9455}{13}$$

Some Formulas

1. **Summation formulas:**

$$\sum_{k=1}^{n} k = \frac{n(n+1)}{2}$$

$$\sum_{k=1}^{n} k^2 = \frac{n(n+1)(2n+1)}{6}$$

2. **Areas:**
   (a) Triangle $A = \dfrac{bh}{2}$
   (b) Circle $A = \pi r^2$
   (c) Rectangle $A = lw$
   (d) Trapezoid $A = \dfrac{h_1 + h_2}{2} b$

3. **Volumes:**
   (a) Rectangular Solid $V = lwh$
   (b) Sphere $V = \dfrac{4}{3}\pi r^3$
   (c) Cylinder $V = \pi r^2 h$
   (d) Cone $V = \dfrac{1}{3}\pi r^2 h$

4. **Distance:**
   (a) Distance between (x_1, y_1) and (x_2, y_2)

$$D = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$