MA 123 — Elem. Calculus Fall 2012 **Name:** Solutions. **Sec.:** ________
EXAM 4 12/10/2012

Do not remove this answer page — you will turn in the entire exam. You have two hours to do this exam. No books or notes may be used. You may use a graphing calculator during the exam, but NO calculator with a Computer Algebra System (CAS) or a QWERTY keyboard is permitted. Absolutely no cell phone use during the exam is allowed.

The exam consists of multiple choice questions. Record your answers on this page. For each multiple choice question, you will need to fill in the box corresponding to the correct answer. For example, if (b) is correct, you must write

[a] [b] [c] [d] [e]

Do not circle answers on this page, but please circle the letter of each correct response in the body of the exam. It is your responsibility to make it CLEAR which response has been chosen. You will not get credit unless the correct answer has been marked on both this page and in the body of the exam.

GOOD LUCK!

1.	a	b	c	d	e	**11.**	a	b	c	d	e
2.	a	b	c	d	e	**12.**	a	b	c	d	e
3.	a	b	c	d	e	**13.**	a	b	c	d	e
4.	a	b	c	d	e	**14.**	a	b	c	d	e
5.	a	b	c	d	e	**15.**	a	b	c	d	e
6.	a	b	c	d	e	**16.**	a	b	c	d	e
7.	a	b	c	d	e	**17.**	a	b	c	d	e
8.	a	b	c	d	e	**18.**	a	b	c	d	e
9.	a	b	c	d	e	**19.**	a	b	c	d	e
10.	a	b	c	d	e	**20.**	a	b	c	d	e

For grading use:

Number Correct	
	(out of 20 problems)

Total	
	(out of 100 points)

Please make sure to list the correct section number on the front page of your exam. In case you forgot your section number, consult the following table. Your section number is determined by your recitation time and location.

Section #	Instructor	Day and Time	Room
001	J. Constable	T, 8:00 am-9:15 am	FB B2
002	J. Constable	T, 9:30 am-10:45 am	NURS 501C
003	J. Constable	T, 11:00 am-12:15 pm	TPC 212
004	L. Davidson	T, 12:30 pm-1:45 pm	CP 111
005	L. Davidson	T, 2:00 pm-3:15 pm	CB 219
006	L. Davidson	T, 3:30 pm-4:45 pm	CB 341
007	W. Hough	R, 8:00 am-9:15 am	FB B2
008	W. Hough	R, 9:30 am-10:45 am	MMRB 243
009	W. Hough	R, 11:00 am-12:15 pm	TPC 212
010	X. Kong	R, 12:30 pm-1:45 pm	Laferty 201
011	X. Kong	R, 2:00 pm-3:15 pm	FPAT 257
012	X. Kong	R, 3:30 pm-4:45 pm	CB 341
013	L. Solus	T, 8:00 am-9:15 am	CB 303
014	K. Effinger	T, 8:00 am-9:15 am	CB 233
015	K. Effinger	T, 11:00 am-12:15 pm	CB 347
016	Q. Liang	T, 12:30 pm-1:45 pm	NURS 501C
017	Q. Liang	T, 2:00 pm-3:15 pm	CB 247
018	Q. Liang	T, 3:30 pm-4:45 pm	CB 245
019	K. Effinger	R, 8:00 am-9:15 am	CB 303
020	L. Solus	R, 8:00 am-9:15 am	CB 233
021	L. Solus	R, 3:30 pm-4:45 pm	CB 214
022	A. Happ	R, 12:30 pm-1:45 pm	NURS 501C
023	A. Happ	R, 2:00 pm-3:15 pm	CB 338
024	A. Happ	R, 3:30 pm-4:45 pm	CB 245
025	F. Smith	T, 12:30 pm-1:45 pm	FPAT 263
026	D. Akers	R, 12:30 pm-1:45 pm	FPAT 263
027	F. Smith	T, 2:00 pm-3:15 pm	FPAT 259
028	D. Akers	R, 2:00 pm-3:15 pm	FPAT 259
029	F. Smith	T, 3:30 pm-4:45 pm	CB 205
030	D. Akers	R, 3:30 pm-4:45 pm	CB 205

Multiple Choice Questions

Show all your work on the page where the question appears.
Clearly mark your answer both on the cover page on this exam
and in the corresponding questions that follow.

1. Evaluate the indefinite integral

Distribute

$$\int t^2(t+5)\,dt.$$

$$= \int t^3 + 5t^2\,dt$$

Power Rule

$$= \frac{1}{3+1}t^{3+1} + \frac{5}{2+1}t^{2+1} + C$$

$$= \frac{1}{4}t^4 + \frac{5}{3}t^3 + C$$

Possibilities:

(a) $\frac{1}{4}t^3 + \frac{5}{3}t^2 + C$
(b) $t^2(t+5) + C$
(c) $\frac{1}{3}t^3\left(\frac{1}{2}t^2 + 5t\right) + C$
(d) $\frac{1}{4}t^4 + \frac{5}{3}t^3 + C$
(e) $4t^4 + 15t^3 + C$

2. Evaluate the definite integral

$$\int_{-2}^{4} 3e^t\,dt. = 3\int_{-2}^{4} e^t\,dt$$

$$= 3\left(e^t\right)\Big|_{-2}^{4}$$

$$= 3(e^4 - e^{-2})$$

Possibilities:

(a) $\ln|4| - \ln|-2|$
(b) $3e^4$
(c) $3e^4 - 3e^{-2}$
(d) $12e^3 - 6e^{-3}$
(e) $12e^3 + 6e^{-3}$

3. Evaluate the indefinite integral

$$\int \left(\frac{1}{t}\right)^5 dt. = \int \frac{1}{t^5}\,dt = \int t^{-5}\,dt$$

$$= \frac{1}{-5+1}t^{-5+1} + C$$

$$= \frac{1}{-4}t^{-4} + C$$

Possibilities:

(a) $\frac{1}{4}t^4 + C$
(b) $\frac{1}{6}t^6 + C$
(c) $-\frac{1}{4}t^{-4} + C$
(d) $\frac{1}{6}t^{-6} + C$
(e) $\frac{1}{5t^5} + C$

4. Suppose

$$f(x) = \begin{cases} x^2 & \text{for } x < 2 \\ x & \text{for } x \geq 2. \end{cases}$$

Evaluate the definite integral

$$\int_0^4 f(x)\,dx.$$

$$= \int_0^2 x^2\,dx + \int_2^4 x\,dx$$

$$= \frac{1}{3}x^3\Big|_0^2 + \frac{1}{2}x^2\Big|_2^4$$

$$= \frac{1}{3}(2^3 - 0^3) + \frac{1}{2}(4^2 - 2^2) = \frac{8}{3} + 6 = \frac{26}{3}$$

Possibilities:

(a) 26/3

(b) 9

(c) 28/3

(d) 29/3

(e) 10

5. Evaluate the integral

$$\int_0^5 \frac{2x}{x^2+1}\,dx$$

Let $u = x^2+1$, then $du = 2x\,dx$

$$\int_0^5 \frac{2x}{x^2+1}\,dx = \int_{0^2+1}^{5^2+1} \frac{2x}{u}\cdot\left(\frac{1}{\cancel{2x}}du\right) = \int_1^{26} \frac{1}{u}\,du$$

$$= \ln(26) - \underbrace{\ln(1)}_{0} = \ln(26)$$

Possibilities:

(a) $\ln(22)$

(b) $\ln(23)$

(c) $\ln(24)$

(d) $\ln(25)$

(e) $\ln(26)$

6. A car travels due east. Its velocity (in miles per hour) at time t is $v(t) = -3t^2 + 20t + 50$. How far does the car travel during the first 4 hours of the trip?

Distance $= \int_0^4 v(t)\,dt = \int_0^4 -3t^2 + 20t + 50\,dt$

$$= -\cancel{3}\cdot\frac{1}{\cancel{3}}t^3\Big|_0^4 + 20\cdot\frac{1}{2}t^2\Big|_0^4 + 50t\Big|_0^4$$

$$= -(4)^3 - -0^3 + 10\cdot4^2 - 10\cdot0^2 + 50\cdot4 - 50\cdot0$$

$$= 296$$

Possibilities:

(a) 256 miles

(b) 266 miles

(c) 276 miles

(d) 286 miles

(e) 296 miles

Factor & cancel

7. Compute $\lim_{t\to4} \frac{t^2-t-12}{t^2-3t-4}$ $= \lim_{t\to4} \frac{(\cancel{t-4})(t+3)}{(\cancel{t-4})(t+1)}$

$= \frac{4+3}{4+1} = \frac{7}{5}$

Possibilities:

(a) 6/5

(b) 7/5

(c) 8/5

(d) 9/5

(e) The limit does not exist.

---

8. Evaluate the limit as n tends to infinity. Note: you will have to use some of the summation formulas (see formula sheet on backpage) to simplify.

$$\lim_{n\to\infty} \frac{1}{n} \sum_{k=1}^{n} \frac{8k}{n} = \lim_{n\to\infty} \frac{8}{n^2} \sum_{k=1}^{n} k$$

$$= \lim_{n\to\infty} \frac{8}{n^2} \cdot \frac{n(n+1)}{2}$$

$$= \lim_{n\to\infty} \frac{4(n+1)}{n} = 4$$

Possibilities:

(a) 0

(b) 4

(c) 8

(d) 9

(e) The limit does not exist or the limit tends to infinity.

---

9. Find the value of x at which

$$F(x) = \int_{-10}^{x} \frac{50}{t^2+1}\,dt$$

takes its maximum value on the interval $[-5, 10]$.

Find Critical Points:

$F'(x) = \frac{50}{x^2+1}$

Now, $F'(x) \neq 0$, and furthermore

$F'(x) > 0$ for all x.

So max. occurs at far right endpoint, $x = 10$.

Possibilities:

(a) -10

(b) 10

(c) -5

(d) 0

(e) 50

10. Given that the area of the ellipse $25x^2 + y^2 = 25$ is 5π, evaluate the integral

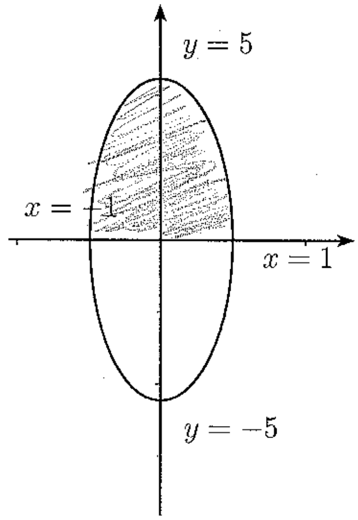
Left + Right half

$$\int_{-1}^{1} \sqrt{25 - 25x^2}\,dx.$$

Top half of ellipse

Hint: It may be useful to interpret the integral as an area.

Integral represents entire top half ellipse. Whole ellipse Area $= 5\pi$. So top half $\frac{5\pi}{2}$.

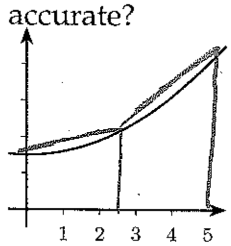


Possibilities:

(a) $\frac{5\pi}{4}$
(b) $\frac{25\pi}{2}$
(c) $\frac{5\pi}{2}$
(d) $\frac{25\pi}{4}$
(e) The value of the integral cannot be determined

---

11. The area under the curve shown below, from $x = 0$ to $x = 5$ is to be approximated by dividing the interval $[0, 5]$ into 2 trapezoids of equal width. Which of the following statements is MOST accurate?



As we saw in Written project 2, concavity effects trapezoid estimates. Increasing & Decreasing does not effect trapezoid estimate.

Possibilities:

(a) The approximate area will be larger than the true area under the curve since the curve is concave up.
(b) It is not possible to determine whether the approximate area is larger or smaller than the true area under the curve.
(c) The approximate area will be smaller than the true area under the curve since the curve is increasing.
(d) The approximate area will be larger than the true area under the curve since the curve is increasing.
(e) The approximate area will be smaller than the true area under the curve since the curve is concave up.

12. Find the value of A which makes $f(x)$ continuous everywhere, where

$$f(x) = \begin{cases} 4x^2, & \text{if } x \leq -5; \\ -3x + A, & \text{if } x > -5 \end{cases}$$

Need $\lim_{x \to -5^-} 4x^2 = \lim_{x \to -5^+} -3x + A.$

Possibilities:

(a) -5

(b) 100

(c) -15

(d) 85

(e) No such value of A exists

$4(-5)^2 = -3(-5) + A$

$100 = 15 + A$

$A = 85.$

13. Suppose that the equation of the tangent line to the graph of $y = f(x)$ at $x = 3$ is given by the equation $y = 6 + 4(x - 3)$. Find $f(3) + f'(3)$.

Slope $= f'(3) = 4.$

$f(3) = 6$

Possibilities:

(a) 7

(b) 8

(c) 9

(d) 10

(e) 11

So $f(3) + f'(3) = 4 + 6 = 10$

14. Which of the following is the correct expression for the derivative $g'(10)$?

Possibilities:

(a) $\lim_{h \to 0} \dfrac{g(10+h) - g(10)}{h}$

(b) $\dfrac{g(10) - g(10+h)}{h}$

(c) $\lim_{h \to 0} \dfrac{g(10-h) - g(10)}{h}$

(d) $\dfrac{g(10+h) - g(10)}{h}$

(e) $\lim_{h \to 0} \dfrac{g(10) - g(10+h)}{h}$

15. Determine the value of x in the interval $[-20, 20]$ for which $f(x)$ attains its minimum value, given that $f'(x) = (x-5)^2 \cdot (x+2)$.
(NOTE: you are given the derivative of $f(x)$, not $f(x)$ itself!)

Min. @ end points or critical points.

Possibilities:

c.p: $f'(x) = 0 \Rightarrow x = 5$ or $x = -2$.

(a) $x = -2$
(b) $x = -20$
(c) $x = 2$
(d) $x = -5$
(e) $x = 5$

	-20		-2		5		20
$f'(x)$		$- - - -$		$+ + + + + +$		$+ + + + + +$	
$f(x)$		↘		↗		↗	

min @ $x = -2$

16. Find the 8^{th} derivative, $f^{(8)}(x)$, where

$$f(x) = e^{17x} + x^2$$

$(x^2)' = 2x$
$(x^2)'' = 2$
$(x^2)''' = 0$
so only need to look at exponential

Possibilities:

$(e^{17x})' = 17e^{17x}$

(a) e^{17x}
(b) $8^{17}e^{17x} + 2$
(c) $17^8 e^{17x}$
(d) $17^8 e^{17x} + 2x$
(e) $e^{136} + 2x$

$(e^{17x})'' = 17^2 e^{17x}$
$(e^{17x})''' = 17^3 e^{17x}$
$\vdots$
so 8^{th} derivative is $17^8 e^{17x}$

17. An expanding rectangle has its length always equal to twice its width. The area is increasing at a rate of 72 square feet per minute. At what rate is the width increasing when the width is 9 feet?

$\frac{dA}{dt} = 72$. $A = L \cdot W \Rightarrow A = 2W \cdot W = 2W^2$
$L = 2W$
$w = 9$

Possibilities:

(a) 18 feet per minute.
(b) 162 feet per minute.
(c) 8 feet per minute.
(d) 2 feet per minute.
(e) 36 feet per minute.

$\frac{dA}{dt} = 2 \cdot 2w \frac{dw}{dt}$

$\frac{dA}{dt} = 4w \frac{dw}{dt}$

so

$72 = 4 \cdot 9 \cdot \frac{dw}{dt}$

$\frac{72}{4 \cdot 9} = \frac{dw}{dt} = 2$.

18. The product of two positive real numbers, x and y is equal to 49. Determine the minimum value of $x + 4y$.

Objective: Minimize $x + 4y$

Possibilities: Constraint: $xy = 49 \Longrightarrow y = 49x^{-1}$

(a) 55/2 So, Objective: $x + 4 \cdot 49x^{-1} = x + 196x^{-1}$

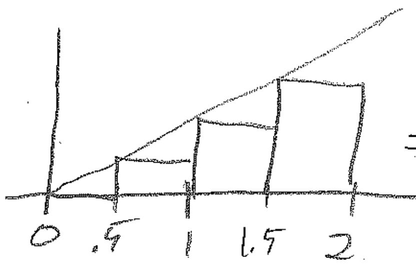
(b) 28

(c) 57/2 $(\text{Objective})' = 1 - 196x^{-2} = 0 \Rightarrow x^2 = 196$

(d) 29 $x = \sqrt{196} = 14$.

(e) 59/2 So Objective $= 14 + 4 \cdot \frac{49}{14} = 28$.

19. Estimate the area under the graph of $f(x) = x^2 + 3x$ for x between 0 and 2. Use a partition that consists of 4 equal subintervals of $[0, 2]$ and use the left endpoint of each subinterval as the sample point.



$.5\left[f(0) + f(.5) + f(1) + f(1.5)\right]$

Possibilities:

$= .5\left[0^2 + 3\cdot 0 + (.5)^2 + 3(.5) + 1^2 + 3\cdot 1 + (1.5)^2 + 3(1.5)\right]$

(a) 433/50

(b) 45/4

(c) 25/2 $= .5 \cdot 12.5 = \frac{1}{2} \cdot \frac{25}{2}$

(d) 14

(e) 25/4 $= \frac{25}{4}$.

20. Suppose $g(3) = 7$ and $g'(3) = -4$. Find $F'(3)$, given that

$$F(x) = \frac{g(x)}{x^2}$$

Possibilities: $F'(x) = \frac{x^2 g'(x) - 2x\, g(x)}{(x^2)^2}$

(a) $-4/3$

(b) $-26/81$ $F'(3) = \frac{3^2 \cdot (-4) - 2 \cdot 3 \cdot 7}{(3^2)^2}$

(c) $-26/27$

(d) $-4/9$

(e) $26/27$

$= \frac{-36 - 42}{81} = \frac{-78}{81} = -\frac{26}{27}$.

Some Formulas

1. **Summation formulas:**

$$\sum_{k=1}^{n} k = \frac{n(n+1)}{2}$$

$$\sum_{k=1}^{n} k^2 = \frac{n(n+1)(2n+1)}{6}$$

2. **Areas:**

   (a) Triangle $A = \frac{bh}{2}$

   (b) Circle $A = \pi r^2$

   (c) Rectangle $A = lw$

   (d) Trapezoid $A = \frac{h_1 + h_2}{2} b$

3. **Volumes:**

   (a) Rectangular Solid $V = lwh$

   (b) Sphere $V = \frac{4}{3}\pi r^3$

   (c) Cylinder $V = \pi r^2 h$

   (d) Cone $V = \frac{1}{3}\pi r^2 h$

4. **Distance:**

   (a) Distance between (x_1, y_1) and (x_2, y_2)

$$D = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$