MA123 — Elem. Calculus Fall 2013
Exam 4 2013-12-16

Name: Key **Sec.:** _____

Do not remove this answer page — you will turn in the entire exam. You have two hours to do this exam. No books or notes may be used. You may use a graphing calculator during the exam, but NO calculator with a Computer Algebra System (CAS) or a QWERTY keyboard is permitted. Absolutely no cell phone use during the exam is allowed.

The exam consists of multiple choice questions. Record your answers on this page. For each multiple choice question, you will need to fill in the box corresponding to the correct answer. For example, if (a) is correct, you must write

(a) (b) (c) (d) (e)

Do not circle answers on this page, but please circle the letter of each correct response in the body of the exam. It is your responsibility to make it CLEAR which response has been chosen. You will not get credit unless the correct answer has been marked on both this page and in the body of the exam.

GOOD LUCK!

1. (a) ● (c) (d) (e)
2. (a) (b) ● (d) (e)
3. ● (b) (c) (d) (e)
4. ● (b) (c) (d) (e)
5. (a) (b) (c) ● (e)
6. (a) (b) (c) ● (e)
7. (a) (b) ● (d) (e)
8. ● (b) (c) (d) (e)
9. (a) (b) (c) (d) ●
10. (a) (b) (c) ● (e)
11. ● (b) (c) (d) (e)
12. (a) ● (c) (d) (e)
13. (a) (b) (c) ● (e)
14. ● (b) (c) (d) (e)
15. (a) (b) ● (d) (e)
16. (a) (b) ● (d) (e)
17. ● (b) (c) (d) (e)
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19. (a) (b) (c) ● (e)
20. (a) ● (c) (d) (e)
21. (a) (b) (c) (d) ●
22. (a) ● (c) (d) (e)
23. (a) (b) ● (d) (e)
24. (a) ● (c) (d) (e)
25. (a) ● (c) (d) (e)

For grading use:

Number Correct	
	(out of 20 problems)

Total	
	(out of 100 points)

Please make sure to list the correct section number on the front page of your exam. In case you forgot your section number, consult the following table. Your section number is determined by your recitation time and location.

Section #	Instructor	Day	Time	Room
001-006	Jack Schmidt	MWF	08:00 am - 08:50 am	KAS 213
001	Jinping Zhuge	T	8:00 am - 9:15 am	FB B9
002	Yiyuan Wu	T	9:30 am - 10:45 am	NURS 501B
003	Devin Willmott	T	8:00 am - 9:15 am	CB 235
004	Tefjol Pllaha	T	8:00 am - 9:15 am	CB 237
005	Tefjol Pllaha	T	2:00 pm - 3:15 pm	CB 347
006	Tefjol Pllaha	T	3:30 pm - 4:45 pm	CB 347
007-012	Jack Schmidt	MWF	09:00 am - 09:50 am	KAS 213
007	Yiyuan Wu	R	8:00 am - 9:15 am	CB 217
008	Jinping Zhuge	R	9:30 am - 10:45 am	DH 323
009	Yiyuan Wu	R	11:00 am - 12:15 pm	EH 202
010	Jinping Zhuge	R	12:30 pm - 1:45 pm	DH 323
011	Dharma Maharjan	R	2:00 pm - 3:15 pm	CB 347
012	Dharma Maharjan	R	3:30 pm - 4:45 pm	CB 347
013-018	Paul Koester	MWF	1:00 pm - 1:50 pm	BS 116
013	Carolyn Troha	T	8:00 am - 9:15 am	CB 345
014	Carolyn Troha	T	9:30 am - 10:45 am	NURS 214
015	Morgan Schreffler	T	11:00 am - 12:15 pm	EH 202
016	Carolyn Troha	T	12:30 pm - 1:45 pm	MMRB 243
017	Morgan Schreffler	T	2:00 pm - 3:15 pm	BH 301
018	Morgan Schreffler	T	3:30 pm - 4:45 pm	CB 235
025-030	Paul Koester	MWF	2:00 pm - 2:50 pm	BS 107
025	Sarah Orchard	T	12:30 pm - 1:45 pm	TPC 212
026	Marie Meyer	R	8:00 am - 9:15 am	CB 240
027	Marie Meyer	T	2:00 pm - 3:15 pm	DH 331
028	Marie Meyer	R	2:00 pm - 3:15 pm	EH 304
029	Sarah Orchard	T	3:30 pm - 4:45 pm	OT OB7
030	Sarah Orchard	R	3:30 pm - 4:45 pm	OT 0B7
401	Brad Schwer	MTR	5:30 pm - 6:45 pm	CB 337

Multiple Choice Questions

Show all your work on the page where the question appears.
Clearly mark your answer both on the cover page on this exam
and in the corresponding questions that follow.

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1. Find the limit as n tends to infinity.

$$\lim_{n\to\infty} \frac{(7n+1)^2}{9n^2+4n+1}$$

Possibilities:

(a) 0

(b) $\frac{49}{9}$

(c) $\frac{7}{9}$

(d) $\frac{14}{9}$

(e) ∞

$$= \lim_{n\to\infty} \frac{49n^2+14n+1}{9n^2+4n+1} \cdot \frac{1/n^2}{1/n^2}$$

$$= \lim_{n\to\infty} \frac{49 + 14/n + 1/n^2}{9 + 4/n + 1/n^2}$$

$$= \frac{49+0+0}{9+0+0} = \frac{49}{9}$$

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2. Evaluate the limit as n tends to infinity. Note: you will have to use some of the summation formulas (see formula sheet on backpage) to simplify.

$$\lim_{n\to\infty} \frac{1}{n} \sum_{k=1}^{n} \frac{6k}{n}$$

Possibilities:

(a) 1

(b) 2

(c) 3

(d) 4

(e) 5

$$= \lim_{n\to\infty} \frac{6}{n^2} \sum_{k=1}^{n} k = \lim_{n\to\infty} \frac{6}{n^2} \cdot \frac{n\cdot(n+1)}{2}$$

$$= \lim_{n\to\infty} \frac{6}{2} \cdot \frac{n^2+n}{n^2} \cdot \frac{1/n^2}{1/n^2} = \lim_{n\to\infty} 3 \cdot \frac{1+1/n}{1} = 3$$

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3. The integral

$$\int_3^8 x^2 \, dx$$

is computed as the limit of the sum

$$\sum_{k=1}^{n} \frac{5}{n}\left(A + \frac{5k}{n}\right)^2$$

What value should be used for A?

Possibilities:

(a) 3

(b) 8

(c) 4

(d) 2

(e) 5

---

4. Evaluate the definite integral

$$\int_2^x \frac{6}{\sqrt{t}} \, dt$$

Possibilities:

(a) $12\sqrt{x} - 12\sqrt{2}$

(b) $3\sqrt{x} - 3\sqrt{2}$

(c) $6\sqrt{x}$

(d) $6\sqrt{x} - 6\sqrt{2}$

(e) $\frac{6}{\sqrt{x}} - \frac{6}{\sqrt{2}}$

$= \int_2^x 6t^{-1/2} dt = \frac{1}{1/2} \cdot 6t^{1/2} \Big|_2^x$

$= 12x^{1/2} - 12 \cdot 2^{1/2}$

$= 12\sqrt{x} - 12\sqrt{2}$

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5. Use the Fundamental Theorem of Calculus to compute the derivative, $F'(x)$, of $F(x)$, if

$$F(x) = \int_1^x t^4 + t^3 + t^2 + 9t + 3 \, dt$$

Possibilities:

(a) $4x^3 + 3x^2 + 2x + 12$

(b) $4x^3 + 3x^2 + 2x + 9$

(c) $x^4 + x^3 + x^2 + 9x$

(d) $x^4 + x^3 + x^2 + 9x + 3$

(e) $\frac{1}{5}x^5 + \frac{1}{4}x^4 + \frac{1}{3}x^3 + \frac{9}{2}x + 3x$

By the FTC,

$F'(x) = \frac{d}{dx} \int_1^x t^4 + t^3 + t^2 + 9t + 3 \, dt$

$= x^4 + x^3 + x^2 + 9x + 3$

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6. Find the value of x at which

$$F(x) = \int_2^x t^4 + t^2 + 7 \, dt$$

takes its minimum value on the interval $[2, 100]$.

Possibilities:

(a) 3

(b) 100

(c) $\frac{346}{15}$

(d) 2

(e) 27

$F'(x) = \frac{d}{dx}\int_2^x t^4 + t^2 + 7 \, dt$

$= x^4 + x^2 + 7 > 0$

The function is increasing over the interval, so it takes its minimum value at the left endpoint.

7. Evaluate the integral

$$\int_0^x (t+3)^9 \, dt$$

$\frac{1}{10}(t+3)^{10}$ has this as its derivative

$= \frac{1}{10}(t+3)^{10}\Big|_0^x$

$= \frac{1}{10}(x+3)^{10} - \frac{1}{10}3^{10}$

$= \frac{1}{10}(x+3)^{10} - \frac{59049}{10}$

Possibilities:

(a) $10(x+3)^{10} - 531441$

(b) $\frac{1}{10}x^{10} - \frac{59049}{10}$

(c) $\frac{1}{10}(x+3)^{10} - \frac{59049}{10}$

(d) $\frac{1}{10}x^{10}$

(e) $\frac{1}{9}(x+3)^9 - \frac{19683}{9}$

8. A train travels along a track and its speed (in miles per hour) is given by $s(t) = 56t$ for the first half hour of travel. Its speed is constant and equal to $s(t) = 28$ after the first half hour. (Here time t is measured in hours.) How far (in miles) does the train travel in the first hour of travel?

Possibilities:

(a) 21 miles

(b) 7 miles

(c) 28 miles

(d) 56 miles

(e) 14 miles

$\int_0^{.5} 56t\,dt + \int_{.5}^1 28\,dt$

$= \frac{1}{2}56t^2\Big|_0^{.5} + 28t\Big|_{.5}^1$

$= 28(.5)^2 - 28(0)^2 + 28(1) - 28(.5)$

$= 7 + 28 - 14 = 21$ miles

9. Evaluate the indefinite integral

$$\int t^2(t+9)\,dt$$

$$= \int t^3 + 9t^2\,dt$$

$$= \frac{1}{4}t^4 + \frac{1}{3}9t^3 + C$$

$$= \frac{1}{4}t^4 + 3t^3 + C$$

Possibilities:

(a) $\frac{1}{3}t^3 + \frac{1}{2}t^2 + C$, for any number C

(b) $\frac{1}{3}t^3 + \frac{9}{2}t^2 + C$, for any number C

(c) $\left(\frac{1}{3}t^3\right)\left(\frac{1}{2}t^2 + 9t\right) + C$, for any number C

(d) $4t^4 + 27t^3 + C$, for any number C

(e) $\frac{1}{4}t^4 + 3t^3 + C$, for any number C

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10. Find the average rate of change of $f(x) = \log(x)$ from $x = 6$ to $x = 8$.

Possibilities:

(a) $\frac{\log(6)+\log(8)}{2}$

(b) $\frac{\log(8)-\log(6)}{\log(6)-\log(8)}$

(c) $\frac{1}{6} - \frac{1}{8}$

(d) $\frac{\log(8)-\log(6)}{8-6}$

(e) $8\log(8) - 8 - 6\log(6) + 6$

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11. Compute $\lim_{t\to 3} \frac{t^2 - 4t + 3}{t^2 + 4t - 21}$

$$= \lim_{t\to 3} \frac{(t-3)(t-1)}{(t-3)(t+7)} = \lim_{t\to 3} \frac{t-1}{t+7} = \frac{3-1}{3+7} = \frac{2}{10} = \frac{1}{5}$$

Possibilities:

(a) $\frac{1}{5}$

(b) $\frac{2}{5}$

(c) $\frac{3}{5}$

(d) $\frac{4}{5}$

(e) The limit does not exist.

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12. Water is evaporating from a pool at a constant rate. The pool is in the shape of a rectangular solid. The length of the pool is 30 feet and the width of the pool is 15 feet. The water in the pool drops 0.2 feet in one day. How fast is the water evaporating in cubic feet per day?

Possibilities:

(a) 88 cubic feet per day

(b) 90 cubic feet per day

(c) 92 cubic feet per day

(d) 94 cubic feet per day

(e) 96 cubic feet per day

$30 \cdot 15 \cdot .2 = 90$

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13. Find an equation for the line with slope 8 passing through the point $(x, y) = (5, 3)$.

Possibilities:

(a) $y = 8x - 5$

(b) $y = \frac{3}{5}x + 8$

(c) $y = 5x + 3$

(d) $y = 8x - 37$

(e) $y = 8x + 3$

$m = 8$

$y - 3 = 8(x - 5)$

$y = 8x - 40 + 3$

$y = 8x - 37$

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14. A train travels from city A to city B, then travels from city B to city C. The train leaves city A at time 1:00pm and arrives at city B at 4:00pm. The train leaves city B at 5:00pm and arrives at city C at 7:00pm. The average velocity of the train, while travelling from A to B, was 40 miles per hour. The average velocity of the train, while travelling from B to C, was 30 miles per hour. What was the average velocity of the train from city A to city C, including the wait at city B?

Possibilities:

(a) 30 miles per hour

(b) 5 miles per hour

(c) 70 miles per hour

(d) 31 miles per hour

(e) 35 miles per hour

Average velocity $= \frac{\text{total distance}}{\text{total time}}$ distance = rate · time

dist(A, B) $= 40 \cdot 3 = 120$ miles

dist(B, C) $= 30 \cdot 2 = 60$ miles

Average velocity $= \frac{120 + 60}{7 - 1} = \frac{180}{6} = 30$ mph

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15. The tangent line to the graph of f at $x = 9$ has equation $y = 3(x - 9) + 4$. Find $f(9)$ and $f'(9)$.

Possibilities:

(a) $f(4) = 9, \quad f'(4) = 3$

(b) $f(9) = 3, \quad f'(9) = 4$

(c) $f(9) = 4, \quad f'(9) = 3$

(d) $f(3) = 4, \quad f'(3) = 9$

(e) $f(4) = 3, \quad f'(4) = 9$

f and the tangent line intersect at $x=9$, so

$f(9) = 3(9-9)+4 = 4$

$f'(x)$ = slope of the tangent line at x

$f'(9) = 3$

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16. The graph of $y = f(x)$ is shown below. The function is differentiable, except at $x =$

Possibilities:

(a) -1

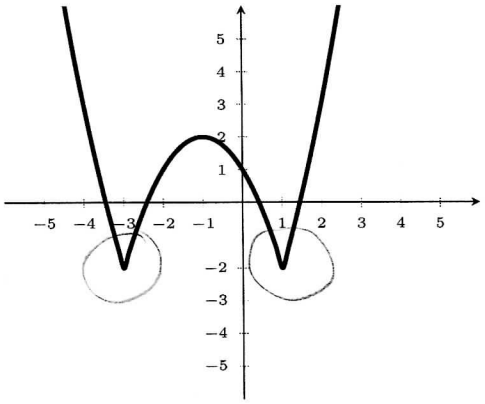
(b) 2

(c) -3 and 1

(d) -2 and 2

(e) -2

At corners, $f'(x)$ DNE



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17. Find the derivative, $f'(x)$, of $f(x) = 6x^5$

Possibilities:

(a) $30x^4$

(b) x^5

(c) $-5x^{(1/6)}$

(d) x^6

(e) $\frac{1}{6}x^6$

$f'(x) = 6 \cdot 5x^4 = 30x^4$

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18. Suppose that $f(x) = \log(g(x))$, but that the formula for $g(x)$ is too complicated to write down. When $x = 5$, the value and derivative of g are measured: $g(5) = 3$, and $g'(5) = 11$. What is $f'(5)$?

Possibilities:

(a) $\frac{11}{5}$
(b) $\frac{1}{5}$
(c) $\frac{3}{11}$
(d) $\frac{5}{11}$
(e) $\frac{11}{3}$

$$f'(x) = \frac{g'(x)}{g(x)}$$

$$f'(5) = \frac{g'(5)}{g(5)} = \frac{11}{3}$$

19. Find the derivative, $f'(x)$, if $f(x) = \sqrt{7x + x^5}$.

Possibilities:

(a) $\frac{1}{2}(7x + x^5)^{(-1/2)}$
(b) $\frac{1}{2}(7 + 5x^4)^{(-1/2)}$
(c) $-\frac{1}{2}(7x + x^5)^{(1/2)}(7 + 4x^5)$
(d) $\frac{1}{2}(7x + x^5)^{(-1/2)}(7 + 5x^4)$
(e) $\frac{1}{2}(7x + x^5)^{(-1/2)}(7 + 5x^4)(5 \cdot 4x^3)$

$$f(x) = (7x + x^5)^{1/2}$$

$$f'(x) = \frac{1}{2}(7x + x^5)^{-1/2}(7 + 5x^4)$$

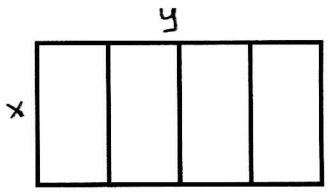
20. Suppose the derivative of $g(t)$ is $g'(t) = -9(t-2)(t-3)(t-7)$. For t in which interval or intervals is g increasing?

Possibilities:

(a) $(4 - \frac{1}{3}\sqrt{21}, 4 + \frac{1}{3}\sqrt{21})$
(b) $(-\infty, 2) \cup (3, 7)$
(c) $(2, 3) \cup (7, \infty)$
(d) $(-9, 2) \cup (3, 7)$
(e) $(-\infty, 4 - \frac{1}{3}\sqrt{21}) \cup (4 + \frac{1}{3}\sqrt{21}, \infty)$

Critical points are at $t = 2$, $t = 3$, $t = 7$

21. A farmer builds a rectangular pen with 4 vertical partitions (5 vertical sides) using 600 feet of fencing. What is the maximum possible total area of the pen?



Possibilities:

(a) 7500

(b) 22500

(c) 600

(d) 300

(e) 9000

$600 = 5x + 2y$

$y = 300 - \frac{5}{2}x$

$A = x(300 - \frac{5}{2}x) = 300x - \frac{5}{2}x^2$

$A' = 300 - 5x = 0$

$x = 60$

$A = 60(300 - \frac{5}{2}(60)) = 60(300 - 150)$

$= 60(150) = 9000$

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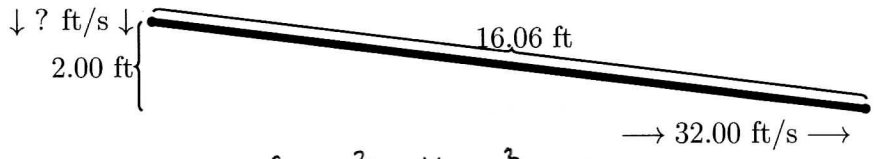
22. A ladder 16.06 feet long rests against a vertical wall. If the bottom of the ladder slides away from the wall at a constant rate of 32.00 feet per second, how fast is the top of the ladder sliding down the wall when the top of the ladder is 2.00 feet above the ground?



Possibilities:

(a) 31.75 feet per second

(b) 255.00 feet per second

(c) 514.00 feet per second

(d) 4.02 feet per second

(e) 0.00 feet per second

$x^2 + y^2 = 16.06^2$

$2x\frac{dx}{dt} + 2y\frac{dy}{dt} = 0$

$x\frac{dx}{dt} + y\frac{dy}{dt} = 0$

$(15.93)(32) + 2\cdot\frac{dy}{dt} = 0$

$\frac{dy}{dt} = \frac{32 \cdot 15.93}{2} \approx 255$ fps

$x^2 = 16.06^2 - y^2$

$x = \sqrt{16.06^2 - y^2}$

when $y = 2$

$x = \sqrt{16.06^2 - 4}$

$= 15.93$

23. Suppose the derivative of $g(t)$ is $g'(t) = 9(t-3)(t-5)$. For t in which interval or intervals is g concave up?

Possibilities:

(a) $(9,3) \cup (4,5)$

(b) $(3,5)$

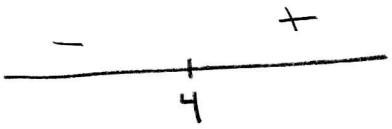
(c) $(4,\infty)$

(d) $(-\infty,3) \cup (5,\infty)$

(e) $(-\infty,4)$

$g'(t) = 9(t^2 - 8t + 15)$

$g''(t) = 9(2t-8) = 0$ when $t = 4$



24. A train travels in a straight line for an hour, with speed measured every 15 minutes. Though the speed varies, we can assume it is a linear function between measurements. Estimate the total distance travelled by the train during this hour.

Possibilities:

(a) about 35.00 miles

(b) about 45.00 miles

(c) about 220.00 miles

(d) about 30.00 miles

(e) about 40.00 miles

time	0.00 hr	0.25 hr	0.50 hr	0.75 hr	1.00 hr
speed	30 mph	40 mph	50 mph	50 mph	50 mph

distance = rate · time

$$= \frac{30+40}{2} \times .25 + \frac{40+50}{2} \times .25 + \frac{50+50}{2} \times .25 + \frac{50+50}{2} \times .25$$

$$= 35 \times .25 + 45 \times .25 + 50 \times .25 + 50 \times .25$$

$$= .25(35+45+50+50) = .25(180) = 45$$

25. Calculate $40 + 45 + 50 + 55 + 60 + 65 + 70 + 75 + 80 + 85 + \ldots + 535$.

Possibilities:

(a) 625

(b) 28750

(c) 820

(d) 21400

(e) 575

$$\sum_{k=8}^{107} 5k = \sum_{k=1}^{107} 5k - \sum_{k=1}^{7} 5k$$

$$= 5\sum_{k=1}^{107} k - 5\sum_{k=1}^{7} k$$

$$= 5\left[\frac{107 \cdot 108}{2}\right] - 5\left[\frac{7 \cdot 8}{2}\right] = 28750$$

Some Formulas

1. Summation formulas:

$$\sum_{k=1}^{n} k = \frac{n(n+1)}{2}$$

$$\sum_{k=1}^{n} k^2 = \frac{n(n+1)(2n+1)}{6}$$

2. Areas:

(a) Triangle $A = \frac{bh}{2}$

(b) Circle $A = \pi r^2$

(c) Rectangle $A = lw$

(d) Trapezoid $A = \frac{h_1 + h_2}{2} b$

3. Volumes:

(a) Rectangular Solid $V = lwh$

(b) Sphere $V = \frac{4}{3}\pi r^3$

(c) Cylinder $V = \pi r^2 h$

(d) Cone $V = \frac{1}{3}\pi r^2 h$

4. Distance:

(a) Distance between (x_1, y_1) and (x_2, y_2)

$$D = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$