MA 123 — Elem. Calculus Spring 2011
EXAM 4 5/2/2011

Name: Solutions **Sec.:** _____

Do not remove this answer page — you will turn in the entire exam. You have two hours to do this exam. No books or notes may be used. You may use a graphing calculator during the exam, but NO calculator with a Computer Algebra System (CAS) or a QWERTY keyboard is permitted. Absolutely no cell phone use during the exam is allowed.

The exam consists of multiple choice questions. Record your answers on this page. For each multiple choice question, you will need to fill in the box corresponding to the correct answer. For example, if (a) is correct, you must write

■ b c d e

Do not circle answers on this page, but please circle the letter of each correct response in the body of the exam. It is your responsibility to make it CLEAR which response has been chosen. You will not get credit unless the correct answer has been marked on both this page and in the body of the exam.

GOOD LUCK!

1. a b ■ d e
2. a b c ■ e
3. a b ■ d e
4. a b c d ■
5. a ■ c d e
6. a b c d ■
7. a b c ■ e
8. ■ b c d e
9. ■ b c d e
10. a b c d ■
11. a ■ c d e
12. ■ b c d e
13. a ■ c d e
14. ■ b c d e
15. ■ b c d e
16. ■ b c d e
17. a ■ c d e
18. ■ b c d e
19. a b c d ■
20. a b c ■ e

For grading use:

Number Correct	**(out of 20 problems)**

Total	**(out of 100 points)**

Please make sure to list the correct section number on the front page of your exam. In case you forgot your section number, consult the following table. If you are enrolled in a lecture with recitation, then your time and location is based on your recitation, not your lecture.

Section #	Instructor	Lectures
001	Koester/Hamilton	T 8:00 - 9:15 am, CP 243
002	Koester/Hamilton	R 8:00 - 9:15 am, CP 243
003	Koester/Hamilton	T 9:30 - 10:45 am, MMRB 243
004	Koester/May	R 9:30 - 10:45 am, CB 342
005	Koester/May	T 11:00 - 12:15 pm, CP 220
006	Koester/May	R 11:30 - 12:15 pm, CP 220
007	Koester/Kyriopoulos	T 9:30 - 10:45 am, CP 367
008	Koester/Kyriopoulos	R 9:30 - 10:45 am, DH 323
009	Koester/Kyriopoulos	T 11:00 - 12:15 pm, FB 307A
010	Koester/Robinson	R 11:00 - 12:15 pm, CP 345
011	Koester/Robinson	T 12:30 - 1:45pm, CB 219
012	Koester/Robinson	R 12:30 - 1:45pm, CB 219
013	Shaw/Taylor	T 11:00 - 12:15 pm, CB 345
014	Shaw/Taylor	R 11:00 - 12:15 pm, MMRB 243
015	Shaw/Taylor	T 12:30 - 1:45 pm, Nurs 201
016	Shaw/Tarr	R 12:30 - 1:45 pm, Nurs 502A
017	Shaw/Tarr	T 2:00 - 3:15 pm, CB 233
018	Shaw/Tarr	R 2:00 - 3:15 pm, CB 245
019	Shaw/Ozbek	T 3:30 - 4:45pm, CP 208
020	Shaw/Ozbek	R 3:30 - 4:45 pm, CP 208
021	Shaw/Ozbek	T 2:00 - 3:15 pm, FB B2
022	Shaw/Zhi	R 2:00 - 3:15 pm, CP 233
023	Shaw/Zhi	T 9:30 - 10:45 am, CP 211
024	Shaw/Zhi	R 9:30 - 10:45 am, CB 341
025	Beth Kelly	MWF 12:00 - 12:50 pm, CP 153
026	John Maki	MWF 2:00 - 2:50 pm, KAS 213

Multiple Choice Questions

Show all your work on the page where the question appears.
Clearly mark your answer both on the cover page on this exam
and in the corresponding questions that follow.

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1. Compute

$$\lim_{n\to\infty} \frac{(3n-5)^2}{n^2+n+2}$$

If the limit tends to $\pm\infty$, select "Limit does not exist".

Possibilities:

(a) 3
(b) 0
(c) 9
(d) 1
(e) Limit does not exist

$$\frac{(3n-5)^2}{n^2+n+2} = \frac{9n^2-30n+25}{n^2+n+2}$$

So $$\lim_{n\to\infty} \frac{9n^2-30n+25}{n^2+n+2} = \lim_{n\to\infty} \frac{9n^2}{n^2}$$

$$= 9$$

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2. The integral

$$\int_4^{10} x^3\,dx$$

is computed as the limit of the sum

$$\sum_{k=1}^{n} \frac{6}{n}\left(A + k\frac{6}{n}\right)^3$$

What value of A must appear in the sum?

Possibilities:

(a) 2
(b) 1
(c) 5
(d) 4
(e) 10

$$\int_4^{10} x^3\,dx = \lim_{n\to\infty} \sum_{k=1}^{n} (a+k\Delta x)^3 \cdot \Delta x$$

$$= \lim_{n\to\infty} \sum_{k=1}^{n} \left((4) + k\cdot\frac{6}{n}\right)^3 \cdot \frac{6}{n}$$

A

---

3. Evaluate the integral

$$\int_0^2 t^3\left(t^2+6\right)\,dt$$

Possibilities:

(a) (176/3)
(b) 120
(c) (104/3)
(d) 80
(e) 152

$= \int_0^2 t^5 + 6t^3\,dt$

$= \frac{1}{6}t^6 + \frac{6}{4}t^4\Big|_0^2$

$= \left(\frac{1}{6}2^6 + \frac{6}{4}\cdot 2^4\right) - \left(\frac{1}{6}0^6 + \frac{6}{4}0^4\right) = \frac{104}{3}$

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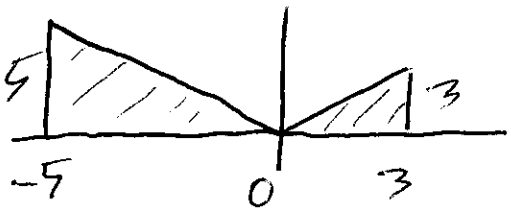
4. Evaluate the integral

$$\int_{-5}^{3} |t|\,dt$$



Possibilities:

(a) 13
(b) 14
(c) 15
(d) 16
(e) 17

= Area of the two triangles

$= \frac{1}{2}\cdot 5\cdot 5 + \frac{1}{2}\cdot 3\cdot 3 = 17$

---

5. Evaluate the integral

$$\int_0^x \frac{1}{(t+3)^2}\,dt$$

Possibilities:

(a) $\frac{1}{x} - \frac{x}{3}$
(b) $\frac{1}{3} - \frac{1}{x+3}$
(c) $\frac{1}{x+3} - \frac{1}{3}$
(d) $\frac{-2}{(x+3)^3}$
(e) $\frac{1}{(x+3)^2} - \frac{1}{3}$

u sub Let $u = t+3$, $du = dt$

$\int_0^x \frac{1}{(t+3)^2}dt = \int_3^{x+3} \frac{1}{u^2}du = \int_3^{x+3} u^{-2}\,du$

$= \frac{1}{-1}\cdot u^{-1}\Big|_3^{x+3} = \left(-\frac{1}{x+3}\right) - \left(-\frac{1}{3}\right)$

$= \frac{1}{3} - \frac{1}{x+3}$

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6. Use the Fundamental Theorem of Calculus to compute the derivative of $F(x)$, if

$$F(x) = \int_4^x \frac{10}{\sqrt{t}}\, dt$$

Your answer should be an expression involving the variable x.

"Derivative of integral is the integrand"

Possibilities:

(a) $F'(x) = 20\sqrt{x} - 40$

(b) $F'(x) = \dfrac{10}{\sqrt{x}} - \dfrac{10}{2}$

(c) $F'(x) = 10x^{-3/2} - \dfrac{10}{2}$

(d) $F'(x) = 5\sqrt{x} - 10$

(e) $F'(x) = \dfrac{10}{\sqrt{x}}$

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7. A rock is thrown down from a cliff with an initial speed of 3 feet per second. The speed of the rock after t seconds is $s(t) = 32t + 3$. If the object lands after 3 seconds, determine the height of the cliff.

speed $= 32t + 3$

Distance travelled from $t=0$ to $t=3$ $= \int_0^3 (32t+3)\,dt$

$= \frac{32}{2}t^2 + 3t \Big|_0^3$

$= (16 \cdot 3^2 + 3 \cdot 3) - (16 \cdot 0^2 + 3 \cdot 0)$

Possibilities:

(a) The cliff is 96 feet high.

(b) The cliff is 135 feet high.

(c) The cliff is 144 feet high.

(d) The cliff is 153 feet high.

(e) The cliff is 3 feet high.

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8. Compute $\lim_{t\to 4} \dfrac{t^2 - 2t - 8}{t^2 - 16}$

$= \lim_{t\to 4} \dfrac{(t-4)(t+2)}{(t-4)(t+4)}$

$= \lim_{t\to 4} \left(\dfrac{t+2}{t+4}\right) = \dfrac{6}{8} = \dfrac{3}{4}$

Possibilities:

(a) 3/4

(b) 1

(c) 5/4

(d) 3/2

(e) The limit does not exist.

---

9. Let $f(x) = 6x^2 - 5x - 5$. Find a value c in the interval $[1, 7]$ so that the average rate of change of $f(x)$ from $x = 1$ to $x = 7$ is equal to the instantaneous rate of change of $f(x)$ at $x = c$.

Inst ROC $= f'(c) = 12c - 5$.

AROC $= \dfrac{f(7) - f(1)}{7 - 1} = 43$

So $12c - 5 = 43$

$12c = 48$

$c = \dfrac{48}{12} = 4$.

Possibilities:

(a) 4

(b) 5

(c) 6

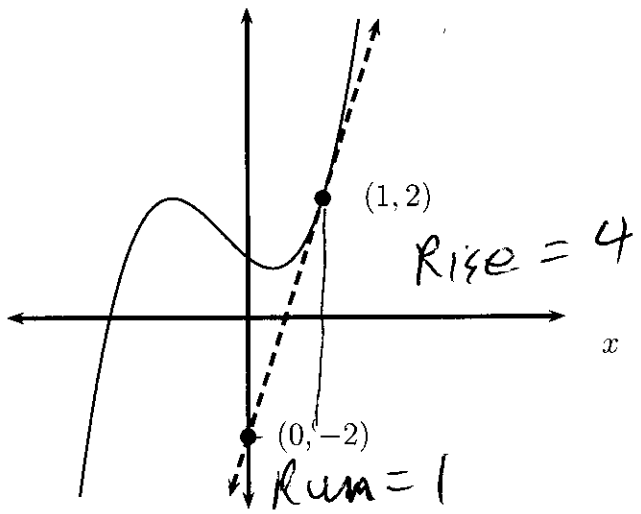
(d) 7

(e) 8

---

10. The graph of $y = g(x)$ is shown, as well as the tangent line to the graph at $x = 1$. Determine $g'(1)$.

$g'(1)$ = Slope tangent Line at $x = 1$. But this Slope is $\dfrac{4}{1} = 4$.



Possibilities:

(a) -2

(b) 1

(c) 2

(d) 1/4

(e) 4

---

11. Determine $f'(4)$ if the equation of the tangent line to $y = f(x)$ at $x = 4$ is

$$y - 10 = 6(x - 4)$$

$f'(4)$ = Slope of tangent Line at $x = 4$.

Possibilities:

(a) -14

(b) 6

(c) 34

(d) 4

(e) 10

---

12. Suppose $g(7) = 25$ and $g'(7) = 16$. Find $F'(7)$, given that

$$F(x) = (g(x))^{3/2}$$

Possibilities:

(a) 120
(b) 6
(c) 75/2
(d) 30
(e) 600

Power-Chain Rule:

$$F'(x) = \frac{3}{2}(g(x))^{1/2} \cdot g'(x)$$

$$F'(7) = \frac{3}{2}(g(7))^{1/2} \cdot g'(7)$$

$$= \frac{3}{2} \cdot (25)^{1/2} \cdot 16 = 120.$$

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13. Find the derivative, $f'(x)$, where

$$f(x) = x\,e^{-5x}$$

Possibilities:

(a) $x\,e^{-5x}$
(b) $e^{-5x} - 5x\,e^{-5x}$
(c) $-5\,e^{-5x-1}$
(d) $-5\,e^{-5x}$
(e) $e^{-5x} - 5x\,e^{-5x-1}$

Product Rule:

$$f'(x) = (x)' \cdot e^{-5x} + x \cdot (e^{-5x})'$$

$$= 1 \cdot e^{-5x} + x \cdot (-5e^{-5x})$$

$$= e^{-5x} - 5x\,e^{-5x}$$

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14. Find the minimum value of $f(x)$ on the interval $[6, 14]$ where $f(x) = \frac{x+8}{x-3}$

Vertical asymptote at $x = 3$, but not in domain $[6, 14]$

Possibilities:

(a) Minimum value is 2
(b) Minimum value is $-11/25$
(c) Minimum value is $-1/11$
(d) Minimum value is 14
(e) Minimum value is 14/3

Min thus occurs at endpoint or critical point

$$f'(x) = \frac{(x-3)-(x+8)}{(x-3)^2} = \frac{-11}{(x-3)^2} \overset{\text{NEVER}}{=} 0,$$

so $f'(x)$ has no critical points on $[6, 14]$.

Check Endpoints:

$f(6) = 14/3$
$f(14) = \frac{22}{11} = 2$

$2 < \frac{14}{3}$ so min value is 2

15. Suppose that the derivative $f'(x) > 0$ for all x in the interval $(7, 10)$. Which statement is definitely true?

Positive Derivative $\Longleftrightarrow$ Increasing Function.

Possibilities:

(a) $f(x)$ is increasing on the interval $(7, 10)$.

(b) $f(x)$ is concave down on the interval $(7, 10)$.

(c) $f(x)$ is decreasing on the interval $(7, 10)$.

(d) The graph of $f(x)$ must be above the x-axis on the interval $(7, 10)$.

(e) $f(x)$ is concave up on the interval $(7, 10)$.

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16. Let $f(x) = x^2 + 72\ln(x)$ for $x > 0$. Find the largest interval on which $f(x)$ is concave up.

Need $f''(x) > 0$.

$f'(x) = 2x + \frac{72}{x} = 2x + 72x^{-1}$

$f''(x) = 2 - 72x^{-2} = 0 \Rightarrow 2x^2 = 72$

$\Rightarrow x^2 = 36$

$\Rightarrow x = 6$ Inf. Point.

	6	
$f''(x)$	− − − − −	+ + + + +

Possibilities:

(a) $(6, \infty)$

(b) $(0, 6)$

(c) $(36, \infty)$

(d) $(0, 36)$

(e) $f(x)$ is never concave up.

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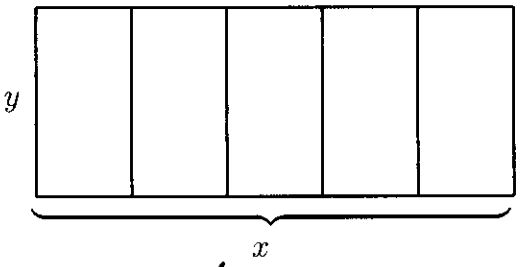
17. A rectangle is to be constructed with 5 vertical partitions (i.e., 6 vertical walls and 2 horizontal walls) as in the figure below. The rectangle is to be constructed with 700 feet of material. Let x denote the length of the horizontal wall and y the length of the vertical wall. Determine the dimensions that will enclose the largest area.

Constraint:

$6y + 2x = 700$

$\Leftrightarrow y = \frac{350}{3} - \frac{x}{3}$



Objective: Maximize $A = xy$

Now, $A = x\cdot\left(\frac{350}{3} - \frac{x}{3}\right)$

$A = \frac{350}{3}x - \frac{x^2}{3}$

$A' = \frac{350}{3} - \frac{2}{3}x = 0$

$\Rightarrow x = 175$

$y = \frac{350}{3} - \frac{175}{3} = \frac{175}{3}$

Possibilities:

(a) $x = 175$ feet and $y = 70$ feet

(b) $x = 175$ feet and $y = 175/3$ feet

(c) $x = 175/2$ feet and $y = 175/2$ feet

(d) $x = 175$ feet and $y = 175$ feet

(e) None. It is possible to enclose an arbitrarily large area.

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18. A stock is increasing at a rate of 6 dollars per share per year. An investor is buying stock at a rate of 14 shares per year. How fast is the value of the investor's stock growing when the price of the stock is 58 dollars per share and the investor owns 50 shares of the stock? (Hint: Write down an expression for the total value, V, of the stock owned by the investor.)

$V = np$ (n = # shares, p = price/share).

$\frac{dV}{dt} = \frac{dn}{dt} \cdot p + n \cdot \frac{dp}{dt}$

$= 14 \cdot 58 + 50 \cdot 6$

Possibilities:

(a) \$1112 per year.

(b) \$2900 per year.

(c) \$300 per year.

(d) \$84 per year.

(e) \$1048 per year.

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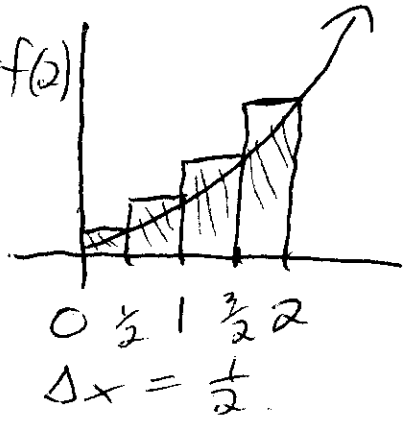
19. Estimate the area under the graph of $f(x) = x^2 + 2x$ for x between 0 and 2. Use a partition that consists of 4 equal subintervals of $[0, 2]$ and use the right endpoint of each subinterval as the sample point.



$\Delta x = \frac{1}{2}$.

Area $= \frac{1}{2} \cdot f(\frac{1}{2}) + \frac{1}{2} f(1) + \frac{1}{2} f(\frac{3}{2}) + \frac{1}{2} f(2)$

$= \frac{1}{2} \cdot \frac{5}{4} + \frac{1}{2} \cdot 3 + \frac{1}{2} \cdot \frac{21}{4} + \frac{1}{2} \cdot 8$

$= \frac{35}{4}$.

Possibilities:

(a) 19/4

(b) 333/50

(c) 35/2

(d) 11

(e) 35/4

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20. Evaluate the sum

$$32 + 36 + 40 + 44 + \ldots + 172 + 176$$

$= 4(8 + 9 + 10 + \ldots + 43 + 44)$ (Factor common 4)

$= 4[(1 + 2 + \ldots + 8 + 9 + \ldots + 44) - (1 + 2 + \ldots + 7)]$

$= 4\left[\frac{44 \cdot 45}{2} - \frac{7 \cdot 8}{2}\right] = 3848$

Possibilities:

(a) 3960

(b) 3816

(c) 3876

(d) 3848

(e) 962

Some Formulas

1. **Summation formulas:**

$$\sum_{k=1}^{n} k = \frac{n(n+1)}{2}$$

$$\sum_{k=1}^{n} k^2 = \frac{n(n+1)(2n+1)}{6}$$

2. **Areas:**

   (a) Triangle $A = \frac{bh}{2}$

   (b) Circle $A = \pi r^2$

   (c) Rectangle $A = lw$

   (d) Trapezoid $A = \frac{b_1 + b_2}{2} h$

3. **Volumes:**

   (a) Rectangular Solid $V = lwh$

   (b) Sphere $V = \frac{4}{3}\pi r^3$

   (c) Cylinder $V = \pi r^2 h$

   (d) Cone $V = \frac{1}{3}\pi r^2 h$

4. **Distance:**

   (a) Distance between (x_1, y_1) and (x_2, y_2)

$$D = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$