MA 123 — Elem. Calculus Spring 2012
EXAM 4 5/3/2012

Name: _____ **Sec.:** _____

Do not remove this answer page — you will turn in the entire exam. You have two hours to do this exam. No books or notes may be used. You may use a graphing calculator during the exam, but NO calculator with a Computer Algebra System (CAS) or a QWERTY keyboard is permitted. Absolutely no cell phone use during the exam is allowed.

The exam consists of multiple choice questions. Record your answers on this page. For each multiple choice question, you will need to fill in the box corresponding to the correct answer. For example, if (b) is correct, you must write

[a] [■] [c] [d] [e]

Do not circle answers on this page, but please circle the letter of each correct response in the body of the exam. It is your responsibility to make it CLEAR which response has been chosen. You will not get credit unless the correct answer has been marked on both this page and in the body of the exam.

GOOD LUCK!

1.	a	b	c	■	e	**11.**	a	■	c	d	e
2.	a	■	c	d	e	**12.**	a	■	c	d	e
3.	a	b	c	■	e	**13.**	a	■	c	d	e
4.	a	■	c	d	e	**14.**	a	b	■	d	e
5.	■	b	c	d	e	**15.**	a	b	c	d	■
6.	■	b	c	d	e	**16.**	a	■	c	d	e
7.	a	b	c	■	e	**17.**	a	■	c	d	e
8.	a	b	■	d	e	**18.**	■	b	c	d	e
9.	a	b	c	■	e	**19.**	a	b	c	■	e
10.	a	■	c	d	e	**20.**	a	b	c	■	e

For grading use:

Number Correct	
	(out of 20 problems)

Total	
	(out of 100 points)

Please make sure to list the correct section number on the front page of your exam. In case you forgot your section number, consult the following table. Your section number is determined by your recitation time and location.

Section #	Instructor	Recitation
001	D. Akers	T 8:00 am - 9:15 am, CB 243
002	D. Akers	R 8:00 am - 9:15 am, CB 243
003	D. Akers	T 12:30 pm - 1:45 pm, TEB 231
004	Q. Liang	R 9:30 am - 10:45 am, NURS 502A
005	Q. Liang	T 11:00 am - 12:15 pm, CB 243
006	Q. Liang	R 11:00 am - 12:15 pm, CB 243
007	D. Corral	T 2:00 pm - 3:15 pm, DH 301
008	D. Corral	R 2:00 pm - 3:15 pm, DH 301
009	D. Corral	T 11:00 am - 12:15 pm, DH 353
010	A. Barra	R 11:00 am - 12:15 pm, DH 353
011	A. Barra	T 12:30 pm - 1:45 pm, MMRB 243
012	A. Barra	R 12:30 pm - 1:45 pm, MMRB 243
013	J. Jung	T 11:00 am - 12:15 pm, TPC 113
014	J. Jung	R 11:00 am - 12:15 pm, TPC 113
015	F. Camacho	T 12:30 pm - 1:45 pm, CB 219
016	J. Jung	R 12:30 pm - 1:45 pm, CB 219
017	F. Camacho	T 2:00 pm - 3:15 pm, FB B8
018	F. Camacho	R 2:00 pm - 3:15 pm, TPC 212
019	S. Hamilton	T 3:30 pm - 4:45 pm, CP 345
020	S. Hamilton	R 3:30 pm - 4:45 pm, BE 301
021	S. Hamilton	T 2:00 pm - 3:15 pm, CB 340
022	J. Constable	R 2:00 pm - 3:15 pm, CB 345
023	J. Constable	T 9:30 am - 10:45 am, L 201
024	J. Constable	R 9:30 am - 10:45 am, L 201
025	M. Shaw	MWF 9:00 am - 9:50 am, CB 110

Multiple Choice Questions

Show all your work on the page where the question appears.
Clearly mark your answer both on the cover page on this exam
and in the corresponding questions that follow.

1. Which of the following is the correct expression for the derivative, $g'(7)$?

Possibilities:

(a) $\dfrac{g(7+h)+g(7)}{h}$

(b) $\dfrac{g(7+h)-g(7)}{h}$

(c) $\lim\limits_{h\to 0}\dfrac{g(7+h)\cdot g(7)}{h}$

(d) $\lim\limits_{h\to 0}\dfrac{g(7+h)-g(7)}{h}$

(e) $\lim\limits_{h\to 0}\dfrac{g(7+h)+g(7)}{h}$

2. Determine the derivative with respect to x of $y = e^{5x^3+4x}$

Possibilities:

(a) e^{15x^2+4}

(b) $(15x^2+4)e^{5x^3+4x}$

(c) $\ln(5x^3+4x)$

(d) $(15x^2+4)e^{5x^3+4x-1}$

(e) e^{5x^3+4x}

$y' = (5x^3+4x)' \cdot e^{5x^3+4x}$

$= (15x^2+4)\, e^{5x^3+4x}$

3. Determine the equation of the line that is tangent to the curve $y = \sqrt{x^2-75}$ at $x = 10$.

Possibilities:

(a) $y = 2(x+10)+5$

(b) $y = 4(x-10)+5$

(c) $y = \dfrac{2x}{\sqrt{x^2-75}}$

(d) $y = 2(x-10)+5$

(e) $y = 4(x-5)+10$

Slope: $y'(10) = \dfrac{2x}{2\sqrt{x^2-75}}\Big|_{x=10} = \dfrac{2\cdot 10}{2\sqrt{10^2-75}}$

$= \dfrac{10}{\sqrt{25}} = 2$

Point: $\sqrt{10^2-75} = \sqrt{25} = 5$

T-Line: $y - 5 = 2(x-10)$

$\Rightarrow y = 2(x-10)+5$

4. A right triangle has base x feet and height $3x$ feet, and the base is increasing at a rate of 3 feet per minute. Determine the rate at which the area of the triangle is increasing, when the base is 6 feet

Possibilities:

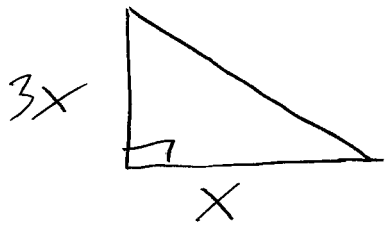
(a) 18 feet per minute.

(b) 54 feet per minute.

(c) 108 feet per minute.

(d) 27 feet per minute.

(e) There is not enough information to solve this problem.



$A = \frac{1}{2}(x)(3x)$

$= \frac{3}{2}x^2$

$\frac{dA}{dt} = \frac{3}{2} \cdot 2x \cdot \frac{dx}{dt}$

$= 3x\frac{dx}{dt}$

So $\frac{dA}{dt} = 3 \cdot 3 \cdot 6 = 54$

5. Determine the indefinite integral

$$\int \left(3x^5 - 7x^2 + \frac{29}{x}\right) dx$$

Possibilities:

(a) $(1/2)x^6 - (7/3)x^3 + 29\ln|x| + C$

(b) $15x^4 - 14x + C$

(c) $(1/2)x^6 - (7/3)x^3 + C$

(d) $(3/5)x^4 - (7/2)x - 29x^{-2} + C$

(e) $(3/5)x^6 - (7/2)x^3 + 29\ln|x| + C$

$3\int x^5 dx - 7\int x^2 dx + 29\int \frac{1}{x} dx$

$= \frac{3}{6}x^6 - \frac{7}{3}x^3 + 29\ln|x| + C$

$= \frac{1}{2}x^6 - \frac{7}{3}x^3 + 29\ln|x| + C$

6. Determine the integral

$$\int_0^x (3t + 18)^3 \, dt$$

Possibilities:

(a) $\frac{1}{12}\left((3x + 18)^4 - 18^4\right)$

(b) x^3

(c) $\frac{1}{4}\left((3x + 18)^4 - 18^4\right)$

(d) $\frac{3}{4}x^4$

(e) $\frac{3}{4}\left((3x + 18)^4 - 18^4\right)$

$u = 3t + 18, \quad du = 3dt$

$\Rightarrow dt = \frac{1}{3}du$

$= \frac{1}{3}\int_{18}^{3x+18} u^3 du = \frac{1}{12}u^4 \Big|_{18}^{3x+18}$

$= \frac{1}{12}(3x+18)^4 - \frac{1}{12} \cdot 18^4$

$= \frac{1}{12}\left[(3x+18)^4 - 18^4\right]$

7. Compute the requested derivative:

$$\frac{d}{dx}\int_{-2}^{x} 7t^6 e^{t^7}\,dt$$

Possibilities:

(a) $42x^6e^{x^7-1}$

(b) $e^{x^7} - e^{-128}$

(c) e^{x^7}

(d) $7x^6e^{x^7}$

(e) $42x^6e^{x^7} + 49x^{12}e^{x^7}$

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8. Determine the definite integral

$$\int_{-2}^{4} |t|\,dt$$

(HINT: Interpret this integral in terms of an area)

Possibilities:

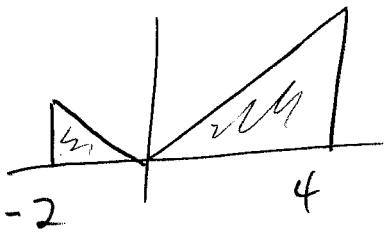
(a) 20

(b) 6

(c) 10

(d) 12

(e) 16



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9. A rock is thrown down from a cliff with an initial speed of 10 feet per second. The speed of the rock after t seconds is $s(t) = 32t + 10$. If the object lands after 3 seconds, determine the height of the cliff.

Possibilities:

(a) The cliff is 114 feet high.

(b) The cliff is 10 feet high.

(c) The cliff is 144 feet high.

(d) The cliff is 174 feet high.

(e) There is not enough information to solve this problem.

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10. Determine the largest interval or collection of intervals on which $f(x) = -x^3 + 30x^2 - 288x + 900$ is concave up.

Need $f''(x) > 0$

$f''(x) = -6x + 60 = -6(x - 10)$

$x = 10$

Possibilities:

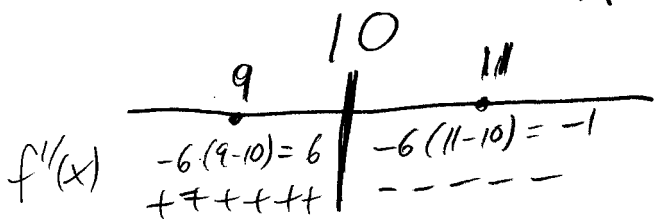
(a) $(10, \infty)$

(b) $(-\infty, 10)$

(c) $(-\infty, 8)$ and $(12, \infty)$

(d) $f(x)$ is never concanve up.

(e) $(8, 12)$



11. The area under the curve $y = 5^x$ from $x = 0$ to $x = 2$ is estimated by the sum of the areas of 4 rectangles of equal width and the heights of the rectangles are determined by the right endpoint. Deteremine the area of the first rectangle.

Possibilities:

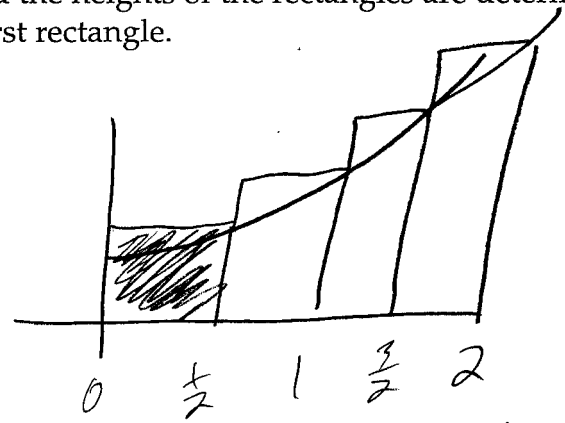
(a) $\sqrt{5}$

(b) $\frac{\sqrt{5}}{2}$

(c) $\frac{5}{2}$

(d) $2\sqrt{5}$

(e) 5



width $= \frac{1}{2}$, Height $= 5^{1/2}$

Area $= \frac{\sqrt{5}}{2}$

12. Determine the exact area under the curve $y = t^2 + 6$ from $t = 0$ to $t = 3$.

Possibilities:

(a) 21

(b) 27

(c) 9

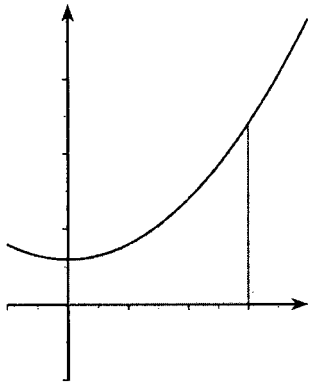
(d) 63/2

(e) 3

Definite integral

$$\int_0^3 t^2 + 6 \, dt$$

$$= \frac{1}{3}t^3 + 6t \Big|_0^3$$



$$= \frac{1}{3} \cdot 27 + 6 \cdot 3 =$$

13. Evaluate the sum

$$15 + 20 + 25 + 30 + \ldots + 750 + 755$$

Possibilities:

(a) 57360
(b) 57365
(c) 57370
(d) 57375
(e) 57380

$= 5[3+4+5+\cdots+151]$

$= 5[1+2+3+\cdots+151-(1+2)]$

$= 5\left[\frac{151\cdot 152}{2} - 3\right] = 57365$

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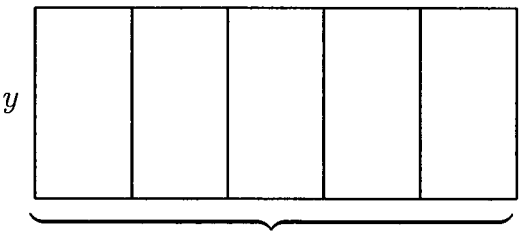
14. A rectangle is to be constructed with 5 vertical partitions (i.e., 6 vertical walls and 2 horizontal walls) as in the figure below. The rectangle is to be constructed with 400 feet of material. Let x denote the length of the horizontal wall and y the length of the vertical wall. Determine the dimensions that will enclose the largest area.

Objective: Maximize Area, xy

Constraint $400 = 2x + 6y$

$200 = x + 3y$



$x = 200 - 3y \Rightarrow A = xy = 200y - 3y^2$

$A' = 200 - 6y = 0$

$\Rightarrow y = \frac{100}{3}$

$x = 200 - \frac{100}{3}\cdot 3$

$= 100$

Possibilities:

(a) $x = 100$ feet and $y = 40$ feet
(b) $x = 50$ feet and $y = 50$ feet
(c) $x = 100$ feet and $y = 100/3$ feet
(d) There is not enough information to solve this problem.
(e) None. It is possible to enclose an arbitrarily large area.

15. Suppose $g'(t) = t^2 + 10t + 9$. Determine the largest interval on which $g(t)$ is decreasing.

Possibilities:

(a) $(5, \infty)$

(b) $g(t)$ is never decreasing.

(c) $(-9, \infty)$

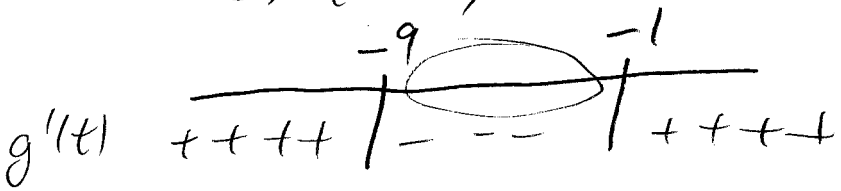
(d) $(-\infty, -1)$

(e) $(-9, -1)$

$g(t) \downarrow$ where $g'(t) > 0$

$g'(t) = (t+9)(t+1) = 0$

$\Rightarrow t = -1, -9$



16. Suppose $f(x) = x^2$, $g(4) = 6$ and $g'(4) = 7$. Determine the instantaneous rate of change of $h(x) = \frac{f(x)}{g(x)}$ at $x = 4$.

Possibilities:

(a) 16/9

(b) $-16/9$

(c) $-8/7$

(d) 4/3

(e) 8/7

$h'(x) = \dfrac{(x^2)' g(x) - x^2 g'(x)}{g(x)^2}$

$h'(x) = \dfrac{2x g(x) - x^2 g'(x)}{g(x)^2}$

$h'(4) = \dfrac{2 \cdot 4 g(4) - 4^2 g'(4)}{g(4)^2} = \dfrac{8 \cdot 6 - 16 \cdot 7}{36}$

$= -\dfrac{64}{36} = -\dfrac{16}{9}$

17. The graph of $y = g(x)$ is shown (solid), as well as the tangent line to the graph (dotted) at $x = 1$. Determine $g'(1)$.

Possibilities:

(a) -1

(b) $-1/2$

(c) $1/3$

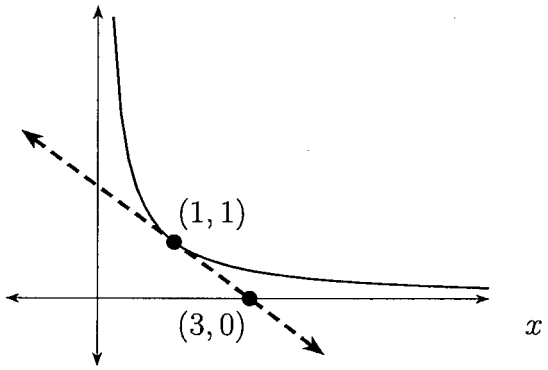
(d) 3

(e) There is not enough information to solve this problem.

$g'(1) =$ Slope tangent line

$= \dfrac{\text{Rise}}{\text{Run}}$

$= \dfrac{1-0}{1-3} = -\dfrac{1}{2}$



18. Suppose $\lim_{x\to 3} g(x) = 12$. Determine the limit

$$\lim_{x\to 3}\left(\frac{x^2-9}{x-3} - \frac{12}{g(x)}\right).$$

$= \lim_{x\to 3}\left[\frac{(x+3)(x-3)}{x-3} - \frac{12}{g(x)}\right]$

$= \lim_{x\to 3}\left[(x+3) - \frac{12}{g(x)}\right] = 3+3-\frac{12}{12}$

$= 5$

Possibilities:

(a) 5
(b) 17
(c) 29
(d) 41
(e) The limit is infinite or the limit does not exist.

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19. Determine the maximum value of $f(x)$ on the interval $[-2, 5]$, where

$$f(x) = \begin{cases} x^2+8, & \text{for } -2 \le x < 3 \\ -4x+29, & \text{for } 3 \le x \le 5 \end{cases}$$

Critical points $\Rightarrow$ $\left(f'(x) = \begin{cases} 2x & -2<x<3 \\ -4 & 3<x<5 \end{cases}\right) = 0$

$\Rightarrow x = 0$, $f'(x)$ DNE $x=3$

Check critical points + end points

$f(-2) = (-2)^2+8 = 12$, $f(3) = -4\cdot 3+29 = 17$

$f(0) = 0^2+8 = 8$, $f(5) = -4\cdot 5+29 = 9$.

Possibilities:

(a) 37
(b) 33
(c) 12
(d) 17
(e) 9

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20. A train leaves station A at 8:00 am and arrives at station B at 11:00 am. The train leaves station B at 11:00 am and arrives at station C at 5:00 pm. The average velocity from station A to station B was 78 miles per hour. The average velocity from station B to station C was 52 miles per hour. Determine the distance between station A and station C.

Distance AC = Dist AB + Dist BC

= Speed · Time + Speed · Time

= 78.3 + 52.6 = 546 miles

mph hr mph hr

Possibilities:

(a) 585 miles
(b) 541 miles
(c) 554 miles
(d) 546 miles
(e) There is not enough information to answer this question.
