MA 123 — Elem. Calculus EXAM 4	Spring 2013 4/30/2013	**Name:** Solutions **Sec.:** ______

Do not remove this answer page — you will turn in the entire exam. You have two hours to do this exam. No books or notes may be used. You may use a graphing calculator during the exam, but NO calculator with a Computer Algebra System (CAS) or a QWERTY keyboard is permitted. Absolutely no cell phone use during the exam is allowed.

The exam consists of multiple choice questions. Record your answers on this page. For each multiple choice question, you will need to fill in the box corresponding to the correct answer. For example, if (b) is correct, you must write

a **b** c d e

Do not circle answers on this page, but please circle the letter of each correct response in the body of the exam. It is your responsibility to make it CLEAR which response has been chosen. You will not get credit unless the correct answer has been marked on both this page and in the body of the exam.

GOOD LUCK!

1.	**a**	b	c	d	e	**11.**	**a**	b	c	d	e
2.	a	b	c	**d**	e	**12.**	**a**	b	c	d	e
3.	a	b	c	**d**	e	**13.**	a	b	**c**	d	e
4.	a	**b**	c	d	e	**14.**	**a**	b	c	d	e
5.	a	**b**	c	d	e	**15.**	a	b	**c**	d	e
6.	a	b	c	**d**	e	**16.**	a	b	c	d	**e**
7.	a	b	**c**	d	e	**17.**	a	**b**	c	d	e
8.	a	b	**c**	d	e	**18.**	a	b	c	d	**e**
9.	**a**	b	c	d	e	**19.**	a	b	c	**d**	e
10.	a	**b**	c	d	e	**20.**	a	b	**c**	d	e

For grading use:

Number Correct	
	(out of 20 problems)

Total	
	(out of 100 points)

Please make sure to list the correct section number on the front page of your exam. In case you forgot your section number, consult the following table. Your section number is determined by your recitation time and location.

Section #	Instructor	Day and Time	Room
001	F. Smith	T, 8:00 am - 9:15 am	CB 213
002	W. Hough	R, 8:00 am - 9:15 am	CB 213
003	D. Akers	T, 12:30 pm - 1:45 pm	CB 342
004	W. Hough	R, 9:30 am - 10:45 am	CP 397
005	D. Akers	T, 11:00 am - 12:15 pm	TPC 212
006	W. Hough	R, 11:00 am - 12:15 pm	TPC 113
007	A. Happ	T, 2:00 pm - 3:15 pm	TPC 109
008	A. Hubbard	R, 2:00 pm - 3:15 pm	L 108
009	A. Happ	T, 11:00 am - 12:15 pm	TPC 113
010	A. Hubbard	R, 11:00 am - 12:15 pm	CB 340
011	A. Happ	T, 12:30 pm - 1:45 pm	TEB 231
012	A. Hubbard	R, 12:30 pm - 1:45 pm	EH 307
013	L. Solus	T, 11:00 am - 12:15 pm	CB 340
014	D. Akers	R, 11:00 am - 12:15 pm	TPC 101
015	L. Solus	T, 12:30 pm - 1:45 pm	OT 0B7
016	F. Smith	R, 12:30 pm - 1:45 pm	FB B4
017	L. Solus	T, 2:00 pm - 3:15 pm	FB B4
018	F. Smith	R, 2:00 pm - 3:15 pm	CB 245
019	X. Kong	T, 3:30 pm - 4:45 pm	BH 303
020	Q. Liang	R, 3:30 pm - 4:45 pm	EGJ 115
021	X. Kong	T, 12:30 pm - 1:45 pm	CB 205
022	X. Kong	R, 2:00 pm - 3:15 pm	CB 233
023	L. Davidson	T, 9:30 am - 10:45 am	OT 0B7
024	L. Davidson	R, 9:30 am - 10:45 am	OT 0B7
026	L. Davidson	R, 8:00 am - 9:15 am	CB 243
027	Q. Liang	T, 9:30 am - 10:45 am	DH 131

You may use the following formula for the derivative of a quadratic function.

$$\text{If} \quad p(x) = Ax^2 + Bx + C, \quad \text{then} \quad p'(x) = 2Ax + B.$$

Multiple Choice Questions

Show all your work on the page where the question appears.
Clearly mark your answer both on the cover page on this exam
and in the corresponding questions that follow.

1. Find the limit as n tends to infinity.

$$\lim_{n\to\infty} \frac{(9n+1)^2}{3n^2+2n+1}$$

$$= \lim_{n\to\infty} \frac{9^2n^2 + 2\cdot 9\cdot n + 1}{3n^2 + 2n + 1}$$

$$= \lim_{n\to\infty} \frac{81n^2 + 18n + 1}{3n^2 + 2n + 1} = \frac{81}{3} = 27$$

Possibilities:

(a) 27
(b) Limit does not exist or is infinite.
(c) 0
(d) 3
(e) 6

2. Evaluate the definite integral

$$\int_5^x \frac{4}{\sqrt{t}}\,dt. = 4\int_5^x t^{-1/2}\,dt$$

$$= 4\frac{1}{(-\frac{1}{2}+1)} t^{-\frac{1}{2}+1}\Big|_5^x$$

$$= 8\sqrt{t}\Big|_5^x = 8\sqrt{x} - 8\sqrt{5}$$

Possibilities:

(a) $\frac{4}{\sqrt{x}} - \frac{4}{\sqrt{5}}$
(b) $4\sqrt{x} - 4\sqrt{5}$
(c) $2\sqrt{x} - 2\sqrt{5}$
(d) $8\sqrt{x} - 8\sqrt{5}$
(e) $4\sqrt{x}$

3. Find the value of x at which

$$F(x) = \int_3^x t^4 + t^2 + 4\,dt$$

takes its minimum value on the interval $[3, 100]$.

Possibilities:

(a) $x = 94$
(b) $x = 348/5$
(c) $x = 100$
(d) $x = 3$
(e) $x = 2$

Find where $F'(x) = 0$,
FTC $\Rightarrow F'(x) = x^4 + x^2 + 4$
$x^4 + x^2 + 4 = 0 \Rightarrow$ Never (Sum of Positives can't be 0)
Min @ endpoint, $x = 3$ OR $x = 100$
But $F'(x) > 0 \Rightarrow F(x) \nearrow$, so min @ $x = 3$

4. The integral

$a=3, b=7, \Delta x = \frac{7-3}{n} = \frac{4}{n}$

$$\int_3^7 x^2\,dx = \lim_{n\to\infty} \sum_{k=1}^{n} (a+k\Delta x)^2 \Delta x$$

is computed as the limit of the sum

$$\sum_{k=1}^{N} \frac{4}{n}\left(A + \frac{4k}{n}\right)^2.$$

$$= \lim_{n\to\infty} \sum_{k=1}^{n} \left(3 + k\cdot\frac{4}{n}\right)^2 \cdot \frac{4}{n}$$

What value should be used for A?

Possibilities:

~~(a) 2~~
(b) 3
(c) 4
(d) 5
(e) 6

5. Evaluate the limit as n tends to infinity. Note: you will have to use some of the summation formulas (see formula sheet on back page) to simplify.

$$\lim_{n\to\infty} \frac{1}{n} \sum_{k=1}^{n} \frac{8k}{n}. = \lim_{n\to\infty} \frac{8}{n^2} \sum_{k=1}^{n} k$$

$$= \lim_{n\to\infty} \frac{\overset{4}{8}}{n^2} \cdot \frac{n(n+1)}{2} = \lim_{n\to\infty} \frac{4n^2+4n}{n^2}$$

$$= 4$$

Possibilities:

~~(a) 8~~
(b) 4
~~(c) 9~~
(d) 0
(e) The limit does not exist or the limit tends to infinity.

6. A car travels due east. Its velocity (in miles per hour) at time t is $v(t) = -3t^2 + 36t$. How far does the car travel during the first 3 hours of the trip?

Distance $= \int_0^3 -3t^2 + 36t \, dt$

$$= -3\cdot\frac{1}{3}t^3 + \frac{36}{2}t^2 \Big|_0^3$$

$$= -t^3 + 18t^2 \Big|_0^3 = (-3^3 + 18\cdot 3^2) - (-0^3 + 18\cdot 0^2)$$

$$= -27 + 162 = 135 \text{ miles}$$

Possibilities:

(a) 105 miles
(b) 115 miles
(c) 125 miles
(d) 135 miles
(e) 145 miles

7. Use the Fundamental Theorem of Calculus to compute the derivative of $F(x)$, where

$$F(x) = \int_1^x (t^4 + t^3 + t^2 + 7t + 6)\, dt.$$

$F'(x) = x^4 + x^3 + x^2 + 7x + 6$

Possibilities:

(a) $4x^3 + 3x^2 + 2x + 13$

(b) $4x^3 + 3x^2 + 2x + 7$

(c) $x^4 + x^3 + x^2 + 7x + 6$

(d) $\frac{1}{5}x^5 + \frac{1}{4}x^4 + \frac{1}{3}x^3 + \frac{7}{2}x + 6x$

(e) $x^4 + x^3 + x^2 + 7x$

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8. Evaluate the integral

$$\int_0^x (t+5)^2\, dt. = \int_{0+5}^{x+5} u^2\, du$$

$u = t+5$

$\frac{du}{dt} = 1 \Rightarrow dt = du$

$= \frac{1}{3}u^3 \Big|_5^{x+5}$

$= \frac{1}{3}(x+5)^3 - \frac{1}{3}5^3$

Possibilities:

(a) $\frac{1}{3}x^3 - \frac{125}{3}$

(b) $\frac{1}{2}(x+5)^2 - \frac{25}{2}$

(c) $\frac{1}{3}(x+5)^3 - \frac{125}{3}$

(d) $3(x+5)^3 - 250$

(e) $\frac{1}{3}x^3$

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9. A train travels along a track and its speed (in miles per hour) is given by $s(t) = 64t$ for the first half hour of travel. Its speed is constant and equal to $s(t) = 32$ after the first half hour. (Here time t is measured in hours.) How far (in miles) does the train travel in the first hour of travel?

Possibilities:

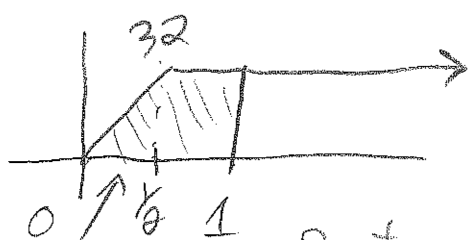
(a) 24 miles.

(b) 16 miles.

(c) 64 miles.

(d) 32 miles.

(e) 8 miles.



$= 8 + 16 = 24$

Triangle Rect

$\frac{1}{2} \cdot \frac{1}{2} \cdot 32 + \frac{1}{2} \cdot 32$

$\frac{1}{2}$ Base Height Base Height

10. Evaluate the indefinite integral

$$\int t^4(t+15)\,dt.$$

$= \int t^5 + 15t^4\,dt$

$= \frac{1}{6}t^6 + \frac{15}{5}t^5 + C$

$= \frac{1}{6}t^6 + 3t^5 + C$

Possibilities:

(a) $6t^6 + 75t^5 + C$

(b) $\frac{1}{6}t^6 + 3t^5 + C$

(c) $\frac{1}{5}t^5 + \frac{1}{2}t^2 + C$

(d) $\left(\frac{1}{5}t^5\right)\left(\frac{1}{2}t^2 + 15t\right) + C$

(e) $\frac{1}{5}t^5 + \frac{15}{4}t^4 + C$

11. Compute $\lim_{t\to 4} \frac{t^2 - t - 12}{t^2 - 16}$.

$= \lim_{t\to 4} \frac{(t-4)(t+3)}{(t-4)(t+4)}$

$= \lim_{t\to 4} \frac{t+3}{t+4} = \frac{7}{8}$

Possibilities:

(a) 7/8

(b) 1

(c) 9/8

(d) 5/4

(e) The limit does not exist.

12. Which of the following is the correct expression for the derivative $g'(8)$?

Possibilities:

(a) $\lim_{h\to 0} \frac{g(8+h) - g(8)}{h}$

(b) $\lim_{h\to 0} \frac{g(8-h) - g(8)}{h}$

(c) $\frac{g(8+h) - g(8)}{h}$

(d) $\frac{g(8) - g(8+h)}{h}$

(e) $\lim_{h\to 0} \frac{g(8) - g(8+h)}{h}$

13. Determine the equation of the tangent line to $f(x) = 4x^2 - 2x + 13$ at $x = 2$.

Point $f(2) = 4\cdot 2^2 - 2\cdot 2 + 13 = 25$

Slope $f'(2) = 8 \cdot 2 - 2 = 14$

Line $y - 25 = 14(x-2)$

Possibilities:

(a) $y = 25(x+2) - 14$
(b) $y = 14(x+2) - 25$
(c) $y = 14(x-2) + 25$
(d) $y = 25(x-2) - 14$
(e) $y = 14(x+2) + 25$

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14. Find the second derivative, $f''(x)$, where

$$f(x) = e^{x^2}.$$

$f'(x) = (x^2)' e^{x^2} = 2x e^{x^2}$

$f''(x) = (2x)' e^{x^2} + 2x (e^{x^2})'$

$= 2e^{x^2} + 2x \cdot 2x e^{x^2}$

$= 2e^{x^2} + 4x^2 e^{x^2}$

Possibilities:

(a) $2e^{x^2} + 4x^2 e^{x^2}$
(b) $2xe^{x^2}$
(c) $4x^2 e^{x^2}$
(d) $2xe^{x^2} + 4x^2 e^{x^2}$
(e) $4xe^{x^2}$

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15. Suppose $f'(x) = x^3 + 8x^2 + 5x + 10$. Find the largest interval or intervals on which $f(x)$ is concave up.

Need $f''(x) > 0$.

$f''(x) = 3x^2 + 16x + 5$

$= (3x+1)(x+5) = 0$

$\Rightarrow 3x + 1 = 0$ or $x + 5 = 0$

$x = -1/3$, $x = -5$

Possibilities:

(a) $(-\infty, -8/3)$
(b) $(-5, -1/3)$
(c) $(-\infty, -5)$ and $(-1/3, \infty)$
(d) $(-8/3, \infty)$
(e) $(-1/3, \infty)$

	-6	-5	-1	$-1/3$	0
$f''(x)$	$(3(-6)+1)(-6+5)$ + + + + + +		$(3(-1)+1)(-1+5)$ − − − − − −		$(3\cdot 0+1)(0+5)$ + + + + + +
$f(x)$	C.U		C.D		C.up

-5 $\quad$ $-1/3$

16. Water is evaporating from a pool at a constant rate. The pool is in the shape of a rectangular solid. The length of the pool is 30 feet and the width of the pool is 20 feet. The depth of the water in the pool decreases by 0.6 feet each day. How fast is the water evaporating, in cubic feet per day, when the depth of the water is 2 feet?

$V = 30 \cdot 20 \cdot h = 600h$

Possibilities:

Know $\frac{dh}{dt} = -0.6$

Want $\frac{dV}{dt}$.

But $\frac{dV}{dt} = 600 \cdot \frac{dh}{dt} = 600(-0.6) = -360$

(a) 352 cubic feet per day
(b) 354 cubic feet per day
(c) 356 cubic feet per day
(d) 358 cubic feet per day
(e) 360 cubic feet per day

17. Two positive real numbers, x and y, satisfy $7x + y = 14$. Determine the maximum value of the product xy.

$y = 14 - 7x$

Possibilities:

$xy = 14x - 7x^2$

$(14x - 7x^2)' = 14 - 14x = 0 \Rightarrow x = 1$

$14 \cdot 1 - 7 \cdot 1^2 = 7$

(a) 13/2
(b) 7
(c) 15/2
(d) 8
(e) 17/2

18. Find the derivative, $f'(x)$, where

$$f(x) = \sqrt{x^2 + \ln(x)}.$$

Possibilities:

$$f'(x) = \frac{(x^2 + \ln(x))'}{2\sqrt{x^2 + \ln(x)}} = \frac{2x + \frac{1}{x}}{2\sqrt{x^2 + \ln(x)}}$$

(a) $\dfrac{2x + 1/x}{\sqrt{x^2 + \ln(x)}}$

(b) $\dfrac{x + e^x}{\sqrt{x^2 + \ln(x)}}$

(c) $\dfrac{1}{2\sqrt{x^2 + \ln(x)}}$

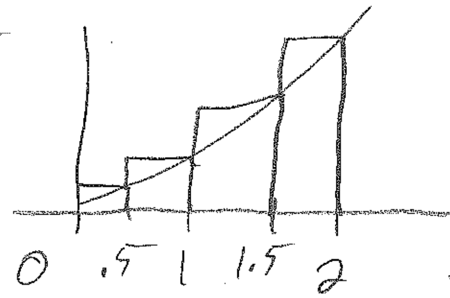
(d) $\dfrac{2x + e^x}{2\sqrt{x^2 + \ln(x)}}$

(e) $\dfrac{2x + 1/x}{2\sqrt{x^2 + \ln(x)}}$

19. Estimate the area under the graph of $f(x) = x^2 + 2x$ for x between 0 and 2. Use a partition that consists of 4 equal subintervals of $[0, 2]$ and use the right endpoint of each subinterval as the sample point.

widths = .5

$x_1 = .5,\ x_2 = 1,\ x_3 = 1.5,\ x_4 = 2$



Heights:

$f(.5) = .5^2 + 2 \cdot .5 = 1.25$

$f(1) = 3$

$f(1.5) = 1.5^2 + 2 \cdot 1.5 = 5.25$

$f(2) = 2^2 + 2 \cdot 2 = 8$

Possibilities:

(a) 3

(b) 333/50

(c) 19/4

(d) 35/4

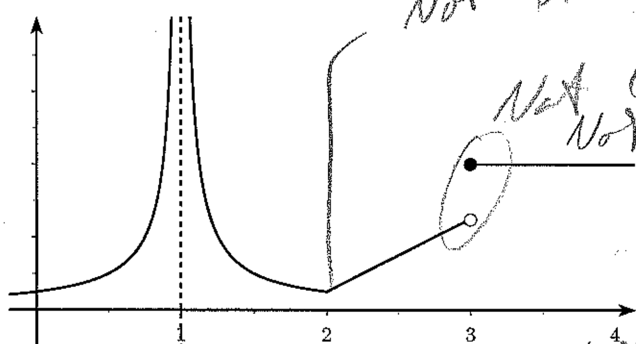
(e) 35/2

Area = $.5(1.25 + 3 + 5.25 + 8) = .5 \cdot 17.5 = \frac{35}{4}$

20. The graph of $y = f(x)$ is shown.

Continuous, Not Diff

Not cont, Not Diff



Not cont, Not, Diff

Which of the following statements are true?

✓ (I) $f(x)$ is neither continuous nor differentiable at $x = 1$.

~~(II)~~ $f(x)$ is differentiable but not continuous at $x = 2$.

✓ (III) $f(x)$ is neither continuous nor differentiable at $x = 3$.

Possibilities:

(a) Only (III) is true

(b) ~~Only (II) is true~~

(c) (I) and (III) are true

(d) Only (I) is true

(e) (I) and (II) are true

Some Formulas

1. Summation formulas:

$$\sum_{k=1}^{n} k = \frac{n(n+1)}{2}$$

$$\sum_{k=1}^{n} k^2 = \frac{n(n+1)(2n+1)}{6}$$

2. Areas:

(a) Triangle $A = \dfrac{bh}{2}$

(b) Circle $A = \pi r^2$

(c) Rectangle $A = lw$

(d) Trapezoid $A = \dfrac{h_1 + h_2}{2} b$

3. Volumes:

(a) Rectangular Solid $V = lwh$

(b) Sphere $V = \dfrac{4}{3}\pi r^3$

(c) Cylinder $V = \pi r^2 h$

(d) Cone $V = \dfrac{1}{3}\pi r^2 h$

4. Distance:

(a) Distance between (x_1, y_1) and (x_2, y_2)

$$D = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$