MA123 — Elem. Calculus	Spring 2014	Name: KEY	Sec.:
Exam 4	2014-05-07		

Do not remove this answer page — you will turn in the entire exam. You have two hours to do this exam. No books or notes may be used. You may use a graphing calculator during the exam, but NO calculator with a Computer Algebra System (CAS) or a QWERTY keyboard is permitted. Absolutely no cell phone use during the exam is allowed.

The exam consists of multiple choice questions. Record your answers on this page. For each multiple choice question, you will need to fill in the box corresponding to the correct answer. For example, if (a) is correct, you must write

(a) (b) (c) (d) (e)

Do not circle answers on this page, but please circle the letter of each correct response in the body of the exam. It is your responsibility to make it CLEAR which response has been chosen. You will not get credit unless the correct answer has been marked on both this page and in the body of the exam.

GOOD LUCK!

1.	(a)	**(b)**	(c)	(d)	(e)	12.	(a)	(b)	(c)	**(d)**	(e)
2.	**(a)**	(b)	(c)	(d)	(e)	13.	(a)	(b)	**(c)**	(d)	(e)
3.	(a)	(b)	**(c)**	(d)	(e)	14.	(a)	(b)	(c)	**(d)**	(e)
4.	(a)	(b)	(c)	**(d)**	(e)	15.	(a)	(b)	(c)	**(d)**	(e)
5.	(a)	**(b)**	(c)	(d)	(e)	16.	**(a)**	(b)	(c)	(d)	(e)
6.	(a)	**(b)**	**(c)**	(d)	(e)	17.	(a)	(b)	**(c)**	(d)	(e)
7.	(a)	(b)	(c)	(d)	**(e)**	18.	(a)	(b)	(c)	(d)	**(e)**
8.	(a)	(b)	**(c)**	(d)	(e)	19.	(a)	(b)	(c)	(d)	**(e)**
9.	(a)	(b)	**(c)**	(d)	(e)	20.	(a)	(b)	(c)	(d)	**(e)**
10.	(a)	**(b)**	(c)	(d)	(e)	21.	(a)	(b)	(c)	(d)	(e)
11.	**(a)**	(b)	(c)	(d)	(e)	22.	(a)	(b)	(c)	**(d)**	(e)

For grading use:

Number Correct	
	(out of 20 problems)

Total	
	(out of 100 points)

Please make sure to list the correct section number on the front page of your exam.
In case you forgot your section number, consult the following table.

Section	Instructor	Day	Time	Room
	Jack Schmidt	MWF	10:00 am	CB 106
001	Wenwen Du	Tu	8:00 am	CB 349
002	Wenwen Du	Th	8:00 am	CB 349
003	Jinping Zhuge	Tu	12:30 pm	CP 201
004	Wenwen Du	Th	9:30 am	CP 211
005	Jinping Zhuge	Tu	11:00 am	TPC 113
006	Jinping Zhuge	Th	11:00 am	CP 103
	Jack Schmidt	MWF	12:00 pm	CB 118
007	Stephen Sturgeon	Tu	2:00 pm	FB 313
008	John Mosley	Th	2:00 pm	FB 313
009	Stephen Sturgeon	Tu	11:00 am	CB 335
010	John Mosley	Th	11:00 am	CB 335
011	Stephen Sturgeon	Tu	12:30 pm	CP 111
012	John Mosley	Th	12:30 pm	CB 233
013	Sarah Orchard	Tu	11:00 am	CP 111
014	Sarah Orchard	Th	11:00 am	CB 334
015	Sarah Orchard	Tu	12:30 pm	CP 103
	Nicholas Nguyen	MWF	2:00 pm	KAS 213
016	Jiaqi Liu	Th	12:30 pm	CB 201
017	Jiaqi Liu	Tu	2:00 pm	CP 345
018	Jiaqi Liu	Th	2:00 pm	CP 345
019	Hao Wang	Tu	3:30 pm	FB B9
020	Hao Wang	Th	3:30 pm	CP 297
021	Fernando Camacho	Tu	12:30 pm	TPC 212
	Drew Butcher	MWF	3:00 pm	BS 107
022	Hao Wang	Th	2:00 pm	BS 109
023	Fernando Camacho	Tu	9:30 am	CB 349
024	Fernando Camacho	Th	9:30 am	CB 349
025	Isaiah Harney	Tu	3:30 pm	CB 345
026	Isaiah Harney	Th	3:30 pm	CB 345
027	Luis Sordo Vieira	Tu	12:30 pm	CP 220
028	Isaiah Harney	Th	2:00 pm	TPC 212

Multiple Choice Questions

Show all your work on the page where the question appears.
Clearly mark your answer both on the cover page on this exam
and in the corresponding questions that follow.

1. Find the limit as n tends to infinity. Here C is a fixed real number.

$$\lim_{n\to\infty} \frac{(7n+1)^2}{9n^2+4n+C}$$

Possibilities:

(a) 0

(b) $\frac{49}{9}$

(c) $\frac{7}{13+C}$

(d) $\frac{7}{9}+C$

(e) ∞

$$\frac{(7n+1)^2}{9n^2+4n+C} = \frac{49n^2+14n+1}{9n^2+4n+C} \to \frac{49}{9} \text{ as } n\to\infty$$

2. Evaluate the limit as n tends to infinity. Note: you will have to use some of the summation formulas (see formula sheet on backpage) to simplify.

$$\lim_{n\to\infty} \frac{1}{n}\sum_{k=1}^{n} \frac{6k}{n} = \lim_{n\to\infty} \frac{6}{n^2} \sum_{k=1}^{n} k = \lim_{n\to\infty} \frac{6}{n^2}\left(\frac{n(n+1)}{2}\right)$$

$$= \lim_{n\to\infty} \frac{3n^2+3n}{n^2} = 3$$

Possibilities:

(a) 3

(b) 2

(c) 1

(d) 4

(e) 5

3. The integral

$$\int_3^8 x^2 \, dx$$

is computed as the limit of the sum

$$\sum_{k=1}^{n} \frac{5}{n}\left(A + \frac{5k}{n}\right)^2$$

What value should be used for A?

$$\int_a^b f(x)\,dx = \lim_{n\to\infty} \sum_{k=1}^{n} f(a+k\Delta x)\,\Delta x$$

$a=3$

Possibilities:

(a) 2
(b) 4
(c) 3
(d) 5
(e) 8

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4. Evaluate the definite integral

$$\int_2^x \frac{6}{\sqrt{t}} \, dt$$

$$\int_2^x \frac{6}{\sqrt{t}}\,dt = 6\int_2^x t^{-1/2}\,dt = 6\left[2t^{1/2}\right]_2^x$$
$$= 6\left[2x^{1/2} - 2\sqrt{2}\right]$$
$$= 12x^{1/2} - 12\sqrt{2}$$

Possibilities:

(a) $6\sqrt{x} - 6\sqrt{2}$
(b) $3\sqrt{x} - 3\sqrt{2}$
(c) $6\sqrt{x}$
(d) $12\sqrt{x} - 12\sqrt{2}$
(e) $\frac{6}{\sqrt{x}} - \frac{6}{\sqrt{2}}$

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5. Use the Fundamental Theorem of Calculus to compute the derivative, $F'(x)$, of $F(x)$, if

$$F(x) = \int_1^x t^4 + t^3 + t^2 + 9t + 3 \, dt$$

If $F(x) = \int_1^x f(t)\,dt$, then $F'(x) = f(x)$,

so

$F'(x) = x^4 + x^3 + x^2 + 9x + 3$

Possibilities:

(a) $4x^3 + 3x^2 + 2x + 12$
(b) $x^4 + x^3 + x^2 + 9x + 3$
(c) $x^4 + x^3 + x^2 + 9x$
(d) $4x^3 + 3x^2 + 2x + 9$
(e) $\frac{1}{5}x^5 + \frac{1}{4}x^4 + \frac{1}{3}x^3 + \frac{9}{2}x + 3x$

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6. Find the value of x at which

$$F(x) = \int_2^x t^4 + t^2 + 7\ dt$$

takes its minimum value on the interval $[2, 100]$.

Possibilities:

(a) 3

(b) 2

(c) 100

(d) $\frac{346}{15}$

(e) 27

$F'(x) = x^4 + x^2 + 7 \geq 0$ for $x \in [2, 100]$

function is increasing, so max is at 100

and min is at x=2

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7. Evaluate the integral

$$\int_0^x (t+3)^9\ dt = \int_3^{x+3} u^9\ du = \left[\frac{u^{10}}{10}\right]_3^{x+3} = \frac{1}{10}(x+3)^{10} - \frac{3^{10}}{10}$$

$u = t+3 \quad t = x \quad u = x+3$

$du = dt \quad t = 0 \quad u = 3$

Possibilities:

(a) $\frac{1}{9}(x+3)^9 - \frac{3^9}{9}$

(b) $10(x+3)^{10} - 9 \cdot 3^{10}$

(c) $\frac{1}{10}x^{10} - \frac{3^{10}}{10}$

(d) $\frac{1}{10}x^{10}$

(e) $\frac{1}{10}(x+3)^{10} - \frac{3^{10}}{10}$

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8. A train travels along a track and its speed (in miles per hour) is given by $s(t) = 56t$ for the first half hour of travel. Its speed is constant and equal to $s(t) = 28$ after the first half hour. (Here time t is measured in hours.) How far (in miles) does the train travel in the first hour of travel?

Possibilities:

(a) 28 miles

(b) 7 miles

(c) 21 miles

(d) 56 miles

(e) 14 miles

$d = \int_0^1 s(t)\,dt = \int_0^{1/2} 56t\ dt + \int_{1/2}^1 28\ dt$

$= [28t^2]_0^{1/2} + [28t]_{1/2}^1$

$= 7 + 14 = 21$

9. Evaluate the indefinite integral

$$\int t^2(t+9)\,dt = \int t^3+9t^2\,dt$$
$$= \frac{1}{4}t^4 + 3t^3 + C$$

Possibilities:

(a) $\frac{1}{3}t^3 + \frac{1}{2}t^2 + C$, for any number C

(b) $\frac{1}{3}t^3 + \dfrac{9}{2}t^2 + C$, for any number C

(c) $\frac{1}{4}t^4 + 3t^3 + C$, for any number C

(d) $4t^4 + 27t^3 + C$, for any number C

(e) $\left(\frac{1}{3}t^3\right)\left(\frac{1}{2}t^2 + 9t\right) + C$, for any number C

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10. Find the average rate of change of $f(x) = \sqrt{x}$ from $x = 36$ to $x = 64$.

$$r = \frac{f(b)-f(a)}{b-a} = \frac{\sqrt{64}-\sqrt{36}}{64-36} = \frac{2}{28} = \frac{1}{14}$$

Possibilities:

(a) $\dfrac{\log(36)+\log(64)}{2}$

(b) $\dfrac{\sqrt{64}-\sqrt{36}}{64-36}$

(c) $\dfrac{1}{36} - \dfrac{1}{64}$

(d) $\dfrac{\sqrt{64}-\sqrt{36}}{\sqrt{36}-\sqrt{64}}$

(e) $\dfrac{1}{2}(64)^{-1/2} - \dfrac{1}{2}(36)^{-1/2}$

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11. Compute $\lim_{t\to 3} \dfrac{t^2-4t+3}{t^2+4t-21}$ $= \lim_{t\to 3} \dfrac{(t-3)(t-1)}{(t-3)(t+7)} = \lim_{t\to 3} \dfrac{t-1}{t+7} = \dfrac{1}{5}$

Possibilities:

(a) $\frac{1}{5}$

(b) $\frac{2}{5}$

(c) $\frac{3}{5}$

(d) $\frac{4}{5}$

(e) The limit does not exist.

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12. Water is evaporating from a pool at a constant rate. The pool is in the shape of a rectangular solid. The length of the pool is 30 feet and the width of the pool is 15 feet. The water in the pool drops 0.2 feet in one day. How fast is the water evaporating in cubic feet per day?

Possibilities:

(a) 88 cubic feet per day

(b) 94 cubic feet per day

(c) 92 cubic feet per day

(d) 90 cubic feet per day

(e) 96 cubic feet per day

$V(t) = (30)(15)\,x(t)$

$V'(t) = (30)(15)\,x'(t)$

$= (30)(15)(.2) = 90$

13. Find an equation for the line with slope 8 passing through the point $(x, y) = (5, 3)$.

Possibilities:

(a) $y = 8x - 5$

(b) $y = \frac{3}{5}x + 8$

(c) $y = 8x - 37$

(d) $y = 5x + 3$

(e) $y = 8x + 3$

$m = 8 \quad pt = (5,3)$

$y - y_0 = m(x - x_0)$

$y - 3 = 8(x - 5)$

$y = 8x - 37$

14. A train travels from city A to city B, then travels from city B to city C. The train leaves city A at time 1:00pm and arrives at city B at 4:00pm. The train leaves city B at 5:00pm and arrives at city C at 7:00pm. The average velocity of the train, while travelling from A to B, was 40 miles per hour. The average velocity of the train, while travelling from B to C, was 30 miles per hour. What was the average velocity of the train from city A to city C, including the wait at city B?

Possibilities:

(a) 31 miles per hour

(b) 5 miles per hour

(c) 70 miles per hour

(d) 30 miles per hour

(e) 35 miles per hour

$V_{ave} = \frac{d}{t} = \frac{120+60}{3+2+1} = \frac{180}{6} = 30$

15. The tangent line to the graph of f at $x = 5$ has equation $y = 2(x - 5) + 7$. Find $f(5)$ and $f'(5)$.

Possibilities:

(a) $f(7) = 2, \quad f'(7) = 5$
(b) $f(2) = 7, \quad f'(2) = 5$
(c) $f(7) = 5, \quad f'(7) = 2$
(d) $f(5) = 7, \quad f'(5) = 2$
(e) $f(5) = 2, \quad f'(5) = 7$

eqn of tan line

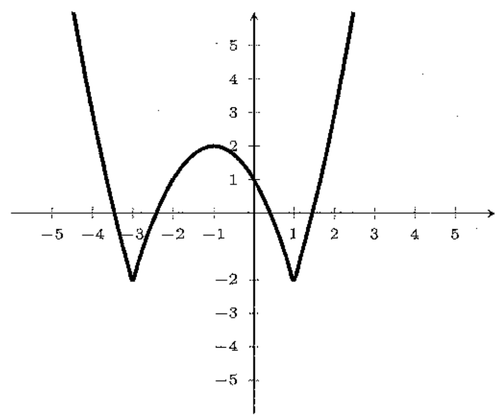
$y = f'(a)(x-a) + f(a)$

$f'(5) = 2 \quad f(5) = 7$

16. The graph of $y = f(x)$ is shown below. The function is differentiable, except at $x =$

Possibilities:

(a) -3 and 1
(b) -1
(c) -2
(d) -2 and 2
(e) 2



17. Find the derivative, $f'(x)$, of $f(x) = 6x^5$

Possibilities:

(a) $\frac{1}{6}x^6$
(b) x^6
(c) $30x^4$
(d) $-5x^{(1/6)}$
(e) x^5

18. Suppose that $f(x) = \log(g(x))$, but that the formula for $g(x)$ is too complicated to write down. When $x = 9$, the value and derivative of g are measured: $g(9) = 11$, and $g'(9) = 5$. What is $f'(9)$?

Possibilities:

(a) $\frac{1}{9}$

(b) $\frac{9}{5}$

(c) $\frac{11}{5}$

(d) $\frac{5}{9}$

(e) $\frac{5}{11}$

$f'(x) = \frac{g'(x)}{g(x)}$ $\quad f'(9) = \frac{g'(9)}{g(9)} = \frac{5}{11}$

19. Find the derivative, $f'(x)$, if $f(x) = \sqrt{3x + x^4}$.

Possibilities:

(a) $\frac{1}{2}(3x + x^4)^{(-1/2)}(3 + 4x^3)(4 \cdot 3x^2)$

(b) $-\frac{1}{2}(3x + x^4)^{(1/2)}(3 + 3x^4)$

(c) $\frac{1}{2}(3 + 4x^3)^{(-1/2)}$

(d) $\frac{1}{2}(3x + x^4)^{(-1/2)}$

(e) $\frac{1}{2}(3x + x^4)^{(-1/2)}(3 + 4x^3)$

$f'(x) = \frac{1}{2}(3x + x^4)^{-\frac{1}{2}} \cdot (3 + 4x^3)$

20. Suppose the derivative of $g(t)$ is $g'(t) = 3(t-5)(t-4)(t-7)$. For t in which interval(s) is g increasing?

Possibilities:

(a) $(3, 4) \cup (5, 7)$

(b) $(-\infty, 4) \cup (5, 7)$

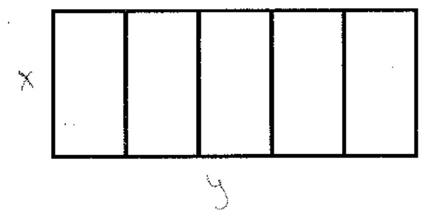
(c) $(\frac{16}{3} - \frac{1}{3}\sqrt{7}, \frac{16}{3} + \frac{1}{3}\sqrt{7})$

(d) $(-\infty, \frac{16}{3} - \frac{1}{3}\sqrt{7}) \cup (\frac{16}{3} + \frac{1}{3}\sqrt{7}, \infty)$

(e) $(4, 5) \cup (7, \infty)$

	0	4 $\frac{9}{2}$	5 6	7 8
$t-5$	$-$	$-$	$+$	$+$
$t-4$	$-$	$+$	$+$	$+$
$t-7$	$-$	$-$	$-$	$+$
g'	$-$	$+$	$-$	$+$

21. A farmer builds a rectangular pen with 5 vertical partitions (6 vertical sides) using 300 feet of fencing. What is the maximum possible total area of the pen?



Possibilities:

(a) 1875

(b) $\frac{11250}{7}$

(c) 300

(d) 5625

(e) 3750

$A = xy$

$300 = 6x + 2y$

$y = 150 - 3x$

$A = 150x - 3x^2$

$A' = 150 - 6x$

$150 - 6x = 0$

$x = 25$

$150(25) - 3(25)^2$

$3{,}750 - 1875$

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22. A ladder 20.00 feet long rests against a vertical wall. If the bottom of the ladder slides away from the wall at a constant rate of 3.00 feet per second, how fast is the top of the ladder sliding down the wall when the top of the ladder is 16.00 feet above the ground?

Possibilities:

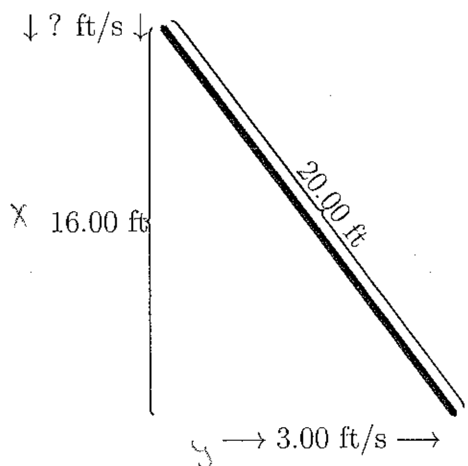
(a) 3.75 feet per second

(b) 2.40 feet per second

(c) 1.80 feet per second

(d) 2.25 feet per second

(e) 0.75 feet per second



$20^2 = [x(t)]^2 + [y(t)]^2$

$0 = 2xx' + 2yy'$

$0 = 16x' + 36 \qquad x' = -2.25$

$20^2 = 16^2 + y^2$

$y = 12$