MA123 Final Exam

12 December 2006

NAME _____ Section _____

Problem	Answer
1	a b c d e
2	a b c d e
3	a b c d e
4	a b c d e
5	a b c d e
6	a b c d e
7	a b c d e
8	a b c d e
9	a b c d e
10	a b c d e
11	a b c d e
12	a b c d e
13	a b c d e
14	a b c d e
15	a b c d e

Instructions. Circle your answer in ink on the page containing the problem and on the cover sheet. After the exam begins, you may not ask a question about the exam. Be sure you have all pages (containing 15 problems) before you begin. You will find a table of logarithms at the end of the exam that you need for Problem 6.

NAME_____

1. Find the average rate of change of the function $G(t) = \left|t^2 - 1\right|$ as t changes from -1 to 2.

   (a) 0
   (b) 1
   (c) 2
   (d) 3
   (e) 4

2. Find the equation of a line tangent to the curve $y = 2x^2 + x + 1$ at $x = 2$.

   (a) $y = 9 + 11(x - 2)$
   (b) $y = 11 + 9(x - 2)$
   (c) $y = 22 + 13(x - 3)$
   (d) $y = 13 + 22(x - 3)$
   (e) $y = 7 + (4x + 1)$

3. Which horizontal line is tangent to the graph of $y = x^3 - x^2 - x + 2$?

   (a) $y = 0$
   (b) $y = 1$
   (c) $y = 2$
   (d) $y = 3$
   (e) $y = 4$

4. What is the largest value of A such that the function $f(t)$ is defined on the interval $(0, A)$ where

$$f(t) = \frac{1}{t^3 - 8}$$

(a) 1
(b) 2
(c) 3
(d) 4
(e) 5

5. Find the limit as x tends to 0 from the left

$$\lim_{x \to 0^-} \frac{|x|}{2x}$$

(a) 1/3
(b) 1/2
(c) 0
(d) $-1/2$
(e) $-1/3$

6. Suppose the position $P(t)$ of an object at time t is given by $t^2 + 1$. Find a value of t at which the instanteous speed of the object equals the average speed on the interval $[0, 1]$.

(a) 1/3
(b) 1/2
(c) 2/3
(d) 3/4
(e) 1

7. Let $[x]$ denote the greatest integer function. Recall the definition:

$$[x] \text{ equals the greatest integer less than or equal to } x \ .$$

How many points are there in the interval $(\frac{1}{2}, \frac{9}{2})$ where the derivative of $[x]$ is not defined?

(a) 1
(b) 2
(c) 3
(d) 4
(e) 5

8. Let

$$f(x) = \begin{cases} -1 & \text{if} \quad x < 0 \\ 2 & \text{if} \quad 0 \leq x < 2 \\ 3 & \text{if} \quad 2 \leq x \end{cases}$$

Evaluate the integral

$$\int_{-2}^{3} f(x)dx$$

(a) 1
(b) 2
(c) 3
(d) 4
(e) 5

9. Evaluate the derivative, $H'(1)$ if

$$H(s) = \frac{2s}{s+1}$$

(a) 2/9
(b) 4/9
(c) 1/2
(d) 3/2
(e) 8/9

10. Use the summation formula below to evaluate $3^3 + 4^3 + \cdots + 40^3$

$$\sum_{k=1}^{n} k^3 = \frac{n^2(n+1)^2}{4}$$

(a) 608391
(b) 608400
(c) 653479
(d) 672400
(e) 672391

11. Evaluate the limit

$$\lim_{n\to\infty} \sum_{k=1}^{n} \frac{1}{n} f(\frac{k}{n})$$

where $f(x) = x^2$. Hint: Draw a picture and relate the limit to an integral.

(a) 1/5
(b) 1/4
(c) 1/3
(d) 1/2
(e) 1

12. Evaluate the limit

$$\lim_{h\to 0} \frac{f(3+h) - f(3)}{h}$$

where

$$f(x) = \sqrt{x+1}$$

(a) 1/6
(b) 1/5
(c) 1/4
(d) 1/3
(e) 1/2

13. Find the area of the rectangle of minimum area with one vertex (corner) at $(0, 0)$ and opposite corner on the ellipse $x^2 + 4y^2 = 4$.

    (a) $3/4$
    (b) $\sqrt{5}/4$
    (c) $\sqrt{7}/4$
    (d) $9/4$
    (e) $\sqrt{11}/4$

14. What is the average of the function $h(t) = t^3 + 1$ on the interval $[1, 4]$?

    (a) $\frac{247}{12}$
    (b) $\frac{257}{12}$
    (c) $\frac{267}{12}$
    (d) $\frac{277}{12}$
    (e) $\frac{279}{12}$

15. Suppose the cost, $C(q)$, of stocking a quantity q of a product equals

$$C(q) = \frac{100}{q} + q$$

    The rate of change of the cost with respect to q is called the marginal cost. When is the marginal cost positive?

    (a) $q > 10$
    (b) $q > 15$
    (c) $q < 20$
    (d) $q < 25$
    (e) $q = 30$

x	$logx$	x	$logx$
2.00	.301	2.50	.398
2.05	.312	2.55	.407
2.10	.322	2.60	.415
2.15	.332	2.65	.423
2.20	.342	2.70	.431
2.25	.352	2.75	.439
2.30	.361	2.80	.447
2.35	.371	2.85	.454
2.40	.382	2.90	.462
2.45	.389	2.95	.470

Geometric Formulas

1. Areas
   (a) Triangle $A = \frac{bh}{2}$
   (b) Circle $A = \pi r^2$
   (c) Rectangle $A = lw$
   (d) Trapazoid $A = \frac{b_1+b_2}{2}h$
2. Volumes
   (a) Rectangular Solid $V = lwh$
   (b) Sphere $V = \frac{4}{3}\pi r^3$
   (c) Cylinder $V = Bh$
   (d) Cone $V = \frac{1}{3}Bh$