MA123 Exam 3

November 14 2007

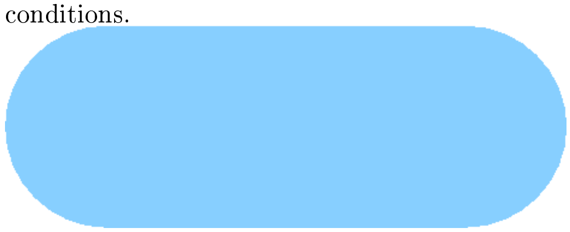
NAME _________________________________ Section ___________

Problem	Answer
1	a b c d e
2	a b c d e
3	a b c d e
4	a b c d e
5	a b c d e
6	a b c d e
7	a b c d e
8	a b c d e
9	a b c d e
10	a b c d e
11	a b c d e
12	a b c d e
13	a b c d e
14	a b c d e
15	a b c d e

Instructions. Circle your answer in ink on the page containing the problem and on the cover sheet. After the exam begins, you may not ask a question about the exam. Be sure you have all pages (containing 15 problems) before you begin. You will find a table and some formulas at the end of the exam.

NAME____

1. A field has the shape of a rectangle with two semicircles attached at opposite sides. Find the radius of the semicircles if the field is to have maximum area, the perimeter of the field equals 100, and the width of the field (twice the radius of the semicircles) is at most 18. The shape of the field is indicated by the figure below. CAUTION: Be sure your answer satisfies all conditions.



The radius equals

   (a) 6
   (b) 7
   (c) 8
   (d) 9
   (e) 10

2. Find the area of the largest rectangle with one corner at the origin and the opposite corner in the first quadrant and on the line $y = 10 - 2x$. Assume the sides of the rectangle are parallel with the axes.

   (a) 73/2
   (b) 67/2
   (c) 55/2
   (d) 49/2
   (e) 25/2

3. A train starts from rest (speed equal to 0 miles per hour) at 12:00 noon. The speed increases at a constant rate until 12:15 when the speed equals 64 miles per hour. How far does the train travel from 12:00 to 12:15?

   (a) 7
   (b) 8
   (c) 9
   (d) 10
   (e) 11

4. A train travels along a straight track at a constant speed of 50 miles per hour. A straight road intersects the track at right angles and a truck is parked on the road one mile from the track. How fast is the train traveling away from the truck when the train is 3 miles past the intersection?

   (a) $10\sqrt{10}$
   (b) $15\sqrt{10}$
   (c) $20\sqrt{10}$
   (d) $5\sqrt{10}$
   (e) $20\sqrt{5}$

5. Water is evaporating from a pool at a constant rate. The area of the pool is 5000 square feet. Assume the sides of the pool drop straight down (perpendicular) from the edge. The water in the pool drops .5 feet in one day. How fast is the water evaporating in cubic feet per day?

   (a) 2000
   (b) 2500
   (c) 3000
   (d) 3500
   (e) 4000

6. Use the table for the function 2^x (last page of exam) to estimate the integral

$$\int_{.1}^{.25} 2^x \, dx$$

   Use three (3) subintervals and the left endpoint of each subinterval to determine the height of the rectangles used in the approximation. The approximate value of the integral is

   (a) .166
   (b) .168
   (c) .172
   (d) .174
   (e) .178

7. Suppose you estimate the integral

$$\int_{10}^{20} (1+x)^2 dx$$

by the sum of the areas of 10 rectangles of equal base length. Use the right endpoint of each base to determine the height. What is the area of the first (left most) rectangle?

(a) 144
(b) 244
(c) 341
(d) 441
(e) 541

8. Evaluate the sum $6 + 8 + 10 + 12 + ... + 100$. The sum equals

(a) 2518
(b) 2530
(c) 2538
(d) 2540
(e) 2544

9. Evaluate the sum

$$\sum_{k=4}^{10} (1+k)$$

(a) 56
(b) 60
(c) 63
(d) 73
(e) 74

10. Suppose the equation

$$\frac{1}{2} + \left(\frac{1}{2}\right)^2 + \left(\frac{1}{2}\right)^3 + \left(\frac{1}{2}\right)^4 + \ldots + \left(\frac{1}{2}\right)^n = 1 - \left(\frac{1}{A}\right)^n$$

holds for all non negative integer values of n. What is the value of the constant A?

(a) 1
(b) 2
(c) 3
(d) 4
(e) 5

11. Evaluate the limit

$$\lim_{n\to\infty} \frac{2n^3}{(n+2)^3}$$

(a) 0
(b) 1
(c) 2
(d) 3
(e) 4

12. Evaluate the limit

$$\lim_{n\to\infty} \frac{1}{n^2} \sum_{k=1}^{n} 2k$$

(a) 0
(b) 1/2
(c) 1
(d) 3/2
(e) 2

13. A point moves along the line $y = 4 + 3x$ so that the y coordinate of the point increases at a constant rate of 2 units per second. How fast is the x coordinate of the point increasing?

(a) 2/3
(b) 1
(c) 3/2
(d) 2
(e) 3

14. You want to estimate the integral,

$$\int_{10}^{30} \frac{1}{x} dx$$

as the sum of areas of rectangles. You break the interval $[10, 30]$ into 20 subintervals of equal length. If you use the left endpoint of each subinterval to determine the height of each rectangle, which estimate is correct? Hint: DRAW A PICTURE!

(a) $\int_{10}^{30} \frac{1}{x} dx \geq \frac{1}{10} + \frac{1}{11} + \frac{1}{12} + \cdots + \frac{1}{29}$
(b) $\int_{10}^{30} \frac{1}{x} dx \leq \frac{1}{10} + \frac{1}{11} + \frac{1}{12} + \cdots + \frac{1}{29}$
(c) $\int_{10}^{30} \frac{1}{x} dx \leq \frac{1}{11} + \frac{1}{12} + \cdots + \frac{1}{29} + \frac{1}{30}$
(d) $\int_{10}^{30} \frac{1}{x} dx \geq \frac{1}{11} + \frac{1}{12} + \cdots + \frac{1}{29} + \frac{1}{30}$
(e) $\int_{10}^{30} \frac{1}{x} dx \leq \frac{1}{10} + \frac{1}{12} + \frac{1}{14} + \cdots + \frac{1}{28} + \frac{1}{30}$

15. If you sell an item at price p, your revenue will equal the price p times the number sold, n. Suppose price is linearly related to the number sold by the equation

$$n = 100 - 10(p - 10)$$

How should you set the price to maximize revenue? The price should equal

(a) 10
(b) 15
(c) 20
(d) 25
(e) 30

Table for the function 2^x.

x	2^x	x	2^x
0	1.000	.50	1.414
.05	1.035	.55	1.464
.10	1.071	.60	1.516
.15	1.109	.65	1.569
.20	1.148	.70	1.625
.25	1.189	.75	1.682
.30	1.231	.80	1.741
.35	1.274	.85	1.803
.40	1.319	.90	1.866
.45	1.366	.95	1.932

Geometric Formulas

1. Areas
   (a) Triangle $A = \frac{bh}{2}$
   (b) Circle $A = \pi r^2$
   (c) Rectangle $A = lw$
   (d) Trapazoid $A = \frac{b_1+b_2}{2}h$
2. Volumes
   (a) Rectangular Solid $V = lwh$
   (b) Sphere $V = \frac{4}{3}\pi r^3$
   (c) Cylinder $V = Bh$
   (d) Cone $V = \frac{1}{3}Bh$
3. Summation

$$\sum_{k=1}^{n} k = \frac{n(n+1)}{2}$$