MA 123 Fall 2023
Elementary Calculus

Exam 3
11/16/23

Name: _____

Student ID #: 9_____ **Sec:** _____

Do not remove this answer page — you will turn in the entire exam. You have two hours to do this exam. No books or notes may be used. You may use an ACT-approved calculator during the exam, but NO calculator with a Computer Algebra System (CAS), networking, or camera is permitted. Absolutely no cell phone use during the exam is allowed.

The exam consists of 2 short answer questions and 18 multiple choice questions. Answer the short answer questions on the back of this page, and record your answers to the multiple choice questions on this page. For each multiple choice question, you will need to fill in the circle corresponding to the correct answer. For example, if (a) is correct, you must shade

(a) (b) (c) (d) (e)

It is your responsibility to make it CLEAR which response has been chosen. **You will not get credit unless the correct answer has been clearly marked on this page.**

GOOD LUCK!

3. (a) (b) (c) (d) (e)

4. (a) (b) (c) (d) (e)

5. (a) (b) (c) (d) (e)

6. (a) (b) (c) (d) (e)

7. (a) (b) (c) (d) (e)

8. (a) (b) (c) (d) (e)

9. (a) (b) (c) (d) (e)

10. (a) (b) (c) (d) (e)

11. (a) (b) (c) (d) (e)

12. (a) (b) (c) (d) (e)

13. (a) (b) (c) (d) (e)

14. (a) (b) (c) (d) (e)

15. (a) (b) (c) (d) (e)

16. (a) (b) (c) (d) (e)

17. (a) (b) (c) (d) (e)

18. (a) (b) (c) (d) (e)

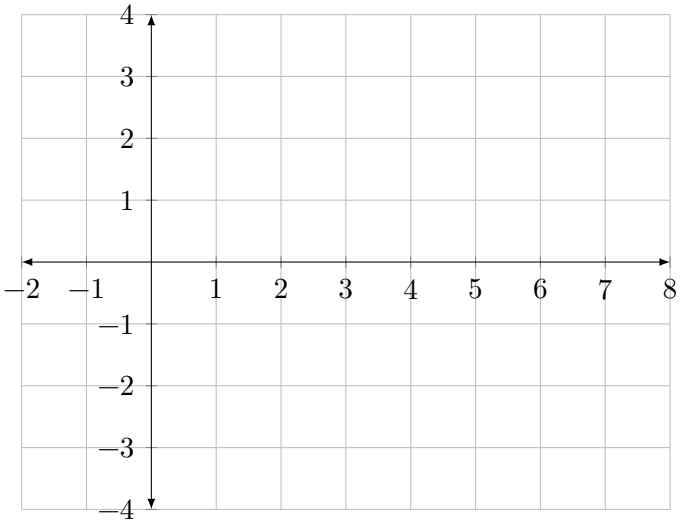
19. (a) (b) (c) (d) (e)

20. (a) (b) (c) (d) (e)

Short Answer Questions

Each question is an opportunity to earn 5 points. Points are earned on the clarity and correctness of your work, not merely on having a correct answer somewhere.

1. Sketch the graph of a continuous function $y = f(x)$ which satisfies the following properties: $f(0) = 1$, $f'(x) < 0$ on $(-\infty, 3)$, $f'(x) > 0$ on $(3, \infty)$, $f''(x) > 0$ on $(-\infty, 3)$ and $f''(x) < 0$ on $(3, \infty)$.



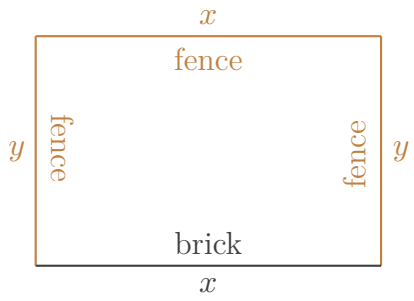
2. Let x and y be two positive numbers such that $xy = 81$. Determine the minimum possible sum of x and y. **You must show all steps of an optimization problem to earn full credit.**

Name: _____

Multiple Choice Questions

Clearly mark your answer on the cover page on this exam for credit.

3. A landscape architect plans to enclose a rectangular garden on the bottom side by a brick wall costing \$40 per foot and on the other three sides by a metal fence costing \$25 per foot. The area of the garden must be 500 square feet, and the architect wants to minimize the cost to enclose the garden. What is the objective function for this optimization problem? In the answer choices below, A denotes area and C denotes cost.



Possibilities:

(a) $2x + 2y = 500$

(b) $C = 40x + 75y$

(c) $C = 65x + 50y$

(d) $A = xy$

(e) $xy = 500$

4. Suppose the derivative of $g(t)$ is $g'(t) = 3t^4 + 12t^3 + 28$. Determine all values of t where $g(t)$ has an inflection point. You may assume that $g(t)$ is defined for all t.

Possibilities:

(a) -3

(b) -2 and 0

(c) -2

(d) 0

(e) -3 and 0

5. Let $f(x) = x^4 - 6x^3 - 108x^2 + 60x + 96$. Determine all intervals on which $f(x)$ is concave down.

Possibilities:

(a) $(-6, 3)$

(b) $(-\infty, -6) \cup (3, \infty)$

(c) $(-\infty, -3) \cup (6, \infty)$

(d) $(-\infty, \infty)$

(e) $(-3, 6)$

---

6. Determine all critical values of the function $f(x) = xe^{32x}$.

Possibilities:

(a) 0

(b) $-\dfrac{1}{32}$ and 0

(c) There are no critical values.

(d) $-\dfrac{1}{32}$

(e) 0 and 32

7. Suppose the derivative of $g(t)$ is $g'(t) = -t^2(t^2+4)(t-6)$. Determine the value of t in the interval $[-70, 70]$ where $g(t)$ takes on its maximum value.

Possibilities:

(a) 0

(b) -2

(c) 6

(d) 70

(e) -70

8. Determine the area of the largest rectangle whose sides are parallel to the coordinate axes, whose bottom-left corner is at $(0, 0)$ and whose top-right corner is on the graph of $y = 972 - 9x^2$.

Possibilities:

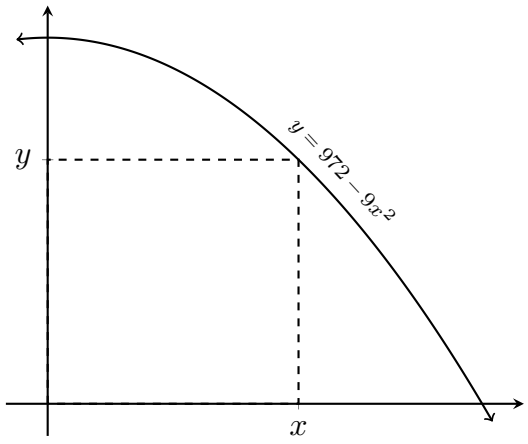
(a) 3888

(b) 2386

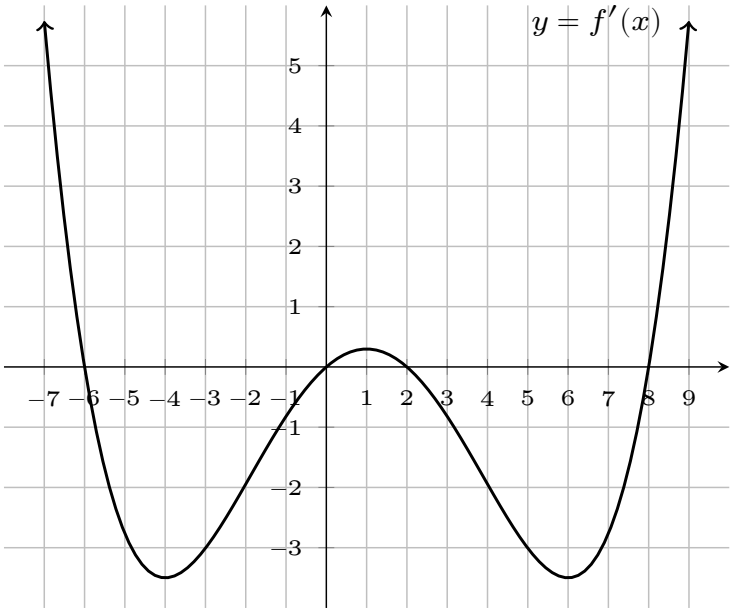
(c) 3181

(d) 4883

(e) 3283



9. The following is the **graph of the derivative**, $f'(x)$, of the function $f(x)$. Determine all intervals on which the original function $f(x)$ is increasing.



Possibilities:

(a) $(-\infty, -6) \cup (0, 2) \cup (8, \infty)$

(b) $(-\infty, \infty)$

(c) $(-4, 1) \cup (6, \infty)$

(d) $(-6, 0) \cup (2, 8)$

(e) $(-\infty, -4) \cup (1, 6)$

10. Determine all intervals on which the function $f(x) = x^3 - 9x^2 + 15x + 8$ is decreasing.

Possibilities:

(a) $(3, \infty)$

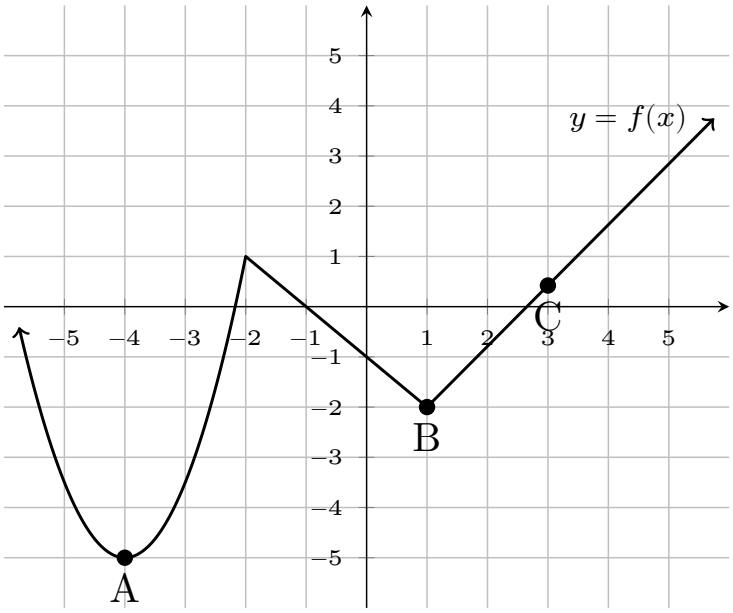
(b) $(-\infty, 1) \cup (5, \infty)$

(c) $(-\infty, 3)$

(d) $(-\infty, \infty)$

(e) $(1, 5)$

11. The graph of $y = f(x)$ is shown below. At which of the labeled points does $f(x)$ attain a local minimum?



Possibilities:

(a) B only

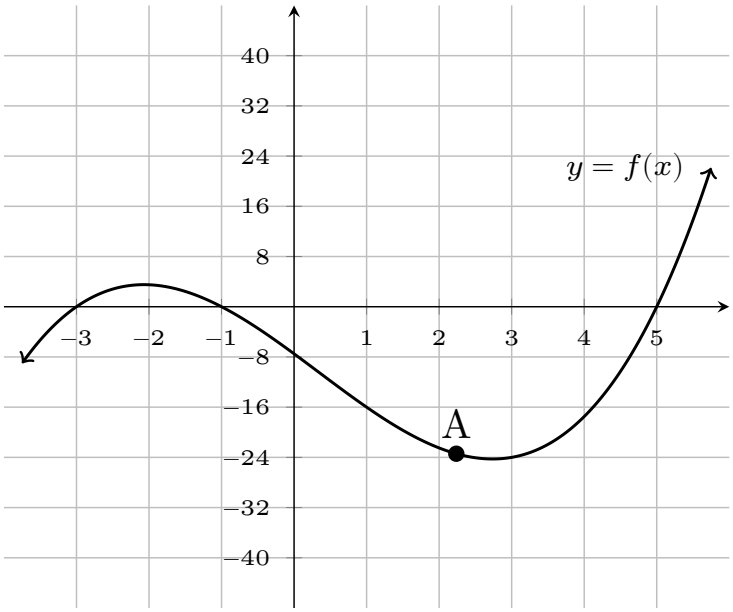
(b) A only

(c) A, B and C

(d) A and B only

(e) B and C only

12. Consider the point labeled A on the graph of the function $y = f(x)$. Use the graph to determine the signs of f' and f'' at A.



Possibilities:

(a) $f' > 0$ and $f'' > 0$

(b) $f' < 0$ and $f'' < 0$

(c) $f' > 0$ and $f'' < 0$

(d) $f' = 0$ and $f'' = 0$

(e) $f' < 0$ and $f'' > 0$

13. Let $f(x) = (x + 18) \cdot \ln(x + 8)$ for $x > -8$. Determine all intervals on which $f(x)$ is concave up.

Possibilities:

(a) $(2, \infty)$

(b) $(-8, 2)$

(c) $(8, 18)$

(d) $(-8, \infty)$

(e) $(-8, 18)$

14. The graph of $y = f(x)$ shown below includes a semicircle and a straight line. Evaluate the definite integral $\int_{-2}^{4} f(x)\,dx$.

Possibilities:

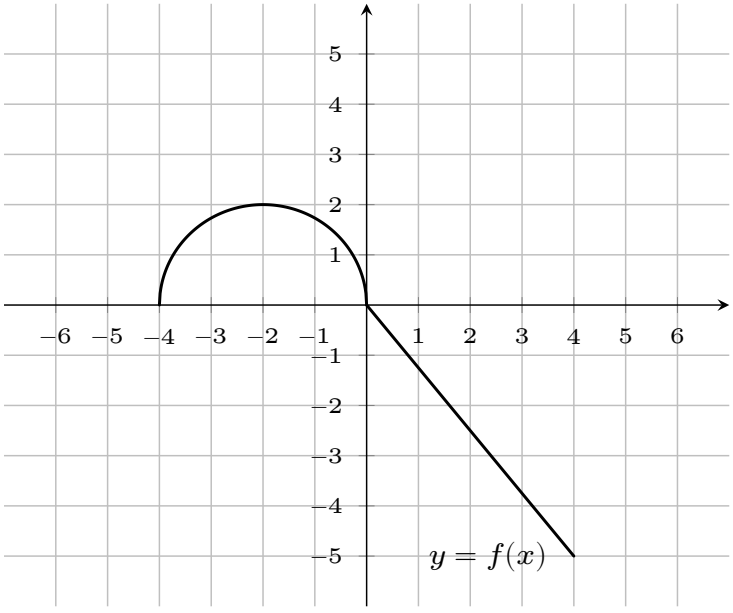
(a) $\pi + 10$

(b) $2\pi + 10$

(c) $-2\pi + 10$

(d) $\pi - 10$

(e) $-\pi + 10$



15. If $\int_1^{17} f(x)\,dx = 26$ and $\int_9^{17} f(x)\,dx = 2$, then determine $\int_1^9 f(x)\,dx$.

Possibilities:

(a) -28

(b) 24

(c) 28

(d) -24

(e) 2

16. Suppose that $\int_6^{25} f(x)\,dx = 9$. Determine the value of $\int_6^{25} (5f(x) + 7)\,dx$.

Possibilities:

(a) 220

(b) 178

(c) 66

(d) 64

(e) 52

17. Determine the average value of $f(x)$ on the interval $[4, 12]$ given that $f(x) = \begin{cases} 50 & \text{if } x < 7, \\ -20 & \text{if } x \geq 7. \end{cases}$

Possibilities:

(a) $-\dfrac{35}{4}$

(b) 15

(c) $\dfrac{25}{4}$

(d) 6

(e) 25

18. The graph of $y = f(x)$ is given below. The numbers shown represent the geometric area of each region. Evaluate the definite integral

$$\int_2^0 f(x)\,dx.$$

Possibilities:

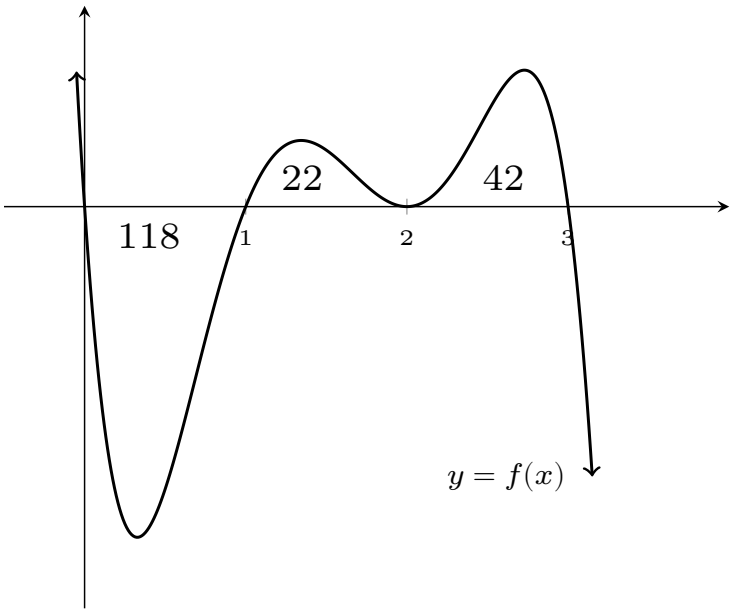
(a) -96

(b) -182

(c) 96

(d) -54

(e) 54



19. Given the function $f(x) = \begin{cases} 7 & \text{if } x < 2, \\ x + 5 & \text{if } x \geq 2, \end{cases}$ evaluate the definite integral $\int_0^8 f(x)\,dx$.

Possibilities:

(a) 254

(b) 134

(c) 170

(d) 74

(e) 53

---

20. The graph of $y = f(x)$ shown below consists of straight lines. Evaluate the definite integral $\int_{-4}^{5} f(x)\,dx$.

Possibilities:

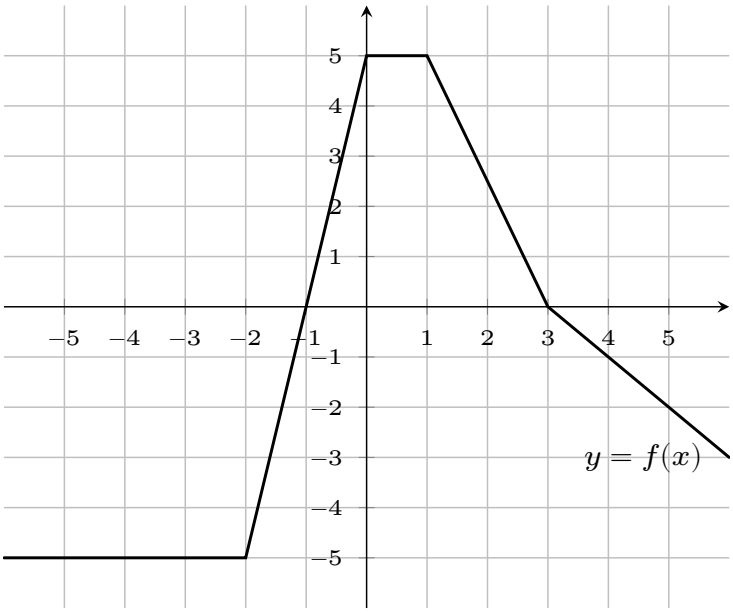
(a) -27

(b) -2

(c) 23

(d) 27

(e) 2



Formulas

Areas:

Circle: $A = \pi r^2$

Triangle: $A = \dfrac{bh}{2}$

Rectangle: $A = lw$

Trapezoid: $A = \dfrac{b_1 + b_2}{2}\, h$

Volumes:

Rectangular Solid: $V = lwh$