MA 123 Fall 2023
Elementary Calculus

Exam 4
12/13/23

Name: _____

Student ID #: 9_____ **Sec:** _____

Do not remove this answer page — you will turn in the entire exam. You have two hours to do this exam. No books or notes may be used. You may use an ACT-approved calculator during the exam, but NO calculator with a Computer Algebra System (CAS), networking, or camera is permitted. Absolutely no cell phone use during the exam is allowed.

The exam consists of 2 short answer questions and 18 multiple choice questions. Answer the short answer questions on the back of this page, and record your answers to the multiple choice questions on this page. For each multiple choice question, you will need to fill in the circle corresponding to the correct answer. For example, if (a) is correct, you must shade

(a) (b) (c) (d) (e)

It is your responsibility to make it CLEAR which response has been chosen. **You will not get credit unless the correct answer has been clearly marked on this page.**

GOOD LUCK!

3. (a) (b) (c) (d) (e)   12. (a) (b) (c) (d) (e)

4. (a) (b) (c) (d) (e)   13. (a) (b) (c) (d) (e)

5. (a) (b) (c) (d) (e)   14. (a) (b) (c) (d) (e)

6. (a) (b) (c) (d) (e)   15. (a) (b) (c) (d) (e)

7. (a) (b) (c) (d) (e)   16. (a) (b) (c) (d) (e)

8. (a) (b) (c) (d) (e)   17. (a) (b) (c) (d) (e)

9. (a) (b) (c) (d) (e)   18. (a) (b) (c) (d) (e)

10. (a) (b) (c) (d) (e)   19. (a) (b) (c) (d) (e)

11. (a) (b) (c) (d) (e)   20. (a) (b) (c) (d) (e)

Each question is an opportunity to earn 5 points. Points are earned on the clarity and correctness of your work, not merely on having a correct answer somewhere.

1. Let x and y be two positive numbers such that $x^2 \cdot y = 100$. Determine the minimum possible sum $S = x + y$. **You must correctly perform a number line test to earn full credit.**

2. Evaluate $\displaystyle\int_1^2 \frac{8x^7}{x^8 + 5}\, dx$. **You must show work to receive credit.**

Multiple Choice Questions

Clearly mark your answer on the cover page on this exam for credit.

3. Let $f(x) = x^3 - 2x - 7$. Determine a value of c between $x = 6$ and $x = 33$ such that the average rate of change of $f(x)$ from $x = 6$ to $x = 33$ is equal to the instantaneous rate of change at $x = c$.
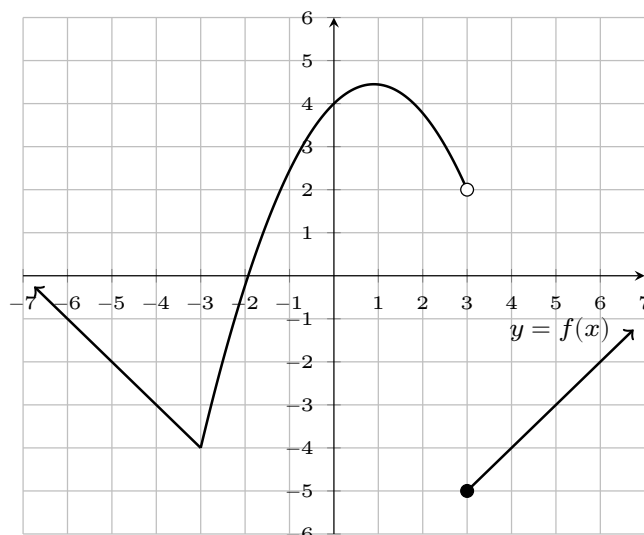
   Possibilities:

   (a) $\dfrac{105}{4}$

   (b) $\dfrac{39}{2}$

   (c) 15

   (d) $\dfrac{51}{4}$

   (e) 21

4. The graph of $y = f(x)$ is shown below. Evaluate $\lim\limits_{x \to 3^-} f(x)$.

   Possibilities:

   (a) 5

   (b) -5

   (c) The limit does not exist.

   (d) 2

   (e) $\dfrac{3}{2}$

5. Let $f(x) = 4x^2 - 36x + 9$. At what value of x does the tangent line to the graph of $y = f(x)$ have slope 5?

Possibilities:

(a) $-\dfrac{31}{8}$

(b) 4

(c) $\dfrac{9}{2} + \dfrac{1}{2}\sqrt{77}$

(d) $\dfrac{41}{2}$

(e) $\dfrac{41}{8}$

6. If $h(t)$ represents the height of an object in feet above ground level at time t seconds and $h(t)$ is given by $h(t) = -16t^2 + 18t + 45$, determine the height of the object at the time when its speed is zero.

Possibilities:

(a) $\dfrac{801}{16}$ feet

(b) 45 feet

(c) 53 feet

(d) $\dfrac{9}{16}$ feet

(e) $\dfrac{1179}{64}$ feet

7. Suppose $g(x) = \sqrt{f(x)}$ and the equation of the tangent line to $f(x)$ at $x = 1$ is

$$y = 64 + 4(x - 1).$$

Determine $g'(1)$.

Possibilities:

(a) 7

(b) $\dfrac{1}{4}$

(c) 6

(d) 16

(e) $\dfrac{1}{16}$

8. The concentration of Acetaminophen in the bloodstream t hours after the drug is administered is given by
$$C(t) = \frac{0.13t}{t^2 + 6}.$$

Determine the instantaneous rate of change of the Acetaminophen concentration with respect to time (in units per hour) at $t = 1$ hour.

Possibilities:

(a) $\dfrac{13}{700}$

(b) $\dfrac{13}{980}$

(c) $-\dfrac{13}{100}$

(d) $\dfrac{117}{100}$

(e) $\dfrac{13}{200}$

9. Let $f(x) = \sqrt[5]{7x + 8}$. Determine the derivative $f'(x)$.

Possibilities:

(a) $\dfrac{7}{5}x^{-4/5}$

(b) $\dfrac{7}{5}(7x + 8)^{-4/5}$

(c) $\dfrac{1}{5}(7x + 8)^{-4/5}$

(d) $-35(7x + 8)^{-6}$

(e) $-\dfrac{7}{4}(7x + 8)^{-4}$

10. The total cost (in dollars) of producing x machines is

$$C(x) = 1700 + 60x - 0.4x^2.$$

Use the marginal cost function to approximate the cost of producing the 71st machine.

Possibilities:

(a) Approximate cost of the 71st machine $\approx$ \$3.60.

(b) Approximate cost of the 71st machine $\approx$ \$56.29.

(c) Approximate cost of the 71st machine $\approx$ \$4.00.

(d) Approximate cost of the 71st machine $\approx$ \$3.20.

(e) Approximate cost of the 71st machine $\approx$ \$2.40.

11. Determine all critical values of the function $f(x) = x^2 e^{45x}$.

Possibilities:

(a) $-\dfrac{2}{45}$ and 0

(b) $-\dfrac{2}{45}$

(c) 0

(d) There are no critical values.

(e) 0 and $\dfrac{2}{45}$

12. Consider the point labeled A on the graph of the function $y = f(x)$. Use the graph to determine the signs of f' and f'' at A.

Possibilities:

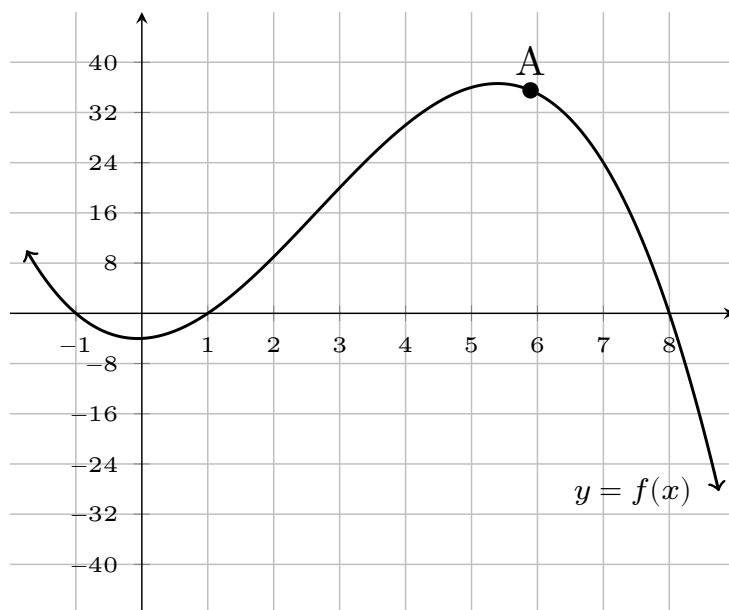(a) $f' > 0$ and $f'' > 0$

(b) $f' > 0$ and $f'' < 0$

(c) $f' < 0$ and $f'' < 0$

(d) $f' < 0$ and $f'' > 0$

(e) $f' = 0$ and $f'' = 0$

13. Determine the average value of $f(x)$ on the interval $[5, 13]$ given that $f(x) = \begin{cases} 30 & \text{if } x < 8, \\ -80 & \text{if } x \geq 8. \end{cases}$

   Possibilities:

   (a) -155

   (b) $-\dfrac{155}{4}$

   (c) -25

   (d) $-\dfrac{55}{4}$

   (e) 6

14. Determine the indefinite integral $\displaystyle\int \frac{108x^8 + 9x^7 + 48x^2}{x^7}\, dx.$

   Possibilities:

   (a) $12x^9 + 16x^3 + 9 + C$

   (b) $54x^2 + 9 - 12x^{-4} + C$

   (c) $12x^9 + 16x^3 + 9x + C$

   (d) $108 - 240x^{-6} + C$

   (e) $54x^2 + 9x - 12x^{-4} + C$

15. Determine the indefinite integral $\int 35x^6 e^{x^7}\, dx$.

Possibilities:

(a) $5e^x + C$

(b) $\frac{1}{5}e^{x^7} + C$

(c) $5e^{x^7} + C$

(d) $245x^{12}e^{x^7} + 210x^5 e^{x^7} + C$

(e) $\frac{1}{5}e^x + C$

16. Let $F(x) = \int_{7x+4}^{1} (t^2 + 9t + 6)\, dt$. Determine $F'(x)$.

Possibilities:

(a) $-7\left((7x+4)^2 + 9(7x+4) + 6\right)$

(b) $2(7x+4) + 9$

(c) $(7x+4)^2 + 9(7x+4) + 6$

(d) $-(7x+4)^2 - 9(7x+4) - 6$

(e) -366

17. Suppose $F(x) = \int_1^x \left(t^2 - 14t + 49\right)^3 dt$. For which value of x does $F'(x) = 0$?

Possibilities:

(a) 0

(b) 1

(c) 8

(d) 7

(e) 5

18. Evaluate $\int_0^2 \left(15e^x - 35x^4\right) dx$.

Possibilities:

(a) $15e^2 - 239$

(b) $5e^3 - 15e - 1120$

(c) $15e^2 - 575$

(d) $15e^2 - 1135$

(e) $5e^3 - 15e - 112$

19. Evaluate $\int_3^{12} f(x)\,dx$, where $f(x) = \begin{cases} x^2 & \text{if } x < 6, \\ 7 & \text{if } x \geq 6. \end{cases}$

Possibilities:

(a) 6

(b) 630

(c) 105

(d) 609

(e) 34

20. Suppose a rock is dropped from a Saturnian cliff. After t seconds, its velocity in feet per second is $v(t) = -72t$, at least until it hits the ground. If the cliff is 64 feet high, how long does it take for the rock to hit the ground?

Possibilities:

(a) $\dfrac{8}{9}$ seconds

(b) $\dfrac{2}{3}$ seconds

(c) $\dfrac{16}{9}$ seconds

(d) $\dfrac{4}{9}$ seconds

(e) $\dfrac{4}{3}$ seconds

Areas:

   Circle: $A = \pi r^2$

   Triangle: $A = \dfrac{bh}{2}$

   Rectangle: $A = lw$

   Trapezoid: $A = \dfrac{b_1 + b_2}{2}\, h$

Volumes:

   Rectangular Solid: $V = lwh$