MA 123 Fall 2025 Elementary Calculus

Exam 3
11/20/25

Name: ______________________________

Student ID #: 9______________ **Sec:** _____

You may remove the formula sheet and multiplication table, but do not remove any other pages from the exam. You have two hours to do this exam. **No books, notes, or calculators may be used. Absolutely no cell phone use during the exam is allowed.**

The exam consists of 2 short answer questions and 18 multiple choice questions. Answer the short answer questions on the back of this page, and record your answers to the multiple choice questions on this page. For each multiple choice question, you will need to fill in the circle corresponding to the correct answer. For example, if (a) is correct, you must shade

(a) (b) (c) (d) (e)

It is your responsibility to make it CLEAR which response has been chosen. **You will not get credit unless the correct answer has been clearly marked on this page.**

GOOD LUCK!

3. (a) (b) (c) (d) (e)   **12.** (a) (b) (c) (d) (e)

4. (a) (b) (c) (d) (e)   **13.** (a) (b) (c) (d) (e)

5. (a) (b) (c) (d) (e)   **14.** (a) (b) (c) (d) (e)

6. (a) (b) (c) (d) (e)   **15.** (a) (b) (c) (d) (e)

7. (a) (b) (c) (d) (e)   **16.** (a) (b) (c) (d) (e)

8. (a) (b) (c) (d) (e)   **17.** (a) (b) (c) (d) (e)

9. (a) (b) (c) (d) (e)   **18.** (a) (b) (c) (d) (e)

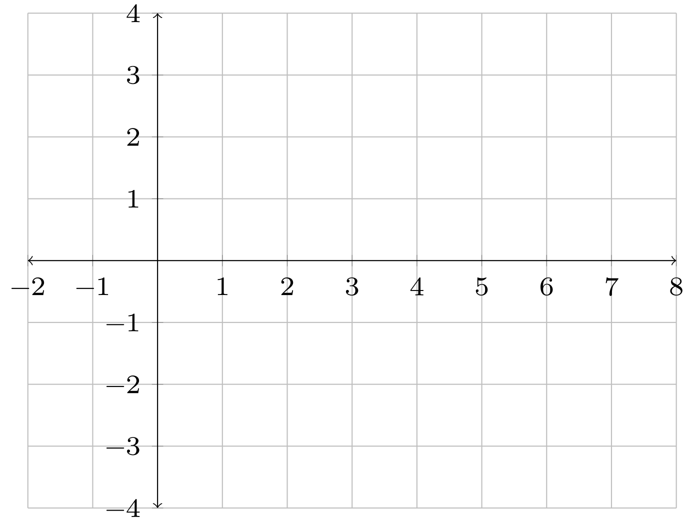
10. (a) (b) (c) (d) (e)   **19.** (a) (b) (c) (d) (e)

11. (a) (b) (c) (d) (e)   **20.** (a) (b) (c) (d) (e)

Short Answer Questions

Each question is an opportunity to earn 5 points. Points are earned on the clarity and correctness of your work, not merely on having a correct answer somewhere.

1. Sketch the graph of a continuous function $y = f(x)$ which satisfies the following properties: $f(1) = 2$, $f'(x) > 0$ on $(-\infty, 3)$, $f'(x) < 0$ on $(3, \infty)$, $f''(x) < 0$ on $(-\infty, 3)$ and $f''(x) > 0$ on $(3, \infty)$.



2. Let x and y be two positive numbers such that $x \cdot y = 36$. Complete the parts below to determine the minimum possible sum of x and y.

   (a) Write the objective function.

   (b) Write the constraint.

   (c) Rewrite the objective function so that it is a function of a single variable.

   (d) Determine the critical values of the objective function and perform an appropriate number line test to determine the minimum possible sum of x and y.

Name: ________________________________

Multiple Choice Questions

Clearly mark your answer on the cover page on this exam for credit.

3. Which of the following would indicate that a function $h(x)$ is concave down on the interval $(1, 6)$?

   Possibilities:

   (a) $h'(x) > 0$ on the interval $(1, 6)$

   (b) $h(x) > 0$ on the interval $(1, 6)$

   (c) $h''(x) < 0$ on the interval $(1, 6)$

   (d) $h'(x) < 0$ on the interval $(1, 6)$

   (e) $h''(x) > 0$ on the interval $(1, 6)$

4. Suppose the derivative of $g(t)$ is $g'(t) = -t^2(t^2 + 25)(t - 3)$. Determine the value of t in the interval $[-15, 15]$ where $g(t)$ takes on its maximum value.

   Possibilities:

   (a) 0

   (b) 3

   (c) -5

   (d) -15

   (e) 15

5. The graph of $y = f(x)$ is given below. At which of the labeled points does $f(x)$ attain a local minimum?

Possibilities:

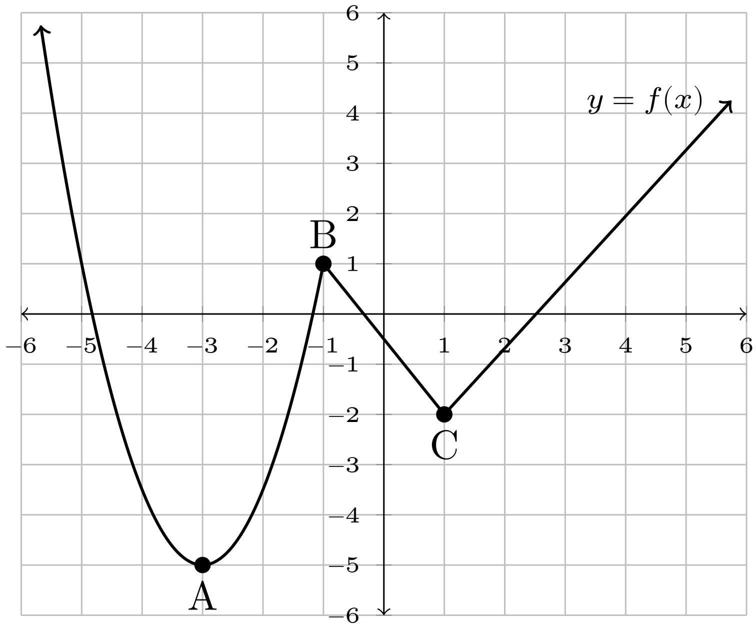
(a) C only

(b) B only

(c) A only

(d) A and C

(e) B and C



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6. The **graph of the derivative of a function** $y = f'(x)$ is given below. Determine all intervals on which the original function $f(x)$ is decreasing.

Possibilities:

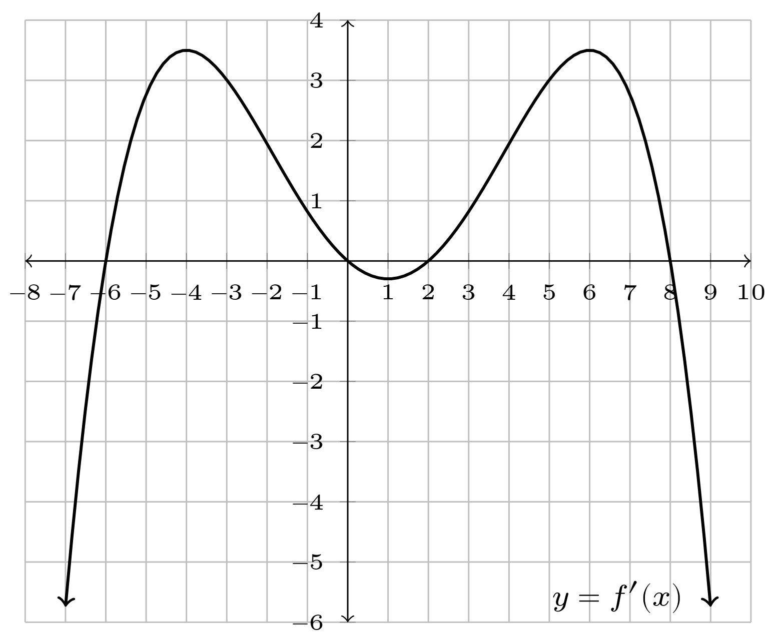
(a) $(-\infty, \infty)$

(b) $(-6, 0) \cup (2, 8)$

(c) $(-\infty, -6) \cup (0, 2) \cup (8, \infty)$

(d) $(-\infty, -4) \cup (1, 6)$

(e) $(-4, 1) \cup (6, \infty)$



7. Consider the point labeled A on the graph of the function $y = f(x)$. Use the graph to determine the signs of f' and f'' at A.

Possibilities:

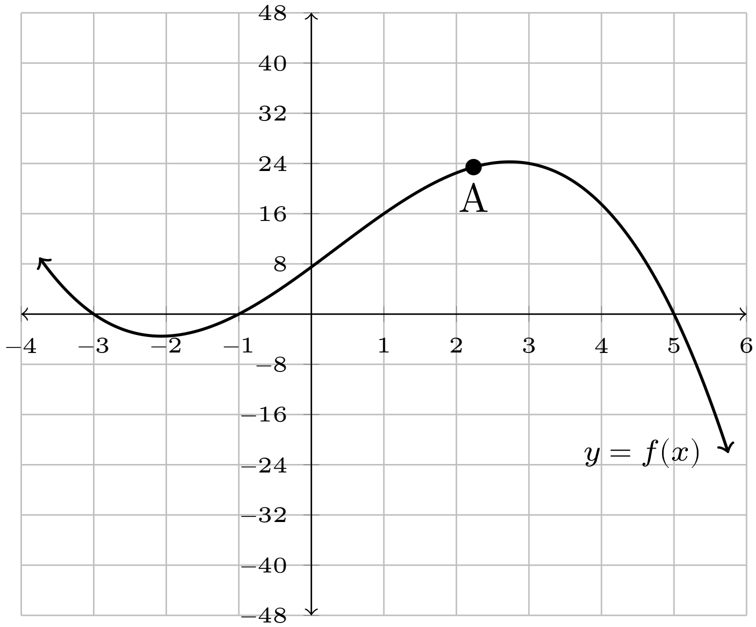
(a) $f' > 0$ and $f'' > 0$

(b) $f' = 0$ and $f'' = 0$

(c) $f' < 0$ and $f'' < 0$

(d) $f' > 0$ and $f'' < 0$

(e) $f' < 0$ and $f'' > 0$



8. Suppose the derivative of $g(t)$ is $g'(t) = t^2 - 4t - 12$. Determine all intervals on which $g(t)$ is concave up.

Possibilities:

(a) $(-\infty, -2) \cup (6, \infty)$

(b) $(2, \infty)$

(c) $(-\infty, \infty)$

(d) $(-2, 6)$

(e) $(-\infty, 2)$

9. Which of the following can you conclude about a differentiable function $h(x)$ if $h'(x) < 0$ on the interval $(-\infty, 7)$ and $h'(x) > 0$ on the interval $(7, \infty)$?

Possibilities:

(a) $h(x)$ has an inflection point at $x = 7$

(b) $h(x)$ does not attain a minimum or a maximum value

(c) $h(x)$ has no inflection points

(d) $h(x)$ attains its maximum value at $x = 7$

(e) $h(x)$ attains its minimum value at $x = 7$

10. Suppose the second derivative of $f(x)$ is $f''(x) = (x + 6)(x - 1)^2$. Determine all values of x where $f(x)$ has an inflection point. You may assume that $f(x)$ is defined for all x.

Possibilities:

(a) -1

(b) 1

(c) -6 and 1

(d) -6

(e) $f(x)$ has no inflection points

11. Determine all intervals on which the function $f(x) = \dfrac{6}{x-4}$ is increasing.

Possibilities:

(a) $(-\infty, 4)$

(b) $(4, \infty)$

(c) $(-\infty, 4) \cup (4, \infty)$

(d) $f(x)$ is never increasing

(e) $(-3, 10)$

12. Determine all critical values of the function $f(x) = 10x^3 + 30x^2$.

Possibilities:

(a) 1

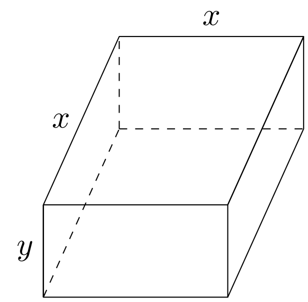
(b) 0 and 1

(c) 0

(d) -3

(e) -2 and 0

13. A person is constructing a box out of two different types of metal. The metal for the top and bottom, which are both square, costs \$7 per square foot and the metal for the sides costs \$21 per square foot. The person wants to minimize the cost to construct the box, which must have a volume of 35 cubic feet. Which of the following is a constraint for this optimization problem? In the answer choices below, C denotes cost and V denotes volume.



Possibilities:

(a) $x^2y = 35$

(b) $V = x^2y$

(c) $2x + y = 35$

(d) $C = 14x^2 + 84xy$

(e) $C = 98x + 84y$

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14. Let $f(x) = (x + 15) \cdot \ln(x + 1)$ for $x > -1$. Determine all intervals on which $f(x)$ is concave down.

Possibilities:

(a) $(13, \infty)$

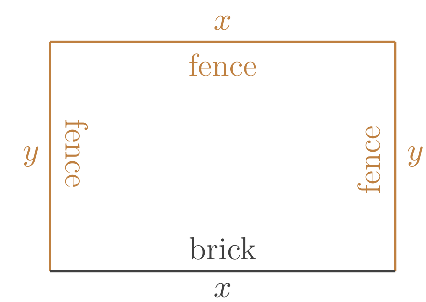
(b) $(1, 15)$

(c) $(-1, 15)$

(d) $(-1, \infty)$

(e) $(-1, 13)$

15. A landscape architect plans to enclose a rectangular garden on one side by a brick wall costing \$45 per foot and on the other three sides by a metal fence costing \$15 per foot. The area of the garden must be 600 square feet, and the architect wants to minimize the cost to enclose the garden. What is the objective function for this optimization problem? In the answer choices below, A denotes area and C denotes cost.



Possibilities:

(a) $C = 60x + 30y$

(b) $2x + 2y = 600$

(c) $C = 45x + 45y$

(d) $xy = 600$

(e) $A = xy$

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16. Given the function $f(x) = \begin{cases} 4 & \text{if } x < 3, \\ x+1 & \text{if } x \geq 3, \end{cases}$ evaluate the definite integral $\displaystyle\int_0^9 f(x)\,dx$.

Possibilities:

(a) 180

(b) 54

(c) 42

(d) 96

(e) 132

17. The graph of $y = f(x)$ is given below. The numbers shown represent the geometric area of each region. Evaluate the definite integral

$$\int_3^1 f(x)\,dx.$$

Possibilities:

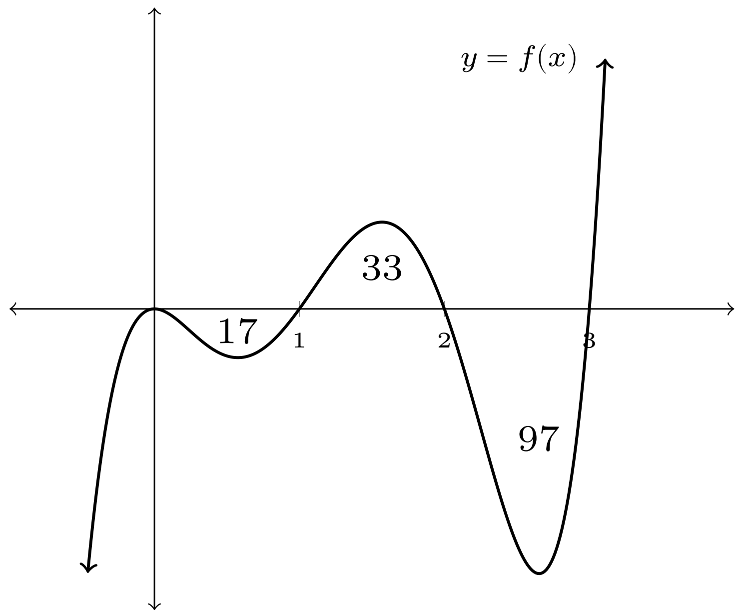
(a) -64

(b) -130

(c) -81

(d) 130

(e) 64



18. The graph of $y = f(x)$ given below consists of straight lines. Evaluate the definite integral $\int_{-5}^{4} f(x)\,dx$.

Possibilities:

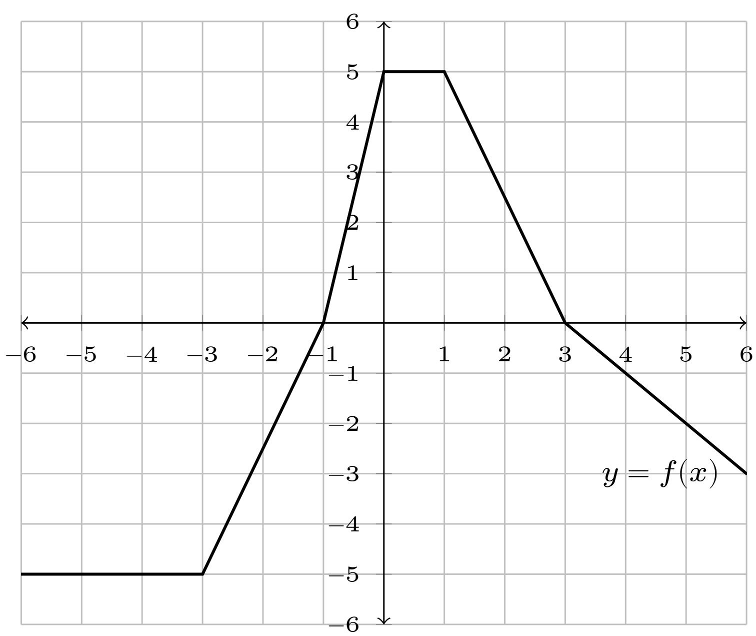
(a) 27

(b) 28

(c) -3

(d) -28

(e) -2



19. Suppose $\int_5^{15} f(x)\,dx = 9$. Determine the value of $\int_5^{15} (3f(x)+7)\ dx$.

Possibilities:

(a) 37

(b) 34

(c) 97

(d) 27

(e) 48

20. If $\int_1^{24} f(x)\,dx = 15$ and $\int_1^{14} f(x)\,dx = 27$, then determine $\int_{14}^{24} f(x)\,dx$.

Possibilities:

(a) -12

(b) 12

(c) 42

(d) -42

(e) -6

Exam 3 Formulas

$\sqrt[n]{x} = x^{1/n}$ $\qquad$ $\dfrac{1}{x^n} = x^{-n}$

If $f(x) = c$, then $f'(x) = 0$ for any constant c.

If $f(x) = x^n$, then $f'(x) = nx^{n-1}$.

$[cf(x)]' = cf'(x)$ for any constant c

$[f(x) \pm g(x)]' = f'(x) \pm g'(x)$

$[f(x)g(x)]' = f'(x)g(x) + f(x)g'(x)$

$$\left[\frac{f(x)}{g(x)}\right]' = \frac{g(x)f'(x) - f(x)g'(x)}{(g(x))^2}$$

$[g(f(x))]' = f'(x)g'(f(x))$

If $f(x) = e^x$, then $f'(x) = e^x$.

If $f(x) = \ln(x)$, then $f'(x) = \dfrac{1}{x}$.

$$\int_a^c f(x)\,dx + \int_c^b f(x)\,dx = \int_a^b f(x)\,dx$$

$\displaystyle\int_a^b kf(x)\,dx = k\int_a^b f(x)\,dx$ for any constant k

$$\int_b^a f(x)\,dx = -\int_a^b f(x)\,dx$$

$$\int_a^b [f(x) \pm g(x)]\,dx = \int_a^b f(x)\,dx \pm \int_a^b g(x)\,dx$$

$A_{\text{circle}} = \pi r^2$

$A_{\text{triangle}} = \dfrac{1}{2}bh$

$A_{\text{rectangle}} = bh$

$A_{\text{trapezoid}} = \dfrac{1}{2}(b_1 + b_2)h$

$V_{\text{rectangular solid}} = lwh$

x	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30
1	1																													
2	2	4																												
3	3	6	9																											
4	4	8	12	16																										
5	5	10	15	20	25																									
6	6	12	18	24	30	36																								
7	7	14	21	28	35	42	49																							
8	8	16	24	32	40	48	56	64																						
9	9	18	27	36	45	54	63	72	81																					
10	10	20	30	40	50	60	70	80	90	100																				
11	11	22	33	44	55	66	77	88	99	110	121																			
12	12	24	36	48	60	72	84	96	108	120	132	144																		
13	13	26	39	52	65	78	91	104	117	130	143	156	169																	
14	14	28	42	56	70	84	98	112	126	140	154	168	182	196																
15	15	30	45	60	75	90	105	120	135	150	165	180	195	210	225															
16	16	32	48	64	80	96	112	128	144	160	176	192	208	224	240	256														
17	17	34	51	68	85	102	119	136	153	170	187	204	221	238	255	272	289													
18	18	36	54	72	90	108	126	144	162	180	198	216	234	252	270	288	306	324												
19	19	38	57	76	95	114	133	152	171	190	209	228	247	266	285	304	323	342	361											
20	20	40	60	80	100	120	140	160	180	200	220	240	260	280	300	320	340	360	380	400										
21	21	42	63	84	105	126	147	168	189	210	231	252	273	294	315	336	357	378	399	420	441									
22	22	44	66	88	110	132	154	176	198	220	242	264	286	308	330	352	374	396	418	440	462	484								
23	23	46	69	92	115	138	161	184	207	230	253	276	299	322	345	368	391	414	437	460	483	506	529							
24	24	48	72	96	120	144	168	192	216	240	264	288	312	336	360	384	408	432	456	480	504	528	552	576						
25	25	50	75	100	125	150	175	200	225	250	275	300	325	350	375	400	425	450	475	500	525	550	575	600	625					
26	26	52	78	104	130	156	182	208	234	260	286	312	338	364	390	416	442	468	494	520	546	572	598	624	650	676				
27	27	54	81	108	135	162	189	216	243	270	297	324	351	378	405	432	459	486	513	540	567	594	621	648	675	702	729			
28	28	56	84	112	140	168	196	224	252	280	308	336	364	392	420	448	476	504	532	560	588	616	644	672	700	728	756	784		
29	29	58	87	116	145	174	203	232	261	290	319	348	377	406	435	464	493	522	551	580	609	638	667	696	725	754	783	812	841	
30	30	60	90	120	150	180	210	240	270	300	330	360	390	420	450	480	510	540	570	600	630	660	690	720	750	780	810	840	870	900