MA123 Final Exam

April 29 2008

NAME ______________________________ Section ___________

Problem	Answer
1	*a b c d e*
2	*a b c d e*
3	*a b c d e*
4	*a b c d e*
5	*a b c d e*
6	*a b c d e*
7	*a b c d e*
8	*a b c d e*
9	*a b c d e*
10	*a b c d e*
11	*a b c d e*
12	*a b c d e*
13	*a b c d e*
14	*a b c d e*
15	*a b c d e*

Instructions. Circle your answer in ink on the page containing the problem and on the cover sheet. After the exam begins, you may not ask a question about the exam. Be sure you have all pages (containing 15 problems) before you begin.

A list of formulas that may be useful for this exam is on the last page (you may tear off the formula page if you wish).

For grading use:

Number of problems correct: _________/15

SCORE: _________/100

NAME____

1. Find a nonnegative number A such that the average rate of change of $F(t) = t^2 - 2t + 1$ from $t = 1$ to $t = A$ equals the instantaneous rate of change of $F(t)$ at $t = 2$.

   (a) $A = 0$
   (b) $A = 2$
   (c) $A = 3$
   (d) $A = 4$
   (e) $A = 5$

2. Find
$$\lim_{x \to 5} \frac{x-5}{x^2-25}$$

   (a) $-\frac{1}{10}$
   (b) $-\frac{1}{5}$
   (c) 0
   (d) $\frac{1}{5}$
   (e) $\frac{1}{10}$

3. Which of the following is true for the function $f(x)$ given by
$$f(x) = \begin{cases} 2x-1 & \text{if } x < -1 \\ x^2+1 & \text{if } -1 \leq x \leq 1 \\ x+1 & \text{if } x > 1 \end{cases}$$

   (a) f is continuous everywhere
   (b) f is continuous everywhere except at $x = -1$ and $x = 1$
   (c) f is continuous everywhere except at $x = -1$
   (d) f is continuous everywhere except at $x = 1$
   (e) None of the above

4. Suppose an object is dropped from a cliff and its speed in ft/sec. after t seconds is given by

$$v(t) = 10t - 5$$

If the object lands after 4 seconds, how high (in ft) is the cliff?
(Hint: how far did the object travel?)

(a) 10
(b) 20
(c) 30
(d) 60
(e) 100

5. Find the limit of $f(x)$ as x tends to 2 from the left if

$$f(x) = \begin{cases} x^3 - 2 & \text{if} \quad x \geq 2 \\ 1 + x^2 & \text{if} \quad x < 2 \end{cases}$$

(a) 5
(b) 6
(c) 7
(d) 8
(e) Does not exist

6. Suppose $f(t) = g(3t)$ and $F(t) = f(t)g(t)$. If $g(1) = 1$, $g(3) = 5$, $g'(1) = 2$ and $g'(3) = 7$, what is $F'(1)$?

(a) 14
(b) 17
(c) 24
(d) 31
(e) 42

7. Suppose the derivative of $G(q)$ is given by $G'(q) = q^2(q+1)^2(q+2)^2$. Find the value of q in the interval $[-5, 5]$ where $G(q)$ takes on its maximum.

   (a) -5
   (b) -2
   (c) -1
   (d) 0
   (e) 5

8. What is the maximum area of the rectangle with sides parallel to the coordinate axes, one corner at the origin, and the opposite corner in the first quadrant on the ellipse given by the equation $2x^2 + y^2 = r^2$?

   (a) r^2
   (b) $\frac{r^2}{\sqrt{2}}$
   (c) $\frac{r^2}{2}$
   (d) $\frac{r^2}{2\sqrt{2}}$
   (e) $\frac{r^2}{4}$

9. Evaluate the limit
$$\lim_{x\to\infty} \frac{(4x+1)^2}{4x^2+3}$$

   (a) 1
   (b) 2
   (c) 4
   (d) 8
   (e) Does not exist

10. Suppose you are given the following data points for a function $f(x)$.

x	1	2	3	4
$f(x)$	2	4	8	12

If f is a linear function on each interval between the given points, find

$$\int_1^4 f(x)\,dx.$$

(Hint: draw a picture)

(a) 13
(b) 19
(c) 20
(d) 26
(e) 40

11. Find

$$\int \frac{x^3+1}{x^2}\,dx$$

(a) $\frac{(x^4/4)+x}{(x^3/3)} + C$
(b) $\frac{x^3+2}{2x} + C$
(c) $\frac{(x^4/4)-x}{(x^3/3)} + C$
(d) $\frac{x^3-2}{2x} + C$
(e) $\frac{x^4+x}{x^3} + C$

12. A stone is dropped into a pond and causes a circular ripple. If the radius of the circle increases at a rate of 0.25 ft/sec., how fast does the area increase (in ft^2/sec.) when the radius equals 0.4 ft.?

(a) 0.2π
(b) 0.4π
(c) 0.8π
(d) 0.16π
(e) 0.64π

13. If

$$F(x) = \int_2^x \left(t^2 + 4t\right)\, dt,$$

find $F'(3)$.

(a) $\frac{49}{3}$
(b) 21
(c) $\frac{64}{3}$
(d) 27
(e) 36

14. Find

$$\lim_{n\to\infty} \frac{1+2+3+4+5+6+\ldots+n}{2n^2}$$

(a) 0
(b) $\frac{1}{8}$
(c) $\frac{1}{4}$
(d) $\frac{1}{2}$
(e) Does not exist

15. Find

$$\int_0^{12} \left(s^2 + 3s + 1\right)\, ds$$

(a) 181
(b) 804
(c) 132
(d) 55
(e) 1

Geometric Formulas

1. Areas
   (a) Triangle $A = \frac{bh}{2}$
   (b) Circle $A = \pi r^2$
   (c) Rectangle $A = lw$
   (d) Trapezoid $A = \frac{a+b}{2}h$
2. Volumes
   (a) Rectangular Solid $V = lwh$
   (b) Sphere $V = \frac{4}{3}\pi r^3$
   (c) Cylinder $V = \pi r^2 h$
   (d) Cone $V = \frac{1}{3}\pi r^2 h$
3. Summation

$$\sum_{k=1}^{n} k = \frac{n(n+1)}{2}$$

$$\sum_{k=1}^{n} k^2 = \frac{n(n+1)(2n+1)}{6}$$