1. Find the **average rate of change** of $f(x)=\sqrt{3x+1}$ from $x=2$ to $x=8$.

$$\text{AROC} = \frac{f(8)-f(2)}{8-2} = \frac{\sqrt{3(8)+1} - \sqrt{3(2)+1}}{6}$$

$$= \frac{\sqrt{25} - \sqrt{7}}{6}$$

$$= \boxed{\frac{5-\sqrt{7}}{6}}$$

2. For what value of b is the function $f(x)$ **continuous** at $x=3$?

$$f(x)=\begin{cases} x^2+b & x\le 3 \\ bx+5 & x>3 \end{cases}$$

$$\lim_{x\to 3^-} f(x) = \lim_{x\to 3^-} (x^2+b) = 3^2+b = 9+b$$

$$\lim_{x\to 3^+} f(x) = \lim_{x\to 3^+} (bx+5) = 3b+5$$

For $\lim_{x\to 3} f(x)$ to exist, we require $3b+5 = 9+b$

$\Rightarrow 2b = 4$

$\Rightarrow \boxed{b=2}$

3. Use the **limit definition of the derivative** to find $f'(x)$ for $f(x)=3x^2+x-5$. (You must use the limit definition to receive credit.)

$$f'(x) = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h}$$

$$= \lim_{h\to 0} \frac{3(x+h)^2 + x+h-5 - (3x^2+x-5)}{h}$$

$$= \lim_{h\to 0} \frac{3(x^2+2xh+h^2) + x + h - 5 - 3x^2 - \cancel{x} \cancel{+5}}{h}$$

$$= \lim_{h\to 0} \frac{\cancel{3x^2} + 6xh + 3h^2 + h - \cancel{3x^2}}{h}$$

$$= \lim_{h\to 0} \frac{h(6x+3h+1)}{h} = \lim_{h\to 0} (6x+3h+1)$$

$$= 6x + 3(0) + 1 = \boxed{6x+1}$$

4. Suppose a **linear function** f satisfies $f(4)=5$ and $f(-2)=3$. Find $f'(2)$.

f includes the points $(4,5)$ and $(-2,3)$.

f is a line with slope $m = \frac{5-3}{4-(-2)} = \frac{2}{6} = \frac{1}{3}$

$f'(2)$ = slope of the tangent line at $x=2$

$= \boxed{\frac{1}{3}}$