Math 123 Recitation Worksheet 1A

1. Simplify the following:
   a. $\left(x^2\right)^7$
   b. $x^2\left(x^3-3x\right)-4x^5+9x^3$
   c. $\dfrac{x^{15}}{x^3}$
   d. $\sqrt[3]{x^{15}}$

2. Solve for y: $8x^3y+2xy-5=6x^2$

3. Plot these points in the $xy-$plane:
   A $(0,0)$ B $(5,0)$ C $(0,-4)$ D $(-3,5)$

4. Without using a calculator, sketch a graph of each of these in the $xy-$plane
   A $y=x$ B $y=3$ C $x=3$ D $y=x^2$ E $y=\dfrac{1}{x}$

5. A line L goes through the points $(-3,7)$ and $(1,-5)$.
   a. Find the slope of L.
   b. Find the equation of L using the point-slope form.
   c. Find the slope of a line perpendicular to L.

1. Consider the function

$$f(x)=\begin{cases}2x+1 & x<1\\ x^2-1 & x\geq 1\end{cases}$$

   a. Evaluate $f(0)$, $f(1)$ and $f(2)$.

   b. Sketch a graph of $y=f(x)$.

2. Find the points of intersection between the graphs of $2x^2+3y^2=66$ and $y=4$.

3. Let $f(x)=\sqrt{4x-3}$.

   a. Evaluate $f(7)$, $f(1)$, $f(10)$ and $f(0)$.

   b. Find the domain of $f(x)$.

   c. Find the range of $f(x)$.

   d. Find $f(x+h)$.

   e. Find the inverse of $f(x)$.

1. A train leaves city A at 10:00 a.m. and arrives at city B at 12:15 p.m. The train leaves city B at 2:00 p.m. and arrives at city C three hours later. The average velocity of the train while traveling from A to B was 45 miles per hour. The distance between city B and city C is 240 miles. What is the average velocity of the train from city A to city C (including the stop)?

2. A train leaves city A at 8:00 a.m. and arrives at city B at 10:00 a.m. The average velocity of the train from A to B was 60 miles per hour. The train leaves city B at 10:00 a.m. and arrives at city C at 1:00 p.m. Find the average velocity of the train from city B to C, given that the average velocity from A to C was 50 miles per hour.

3. Let $f(x)=\dfrac{3}{x^2+1}$.

   a. Find the average rate of change of $f(x)$ from $x=0$ to $x=2$.

   b. Draw the graph of $y=f(x)$ (a graphing calculator can help). Show how to represent your answer to part (a) on the graph.

4. Find a positive number A so that the average rate of change of $g(x)=3x^2-1$ from $x=2$ to $x=A$ is equal to 33.

1. Let $g(x) = x^2 - 4x$.
   a. Find the value of x for which the tangent line to $y = g(x)$ has slope equal to 6.
   b. Find the value of $g(x)$ at the point where the tangent line to $y = g(x)$ is parallel to $y = 2x + 5$.
   c. Find a value of x so that the instantaneous rate of change of g at x is equal to the average rate of change of g from $x = -1$ to $x = 3$.

2. An object is launched up in the air. The height of the object after t seconds is $P(t)$ feet, where $P(t) = -16t^2 + 256t + 64$.
   a. When is the object at its greatest height? (Hint: What must be true about the velocity of the object when it is at the greatest height?)
   b. What is the maximum height of the object?

3. Suppose $q(x) = 3x^2 - 12x + 8$ and $p(x) = 3x^2 - 12x + 5$.
   a. Find $q'(x)$ and $q'(1)$.
   b. Find the equation of the tangent line to $y = q(x)$ at $x = 1$.
   c. Find $p'(x)$ and $p'(1)$.
   d. Find the equation of the tangent line to $y = p(x)$ at $x = 1$.
   e. What do you notice when you compare your answers? Draw the graphs of $y = p(x)$ and $y = q(x)$ and explain what you've found.

1. Find each of the following limits.

   a. $\lim_{t\to 3}(4t+7)$

   b. $\lim_{x\to 1}\dfrac{x^2-5x+6}{x^2-3x+1}$

2. Let $f(x)=\begin{cases} x^2+2 & x\le 1 \\ -3x+1 & x>1 \end{cases}$

   Sketch the graph of $y=f(x)$ and use it to find the following:

   a. $f(1)$

   b. $\lim_{x\to 1^-} f(x)$

   c. $\lim_{x\to 1^+} f(x)$

   d. $\lim_{x\to 1} f(x)$

   e. $f(2)$

   f. $\lim_{x\to 2^-} f(x)$

   g. $\lim_{x\to 2^+} f(x)$

   h. $\lim_{x\to 2} f(x)$

3. Sketch a graph of $y=|x|$ and use it to find $\lim_{x\to 0^-} f(x)$, $\lim_{x\to 0^+} f(x)$ and $\lim_{x\to 0} f(x)$.

4. Sketch a graph of $y=\dfrac{|x|}{x}$ and use it to find $\lim_{x\to 0^-} f(x)$, $\lim_{x\to 0^+} f(x)$ and $\lim_{x\to 0} f(x)$.

1. Compute each of the following limits.

   a. $\lim_{x\to 2}\dfrac{x^2-5x+6}{x^2-3x+2}$

   b. $\lim_{x\to 2}\dfrac{x^2-4x+4}{x^2-4}$

   c. $\lim_{h\to 0}\dfrac{(5+2h)^2-25}{h}$

   d. $\lim_{t\to 0}\left(\dfrac{2}{t}+\dfrac{7t-4}{2t}\right)$

   e. $\lim_{h\to 0}\dfrac{(x+h)^2-x^2}{h}$

   f. $\lim_{x\to 0}\dfrac{x^2-3x}{x^2-6x}$

   g. $\lim_{x\to 5}\dfrac{x^2+1}{x-5}$

   h. $\lim_{x\to 0^+}\dfrac{27x}{\sqrt{x}}$

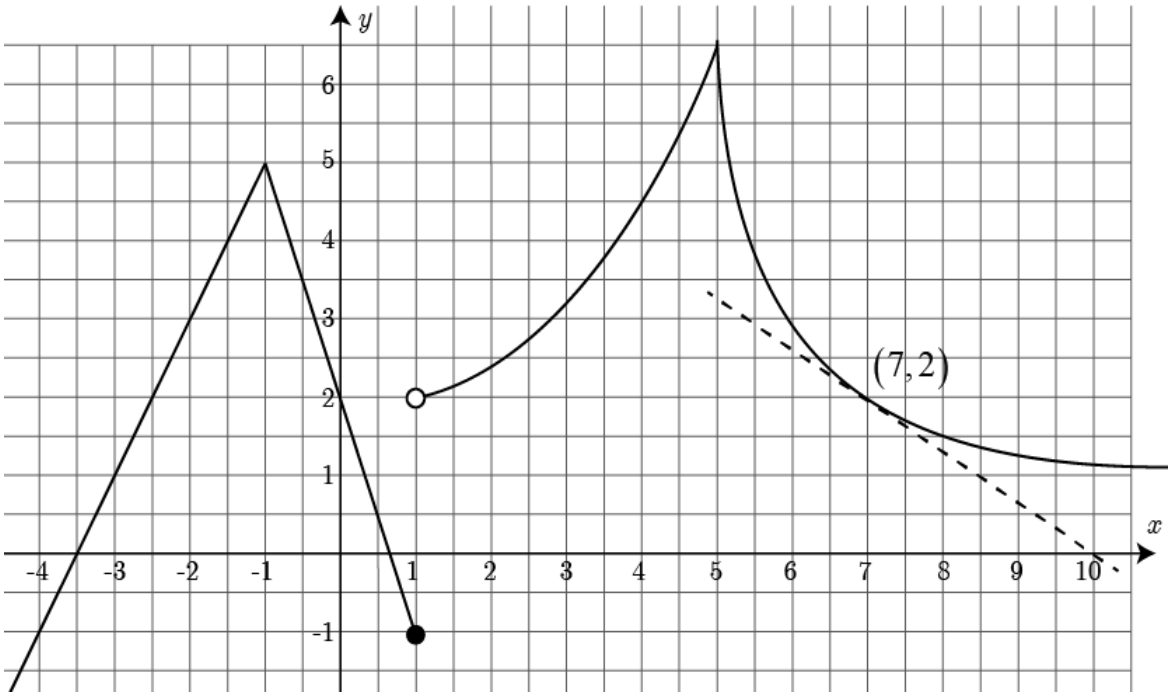
2. Refer to Recitation Worksheet 3A problem 2.
   a. Is $f(x)$ continuous at $x=1$?
   b. Is $f(x)$ continuous at $x=2$?

3. Refer to Recitation Worksheet 3A problem 3. Is $y=|x|$ continuous at $x=0$?

4. Let $g(x)=\begin{cases} x-1 & x<2 \\ x^2-A^2 & x\geq 2 \end{cases}$

   a. Sketch the graph of $y=g(x)$ using $A=0$. Is $g(x)$ continuous?
   b. Sketch the graph of $y=g(x)$ using $A=1$. Is $g(x)$ continuous?
   c. Sketch the graph of $y=g(x)$ using $A=2$. Is $g(x)$ continuous?
   d. Do you think there is a real value of A which makes $g(x)$ continuous? If so, what is A? If not, why not?

1. The graph of $y = f(x)$ is shown below, and the tangent line at $x = 7$ is indicated.



   a. Find $f'(-2)$, $f'(0)$, and $f'(7)$.

   b. For which values of x is $f(x)$ not continuous?

   c. For which values of x is $f(x)$ not differentiable?

2. Let $g(x) = |x^2 + 2x - 15|$. Find all points where $g(x)$ is not differentiable.

On these problems you will use the **limit definition of the derivative**,

$f'(x)=\lim_{h\to 0}\dfrac{f(x+h)-f(x)}{h}$, to compute each of the following derivatives.

For each function do the following steps:

(a) Find the difference quotient $\dfrac{f(x+h)-f(x)}{h}$

(b) Simplify your answer to part (a) using algebra

(c) Take the limit as $h\to 0$ to compute $f'(x)$.

1. $f(x)=x^2+5x$
2. $f(x)=\sqrt{x+5}$
3. $f(x)=\dfrac{1}{x+3}$
4. $f(x)=\sqrt{3x-2}$
5. $f(x)=\dfrac{7}{x-4}$