I. First rewrite the function in the form $y = ax^n$. Then find the derivative.

1. $y = \dfrac{5}{x^3}$
2. $y = \sqrt[3]{x^{10}}$
3. $y = \dfrac{1}{5x^3}$
4. $y = \dfrac{7}{6\sqrt[5]{x^8}}$

II. Rewrite if necessary until you have the sum of a few terms, each of the form ax^n. Then find the derivative. (**Do not** use the product or quotient rule for these.)

5. $y = \dfrac{x^3 - 3x^2 + 5x + 2}{x^2}$
6. $y = x^2\left(x^3 + \sqrt{x} - \dfrac{1}{x^9} + 15\right)$

III. Find the derivative. You *will* want the product or quotient rule. **Do not simplify** your answer.

7. $y = (3x^2 + 2x - 3)(5x^7 + 4x^3 - 2x + 1)$
8. $y = \dfrac{8x^4 + 17}{7x^3 + 2x - 1}$

IV. Suppose the functions $f(x)$ and $g(x)$ and their derivatives have the following values at $x = 1$:

x	$f(x)$	$g(x)$	$f'(x)$	$g'(x)$
1	6	2	-7	5

9. Find $h'(1)$ if $h(x) = f(x)g(x)$

10. Find $h'(1)$ if $h(x) = \dfrac{f(x) + g(x)}{3x + 1}$.

I. Find the derivative of each of the following. Do not simplify your answers.

1. $y = \dfrac{5}{\sqrt[7]{3x-5}}$ (Rewrite first!)

2. $y = \left(x^3+6\right)^{23}$

3. $y = \left(\left(x^2+1\right)^4+3\right)^6+5x+10$

II. Suppose f and g and their first derivatives have the following values at $x=2$ and $x=4$:

x	$f(x)$	$g(x)$	$f'(x)$	$g'(x)$
2	5	4	7	-3
4	1	-2	9	8

a. Find $h'(2)$ if $h(x) = \sqrt{f(x)+g(x)}$

b. Find $h'(2)$ if $h(x) = f\left(g(x)\right)$

III. Suppose f and g and their first derivatives have the following values at $x=1$ and $x=2$:

x	$f(x)$	$g(x)$	$f'(x)$	$g'(x)$
1	6	1	-7	$1/2$
2	3	-1	$1/2$	-4

Find $h'(2)$ if $h(x) = f\left(x+g(x)\right)$.

Then find the equation of the tangent line to the graph of $y=h(x)$ at $x=2$.

IV. Find the third derivative of $y=\sqrt{3x+2}$.

I. Find the derivative of each of the following. **Do not simplify** your answers.

1. $y = x^4 + x^e + e^x + e^\pi + \ln x + \ln 7$
2. $y = (3x + \ln x)e^x$
3. $y = \dfrac{\ln x}{x^3 - 2x}$
4. $y = e^{x^4 + 2x^3 + 7}$
5. $y = \ln\left(x^3 + 5x - 2\right)$
6. $y = \sqrt{\ln(8x + 20)}$
7. $y = \ln\left(\ln\left(x^2 + e^x\right)\right)$
8. $y = \ln\left(x^5 e^x\right)$ (simplify with logarithm properties before you differentiate.)

II. Find **and simplify** the **second** derivative of $y = e^x(5x + 2)$.

III. Suppose $g(4) = 7$ and $g'(4) = -6$. Find $h'(4)$ if $h(x) = \ln\left(x^2 + g(x)\right)$.

1. Suppose \$10,000 is invested at an annual interest rate of 5% compounded continuously.
   a. How long will it take for the investment to double in value?
   b. How long will it take for the investment to triple in value?

2. A recent college graduate decides he would like to have \$20,000 in five years to make a down payment on a home.
   a. How much money will he need to invest today in order to have \$20,000 in five years, given that he can invest at an annual interest rate of 4% compounded continuously?
   b. Suppose instead the best interest rate he can find is only 2.5% (instead of 4%). Now how much will he need to invest?
   c. Suppose the interest rate is 4% again, but now he would like to have the \$20,000 in only four years. How much does he need to invest?

3. The half-life of caffeine is 5 hours. This means the amount of caffeine in your bloodstream is reduced by 50% every five hours. A grande French Roast has 330 mg of caffeine. Let $Q(t)$ denote the amount of caffeine in your system t hours after drinking your grande French Roast. (For simplicity, assume the entire drink is consumed instantly.)
   a. How many milligrams of caffeine will be in your system after 5 hours? after 10 hours?
   b. Let $Q(t) = Q_0 e^{-kt}$. Find Q_0 and k.
   c. How many milligrams of coffee will be in your system after 2 hours?

4. A bacteria culture triples in size every 7 hours. Three hours from now, the culture will have 8000 bacteria. If $Q(t)$ denotes the number of bacteria at time t, then $Q(t) = Q_0 e^{kt}$. Find Q_0 and k.

5. The graph of $y = f(x)$ passes through the point $(0,4)$. The slope of f at any point P is three times the $y-$coordinate of P. Find an expression for $f(x)$, and find $f(2)$.

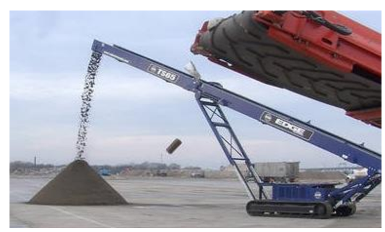
1. Two trains leave a station at 1:00 p.m. One train travels north at 70 miles per hour. The other train travels east at 50 miles per hour. How fast is the distance between the two trains changing at 4:00 p.m.?

2. Suppose the height of a certain right triangle is always twice its base. Suppose the base of the triangle is expanding at a rate of 5 inches per second. When the base of the triangle is 15 inches long,
   a. how fast is the height of the triangle increasing?
   b. how fast is the length of the hypotenuse of the triangle increasing?
   c. how fast is the area of the triangle increasing?

3. A spherical balloon is being filled at a rate of 100 cubic centimeters per minute. When the radius is 50 cm,
   a. how fast is the radius increasing?
   b. how fast is the surface area increasing?

4. The price of a share of stock is increasing at a rate of $7 per year. An investor is buying stock at a rate of 20 shares per year. Find the rate at which the value of the investor's stock is increasing when the current price of the stock is $40 per share, and the investor owns 100 shares.

5. Sand falls from a conveyor belt at the rate of 9 m^3/min onto the top of a conical pile. The height of the pile is always three-fourths of the radius of the base. How fast is the radius changing when the pile is 4 m high?



(Image by Stephen Farrel, nytimes.com)

Useful Formulas:

Area of a triangle, $A = \frac{1}{2}bh$.

Volume of a sphere: $V = \frac{4}{3}\pi r^3$. Surface area of a sphere: $SA = 4\pi r^2$.

Volume of a cone: $V = \frac{1}{3}\pi r^2 h$.

1. Let $f(x) = x^3 + 3x^2 - 45x + 18$.
   a. Find the critical numbers (the $x-$values where $f'(x) = 0$ or $f'(x)$ DNE), if any.
   b. Use your answer to part (a) find the maximum and minimum values of $f(x)$ on the interval $[2,5]$.

2. Let $g(x) = \ln(x^2 - 8x + 20)$. Find the critical numbers, if any, and use them to find maximum and minimum values of $f(x)$ on the interval $[0,10]$.

3. Let $h(x) = \begin{cases} x^2 + 2x + 3 & x \le 1 \\ x^2 - 4x + 9 & x > 1 \end{cases}$.
   a. Is $h(x)$ continuous at $x = 1$?
   b. Is $h(x)$ differentiable at $x = 1$?
   c. Find the critical numbers of $h(x)$. (Hint: there are three.)
   d. Find the maximum and minimum values of $h(x)$ on the interval $[0,2]$.
   e. Find the maximum and minimum values of $h(x)$ on the interval $[-2,5]$.

1. On the same graph, plot both $f(x)=x^3-3x-5$ and its derivative on the interval $[-4,4]$. What do you notice? In particular, what appears to be true about $f(x)$ when its derivative is zero? What appears to be true about $f(x)$ when its derivative is positive? is negative?

2. Let $g(x)=\dfrac{x+4}{x+9}$.
   a. Find the critical numbers of $g(x)$, if any.
   b. Find the maximum and minimum value of $g(x)$ on the interval $[1,6]$.

3. Let $h(x)=e^x(x-5)$.
   a. Find the critical numbers of $h(x)$, if any.
   b. Find the maximum and minimum value of $h(x)$ on the interval $[0,6]$.

4. Let $g(x)=x^2+3x+1$. Find a value c in the interval $[3,9]$ such that $g'(c)$ equals the average rate of change of $g(x)$ on the interval $[3,9]$.