1. Suppose $f(x) = (x-1)(x-4)(x-9) = x^3 - 14x^2 + 49x - 36$. Find the intervals on which $f(x)$ is increasing and the intervals on which $f(x)$ is decreasing.

2. Suppose $g'(x) = (x-1)(x-4)(x-9) = x^3 - 14x^2 + 49x - 36$. Find the intervals on which $g(x)$ is increasing and the intervals on which $g(x)$ is decreasing.

3. Suppose $h(x) = \frac{1}{(2x-10)^2}$. Find the largest value of A for which the function $h(x)$ is increasing for all x in the interval $(-\infty, A)$.

4. Suppose $f'(x) = \frac{-5}{(x-3)^2}$. Find the value of x in the interval $[-20, 2]$ on which $f(x)$ takes its maximum.

5. Suppose we know that $g(8) = -3$. In addition, you are given that $g(x)$ is continuous everywhere, and is increasing on the interval $(-\infty, 10)$ and decreasing on the interval $(10, \infty)$. Which of the following are possible, and which are not possible? *Hint*: draw a graph in each case.
   a. g has a local minimum at $x = 8$
   b. g has a local maximum at $x = 10$
   c. $g(0) = -5$
   d. $g(0) = 5$
   e. $g(0) = -6$ and $g(1) = -4$
   f. $g(0) = -4$ and $g(1) = -6$
   g. $g(0) = -4$ and $g(12) = -4$

6. Sketch the graph of a function which is continuous and differentiable everywhere, is increasing on the intervals $(-\infty, -2)$ and $(5, 7)$, and is decreasing on the intervals $(-2, 5)$ and $(7, \infty)$.

1. Suppose $f(x)=(x-1)(x-4)(x-9)=x^3-14x^2+49x-36$. Find the intervals on which $f(x)$ is concave up and the intervals on which $f(x)$ is concave down.

2. Suppose $g'(x)=(x-1)(x-4)(x-9)=x^3-14x^2+49x-36$. Find the intervals on which $g(x)$ is concave up and the intervals on which $g(x)$ is concave down.

3. Suppose $h(x)=xe^x$. Find intervals where $h(x)$ is concave up and the intervals on which $h(x)$ is concave down.

4. Sketch the graph of a continuous function $y=f(x)$ which satisfies the following:

$f'>0$ for x in $(-\infty,-1)$ and $(3,5)$; $f'<0$ for x in $(-1,3)$ and $(5,\infty)$

$f''>0$ for x in $(2,5)$ and $(5,\infty)$; $f''<0$ for x in $(-\infty,2)$

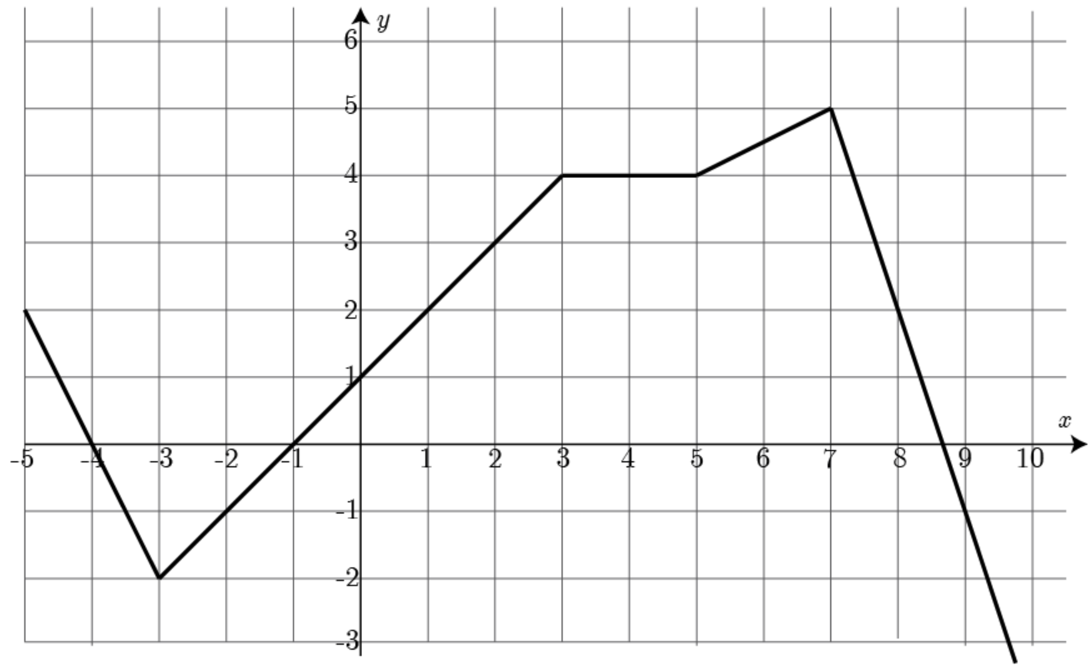
$f(0)=5$, $f(3)=1$

1. The product of two positive real numbers x and y is 24. Find the minimal value of the expression $3x + 2y$.

2. Stacy has $400 to spend on materials for a fencing project. She needs to fence in a rectangular portion of her yard. For the fencing along the front and back she can use cheap materials costing $5 per foot. However, for the sides (which are visible to the neighbors) she must use a more expensive type of fencing which costs $15 per foot. What dimensions should the fence be in order to enclose the largest area possible?

3. A manufacturer has been selling 1000 televisions a week at $450 each. A survey indicates that for each $10 the price is lowered, the number of sets sold will increase by 100 per week. How large a rebate should the company offer the buyer in order to maximize its revenue?

1. Compute each integral using geometry, given the graph of $y = f(x)$ below:



   a. $\int_3^5 f(x)\,dx$

   b. $\int_0^7 f(x)\,dx$

   c. $\int_{-4}^{-1} f(x)\,dx$

   d. $\int_{-3}^{3} f(x)\,dx$

   e. $\int_{-5}^{8} f(x)\,dx$

2. Evaluate each integral by interpreting it in terms of areas. Include a sketch of the graph of the integrand, shading the appropriate area.

   a. $\int_{-1}^{1} (1-|x|)\,dx$

   b. $\int_0^5 (8-2x)\,dx$

   c. $\int_{-6}^{0} \sqrt{36-x^2}\,dx$

   d. $\int_0^6 \left(6-\sqrt{36-x^2}\right)dx$

1. Suppose $\int_2^{12} g(x)\,dx = 5$ and $\int_4^{12} g(x)\,dx = 9$. Find the value of $\int_2^4 3g(x)\,dx$.

2. Suppose that $\int_1^9 f(x)\,dx = -2$ and $\int_1^7 f(x)\,dx = 4$. Find the following values.

   a. $\int_7^1 5f(x)\,dx$

   b. $\int_7^9 f(x)\,dx$

   c. $\int_1^7 \left(4f(x) - 2\right)dx$

3. Suppose we are given $f(x) = \begin{cases} 3 & x \le 4 \\ 15 - 3x & x > 4 \end{cases}$.

   a. Sketch the graph of $y = f(x)$.

   b. Use your graph to evaluate $\int_1^6 f(x)\,dx$

   c. Find the average value of $f(x)$ on the interval [1, 6].

1. Estimate the area under the curve $y = x^2$ on the interval $[0,4]$ in five different ways:

   a. Divide $[0,4]$ into four equal subintervals, and use the left endpoint on each subinterval as the sample point.

   b. Divide $[0,4]$ into four equal subintervals, and use the right endpoint on each subinterval as the sample point.

   c. Divide $[0,4]$ into four equal subintervals, and use the midpoint of each subinterval as the sample point.

   d. Divide $[0,4]$ into eight equal subintervals, and use the left endpoint on each subinterval as the sample point.

   e. Divide $[0,4]$ into eight equal subintervals, and use the right endpoint on each subinterval as the sample point.

   For each of the above, draw a rough sketch. Use your sketch to help determine which estimates will give areas that are larger than the desired area, and which will give areas smaller than the desired area.

1. Suppose we estimate the area under the graph $f(x) = 2^x$ from $x = 1$ to $x = 16$ by partitioning the interval into 30 equal subintervals and using the right endpoint of each interval to determine the height of the rectangle. What is the area of the 12th rectangle?

2. A Mustang can accelerate from 0 to 88 feet per second in 5 seconds (i.e., 0 to 60 miles per hour in 5 seconds). The velocity of the Mustang is measured each second and recorded in the table below. You should assume the velocity is increasing throughout the entire 5 second period. The distance traveled equals the area under the velocity curve. You can estimate this area using left endpoints or right endpoints.

t	0	1	2	3	4	5
$v(t)$	0	22	52	73	81	88

   a. Draw a picture to help you decide which will give an overestimate of the distance traveled and which will give an underestimate of the distance traveled.
   b. What is the longest distance the Mustang could have traveled from $t = 0$ to $t = 5$?
   c. What is the shortest distance the Mustang could have traveled from $t = 0$ to $t = 5$?

3. A train travels in a straight westward direction along a track. The velocity of the train varies, but is measured at regular time intervals of 1/10 hour. The measurements for the first half hour are

time	0	0.1	0.2	0.3	0.4	0.5
velocity	0	8	13	17	20	22

   Estimate the distance traveled by the train over the first half hour assuming that the speed of the train is a linear function on each of the subintervals. The velocity in the table is given in miles per hour.