`Chapter 3: Practice/review problems`

The collection of problems listed below contains questions taken from previous MA123 exams.

Limits and one-sided limits

[1]. Suppose $H(t) = t^2 + 5t + 1$. Find the limit $\lim_{t\to 2} H(t)$.

$\boxed{\textbf{(a)}}$ 15 **(b)** 1 **(c)** 9 **(d)** 6 **(e)** $2t + 5$

[2]. Find the limit $\lim_{t\to 2} \dfrac{t^2 - 4}{t - 2}$.

(a) 2 $\boxed{\textbf{(b)}}$ 4 **(c)** 6 **(d)** 8 **(e)** The limit does not exist

[3]. Find the limit $\lim_{x\to 5} \dfrac{x - 5}{x^2 - 25}$.

(a) $-\dfrac{1}{10}$ **(b)** $-\dfrac{1}{5}$ **(c)** 0 **(d)** $\dfrac{1}{5}$ $\boxed{\textbf{(e)}}$ $\dfrac{1}{10}$

[4]. Compute $\lim_{x\to 3} \dfrac{x^2 - 7x + 12}{x - 3}$.

(a) 0 **(b)** 1 $\boxed{\textbf{(c)}}$ -1 **(d)** 2 **(e)** The limit does not exist

[5]. Find $\lim_{r\to 1} \dfrac{r^2 - 3r + 2}{r - 1}$.

(a) 1 **(b)** 0 $\boxed{\textbf{(c)}}$ -1 **(d)** 2 **(e)** The limit does not exist

[6]. Find the limit or state that it does not exist: $\lim_{x\to 4} \dfrac{x^2 + x - 20}{x - 4}$.

(a) 8 **(b)** -20 **(c)** -15 $\boxed{\textbf{(d)}}$ 9 **(e)** Does Not Exist

[7]. Compute $\lim_{x\to 0} \left(\dfrac{2x^2 - 3x + 4}{x} + \dfrac{5x - 4}{x} \right)$.

(a) 5 **(b)** 4 **(c)** 3 $\boxed{\textbf{(d)}}$ 2 **(e)** 1

[8]. Compute $\lim_{h\to 0} \dfrac{(h + 4)^2 - 16}{h}$.

(a) 4 **(b)** 5 **(c)** 6 **(d)** 7 $\boxed{\textbf{(e)}}$ 8

[9]. Find the limit $\lim_{t\to 0^+} \frac{\sqrt{t^3}}{\sqrt{t}}$.

$\boxed{\textbf{(a)}}$ 0 **(b)** 1 **(c)** 2 **(d)** 3 **(e)** The limit does not exist

[10]. Find the limit as x tends to 0 from the left $\lim_{x\to 0^-} \frac{|x|}{2x}$.

(a) 1/3 **(b)** 1/2 **(c)** 0 $\boxed{\textbf{(d)}}$ −1/2 **(e)** −1/3

[11]. Find the limit $\lim_{h\to 0^-} \frac{|4h|}{h}$.

(**Hint:** Evaluate the quotient for some negative values of h close to 0.)

(a) 0 **(b)** 2 **(c)** −2 **(d)** 4 $\boxed{\textbf{(e)}}$ −4

[12]. Compute $\lim_{x\to 3^-} \frac{|4x-12|}{x-3}$.

(a) 4 $\boxed{\textbf{(b)}}$ −4 **(c)** 0 **(d)** Doesn't exist **(e)** Cannot be determined

[13]. Find the limit of $f(x)$ as x tends to 2 from the left if $f(x) = \begin{cases} 1+x^2 & \text{if } x<2 \\ x^3 & \text{if } x \geq 2 \end{cases}$

$\boxed{\textbf{(a)}}$ 5 **(b)** 6 **(c)** 7 **(d)** 8 **(e)** 9

[14]. Find the limit of $f(x)$ as x tends to 2 from the left if $f(x) = \begin{cases} x^3-2 & \text{if } x \geq 2 \\ 1+x^2 & \text{if } x<2 \end{cases}$

$\boxed{\textbf{(a)}}$ 5 **(b)** 6 **(c)** 7 **(d)** 8 **(e)** Does not exist

[15]. For the function $f(x) = \begin{cases} 4x^2-1 & \text{if } x<1 \\ 3x+2 & \text{if } x \geq 1 \end{cases}$

Find $\lim_{x\to 1^+} f(x)$.

$\boxed{\textbf{(a)}}$ 5 **(b)** 3 **(c)** 1 **(d)** 0 **(e)** The limit does not exist

[16]. Let $f(x) = \begin{cases} x^2+8x+15 & \text{if } x \leq 2 \\ 4x+7 & \text{if } x > 2. \end{cases}$

Find $\lim_{x\to 2^+} f(x)$.

$\boxed{\textbf{(a)}}$ 15 **(b)** 20 **(c)** 30 **(d)** 35 **(e)** The limit does not exist

[17]. Let $f(x) = \begin{cases} -5x+7 & \text{if } x < 3 \\ x^2 - 16 & \text{if } x \geq 3. \end{cases}$

Find $\lim_{x \to 3^+} f(x)$.

(a) 6 **(b)** -6 $\boxed{\textbf{(c)}}$ -7 **(d)** -8 **(e)** The limit does not exist

[18]. Suppose $f(t) = \begin{cases} -t & \text{if } t < 1 \\ t^2 & \text{if } t \geq 1 \end{cases}$

Find the limit $\lim_{t \to 1} f(t)$.

(a) -1 **(b)** 1 **(c)** 0 **(d)** 2 $\boxed{\textbf{(e)}}$ The limit does not exist

[19]. Suppose $f(t) = \begin{cases} (-t)^2 & \text{if } t < 1 \\ t^3 & \text{if } t \geq 1 \end{cases}$

Find the limit $\lim_{t \to 1} f(t)$.

(a) -2 **(b)** -1 $\boxed{\textbf{(c)}}$ 1 **(d)** 2 **(e)** The limit does not exist

[20]. Suppose the total cost, $C(q)$, of producing a quantity q of a product equals a fixed cost of \$1000 plus \$3 times the quantity produced. So total cost in dollars is

$$C(q) = 1000 + 3q.$$

The average cost per unit quantity, $A(q)$, equals the total cost, $C(q)$, divided by the quantity produced, q. Find the limiting value of the average cost per unit as q tends to 0 from the right. In other words find

$$\lim_{q \to 0^+} A(q)$$

(a) 0 **(b)** 3 **(c)** 1000 **(d)** 1003 $\boxed{\textbf{(e)}}$ The limit does not exist

Limits at infinity

[21]. Find the limit $\lim_{t \to \infty} \frac{3}{1+t^2}$.

$\boxed{\textbf{(a)}}$ 0 **(b)** 1 **(c)** 2 **(d)** 3 **(e)** The limit does not exist

[22]. Find the limit $\lim_{x \to \infty} \frac{x^2+x+1}{(3x+2)^2}$.

(a) 1 **(b)** 1/3 **(c)** 0 $\boxed{\textbf{(d)}}$ 1/9 **(e)** The limit does not exist

[23]. Find the limit $\lim_{s \to \infty} \frac{s^4+s^2+13}{s^3+8s+9}$.

(a) 0 **(b)** 1 **(c)** 2 **(d)** 3 $\boxed{\textbf{(e)}}$ The limit does not exist

[24]. Find the limit $\lim_{x\to\infty} \dfrac{2x^2}{(x+2)^3}$.

$\boxed{\textbf{(a)}}$ 0   **(b)** 1   **(c)** 2   **(d)** 3   **(e)** The limit does not exist

[25]. Suppose the total cost, $C(q)$, of producing a quantity q of a product is given by the equation

$$C(q) = 5000 + 5q.$$

The average cost per unit quantity, $A(q)$, equals the total cost, $C(q)$, divided by the quantity produced, q. Find the limiting value of the average cost per unit as q tends to ∞. In other words find

$$\lim_{q\to\infty} A(q)$$

$\boxed{\textbf{(a)}}$ 5   **(b)** 6   **(c)** 5000   **(d)** 5006   **(e)** The limit does not exist

Continuity and differentiability

[26]. Suppose $f(t) = \begin{cases} Bt & \text{if } t \le 3 \\ 5 & \text{if } t > 3 \end{cases}$

Find a value of B such that the function $f(t)$ is continuous for all t.

(a) 3/5   **(b)** 4/5 $\boxed{\textbf{(c)}}$ 5/3   **(d)** 5/4   **(e)** 5/2

[27]. Suppose that $f(x) = \begin{cases} A + x & \text{if } x < 2 \\ 1 + x^2 & \text{if } x \ge 2 \end{cases}$

Find a value of A such that the function $f(x)$ is continuous at the point $x = 2$.

(a) $A = 8$ **(b)** $A = 1$ **(c)** $A = 2$ $\boxed{\textbf{(d)}}$ $A = 3$ **(e)** $A = 0$

[28]. Suppose $f(t) = \begin{cases} t & \text{if } t \le 3 \\ A + \dfrac{t}{2} & \text{if } t > 3 \end{cases}$

Find a value of A such that the function $f(t)$ is continuous for all t.

(a) 1/2   **(b)** 1 $\boxed{\textbf{(c)}}$ 3/2   **(d)** 2   **(e)** 5/2

[29]. Consider the function $f(x) = \begin{cases} 2x^2 + 3 & \text{if } x \le 3 \\ 3x + B & \text{if } x > 3 \end{cases}$.

Find a value of B such that $f(x)$ is continuous at $x = 3$.

(a) 6   **(b)** 9 $\boxed{\textbf{(c)}}$ 12   **(d)** 15   **(e)** There is no such value of B.

[30]. Find all values of a such that the function $f(x) = \begin{cases} x^2 + 2x & \text{if } x < a \\ -1 & \text{if } x \ge a \end{cases}$ is continuous everywhere.

$\boxed{\textbf{(a)}}$ $a = -1$ only   **(b)** $a = -2$ only   **(c)** $a = -1$ and $a = 1$

(d) $a = -2$ and $a = 2$ **(e)** all real numbers

[31]. Which of the following is true for the function $f(x)$ given by

$$f(x) = \begin{cases} 2x-1 & \text{if } \; x < -1 \\ x^2+1 & \text{if } \; -1 \le x \le 1 \\ x+1 & \text{if } \; x > 1 \end{cases}$$

(a) f is continuous everywhere

(b) f is continuous everywhere except at $x = -1$ and $x = 1$

$\boxed{\textbf{(c)}}$ f is continuous everywhere except at $x = -1$

(d) f is continuous everywhere except at $x = 1$

(e) None of the above

[32]. Which of the following is true for the function $f(x) = |x-1|$?

(a) f is differentiable at $x = 1$ and $x = 2$.

(b) f is differentiable at $x = 1$, but not at $x = 2$.

$\boxed{\textbf{(c)}}$ f is differentiable at $x = 2$, but not at $x = 1$.

(d) f is not differentiable at either $x = 1$ or $x = 2$.

(e) None of the above.