`Chapter 6: Practice/review problems`

The collection of problems listed below contains questions taken from previous MA123 exams.

Extreme values problems on a closed interval

[1]. Suppose $f(t) = \begin{cases} \sqrt{4-t} & \text{if} \quad t < 4 \\ \sqrt{t-4} & \text{if} \quad t \geq 4. \end{cases}$

Find the minimum of $f(t)$ on the interval $[0, 6]$.

$\boxed{\textbf{(a)}}$ 0 **(b)** 2 **(c)** 4 **(d)** 6 **(e)** 8

[2]. Let $g(s) = \dfrac{s-1}{s+1}$. Find the maximum of $g(s)$ on the interval $[0, 2]$.

(a) $-1/3$ **(b)** 0 $\boxed{\textbf{(c)}}$ $1/3$ **(d)** $2/3$

(e) Neither the maximum nor the minimum exists on the given interval.

[3]. Suppose $f(t) = \begin{cases} t^2 - 2t + 2 & \text{if} \quad t < 1 \\ t^3 & \text{if} \quad t \geq 1. \end{cases}$

Find the minimum of $f(t)$ on the interval $[0, 2]$.

(a) -1 **(b)** 0 $\boxed{\textbf{(c)}}$ 1 **(d)** 2 **(e)** 8

[4]. Let $f(x) = 3x^2 + 6x + 4$. Find the maximum value of $f(x)$ on the interval $[-2, 1]$.

(a) 5 **(b)** 7 **(c)** 9 $\boxed{\textbf{(d)}}$ 13 **(e)** -1

[5]. Let $G(x) = \begin{cases} (x-3)+6 & \text{if } x \geq 3 \\ -(x-3)+6 & \text{if } x < 3. \end{cases}$

Find the minimum of $G(x)$ on the interval $[-10, 10]$.

(a) 3 **(b)** 1 **(c)** -6 **(d)** 19 $\boxed{\textbf{(e)}}$ 6

[6]. Let $g(s) = \dfrac{1}{s+1}$. Find the maximum of $g(s)$ on the interval $[0, 2]$.

(a) -1 **(b)** 0 $\boxed{\textbf{(c)}}$ 1 **(d)** 2

(e) Neither the maximum nor the minimum exists on the given interval.

[7]. Find the minimum value of $f(x) = x^3 - 3x + 3$ on the interval $[-2, 4]$.

(a) 2 $\boxed{\textbf{(b)}}$ 1 **(c)** 0 **(d)** -1 **(e)** -2

[8]. Find the maximum of $g(t) = |t+4| + 10$ on the interval $[-12, 12]$.

(a) 19 **(b)** 20 **(c)** 24 $\boxed{\textbf{(d)}}$ 26 **(e)** 28

[9]. Find the minimum value of $f(x) = \sqrt{x^2 - 2x + 16}$ on the interval $[0, 5]$.

(a) 1 **(b)** 2 $\boxed{\textbf{(c)}}$ $\sqrt{15}$ **(d)** $\sqrt{12}$ **(e)** 0

[10]. Let $f(x) = |x^2 - 1| + 2$. Find the minimum of $f(x)$ on the interval $[-3, 3]$.

(a) 3 **(b)** 0 **(c)** 1 $\boxed{\textbf{(d)}}$ 2 **(e)** -1

[11]. Suppose $f(t) = 2t^3 - 9t^2 + 12t + 31$. Find the value of t in the interval $[0, 3]$ where $f(t)$ takes on its minimum.

$\boxed{\textbf{(a)}}$ 0 **(b)** 1 **(c)** 2 **(d)** 3

(e) Neither the maximum nor the minimum exists on the given interval.

[12]. Let $Q(t) = t^2$. Find a value A such that the average rate of change of $Q(t)$ from 1 to A equals the instantaneous rate of change of $Q(t)$ at $t = 2A$

(a) 1 $\boxed{\textbf{(b)}}$ $\frac{1}{3}$ **(c)** $\frac{1}{4}$ **(d)** $\frac{1}{5}$ **(e)** Does not exist

Mean Value Theorem problems

[13]. Find the value of A such that the average rate of change of the function $g(s) = s^3$ on the interval $[0, A]$ is equal to the instantaneous rate of change of the function at $s = 1$.

(a) $\sqrt{2}$ $\boxed{\textbf{(b)}}$ $\sqrt{3}$ **(c)** $\sqrt{5}$ **(d)** $\sqrt{6}$ **(e)** $\sqrt{12}$

[14]. Suppose $k(s) = s^2 + 3s + 1$. Find a value c in the interval $[1, 3]$ such that $k'(c)$ equals the average rate of change of $k(s)$ on the interval $[1, 3]$.

(a) -1 **(b)** 0 **(c)** 1 $\boxed{\textbf{(d)}}$ 2 **(e)** 3

[15]. Let $k(x) = x^3 + 2x$. Find a value of c between 1 and 3 such that the average rate of change of $k(x)$ from $x = 1$ to $x = 3$ is equal to the instantaneous rate of change of $k(x)$ at $x = c$.

(a) 30 **(b)** 15 **(c)** $\sqrt{\frac{28}{3}}$ $\boxed{\textbf{(d)}}$ $\sqrt{\frac{13}{3}}$ **(e)** 60

Increasing/decreasing problems

[16]. Which function is always increasing on $(0, 2)$

$\boxed{\textbf{(a)}}$ $\sqrt{x} + x^2$ **(b)** $x + (1/x)$ **(c)** $x^3 - 3x$

(d) $7 - |x|$ **(e)** $(x-1)^4$

[17]. Suppose that a function $f(x)$ has derivative $f'(x) = x^2 + 1$. Which of the following statements is true about the graph of $y = f(x)$?

$\boxed{\textbf{(a)}}$ The function is increasing on $(-\infty, \infty)$

(b) The function is decreasing on $(-\infty, \infty)$

(c) The function is increasing on $(-\infty, -1)$ and $(1, \infty)$, and the function is decreasing on $(-1, 1)$.

(d) The function is increasing on $(-\infty, 0)$, and the function is decreasing on $(0, \infty)$.

(e) The function is decreasing on $(-\infty, 0)$, and the function is increasing on $(0, \infty)$.

[18]. Find the largest value of A such that the function $g(s) = s^3 - 3s^2 - 24s + 1$ is increasing on the interval $(-5, A)$.

(a) -4 $\boxed{\textbf{(b)}}$ -2 **(c)** 0 **(d)** 2 **(e)** 4

[19]. Let $f(x) = e^{-x^2}$. Find the intervals where $f(x)$ is decreasing.

(a) $(-\infty, 0)$ $\boxed{\textbf{(b)}}$ $(0, \infty)$ **(c)** $(-\infty, -1)$

(d) $(1, \infty)$ **(e)** $(-1, 1)$

[20]. Let $f(x) = x \ln x$. Find the intervals where $f(x)$ is increasing.

(a) $(0, \infty)$ **(b)** $(1, \infty)$ **(c)** (e, ∞)

$\boxed{\textbf{(d)}}$ $(1/e, \infty)$ **(e)** $(1/e, e)$

[21]. Suppose the cost, $C(q)$, of stocking a quantity q of a product equals $C(q) = \dfrac{100}{q} + q$. The rate of change of the cost with respect to q is called the marginal cost. When is the marginal cost positive?

$\boxed{\textbf{(a)}}$ $q > 10$ **(b)** $q > 15$ **(c)** $q < 20$ **(d)** $q < 25$ **(e)** $q = 30$

[22]. For which values of t is the function $t^3 - 2t + 1$ increasing?

$\boxed{\textbf{(a)}}$ $t > \sqrt{2/3}$ or $t < -\sqrt{2/3}$ **(b)** $-\sqrt{2/3} < t < \sqrt{2/3}$ **(c)** $0 < t < \sqrt{4/3}$

(d) $-\sqrt{4/3} < t < 0$ **(e)** Never

[23]. Suppose that $g'(x) = x^2 - x - 6$. Find the interval(s) where $g(x)$ is increasing.

(a) $(-1, 2)$ $\boxed{\textbf{(b)}}$ $(-\infty, -2)$ and $(3, \infty)$ **(c)** $(-\infty, -1)$ and $(2, \infty)$

(d) $(-2, 3)$ **(e)** It cannot be determined from the information given

[24]. Let $f(x) = xe^{2x}$. Then f is decreasing on the following interval.

$\boxed{\textbf{(a)}}$ $(-\infty, -1/2)$ **(b)** $(-1/2, \infty)$ **(c)** $(-\infty, 1/2)$

(d) $(1/2, \infty)$ **(e)** $(-\infty, 0)$

[25]. Find the interval(s) where $f(x) = -x^3 + 18x^2 - 105x + 4$ is increasing.
(Note that the coefficient of x^3 is -1, so compute carefully.)

(a) $(-\infty, 5)$ and $(7, \infty)$ $\boxed{\textbf{(b)}}$ $(5, 7)$ **(c)** $(-\infty, -5)$ and $(7, \infty)$

(d) $(-5, 7)$ **(e)** $(-7, 5)$

[26]. Suppose that $f(x) = xg(x)$, and for all positive values of x the function $g(x)$ is negative (i.e., $g(x) < 0$) and decreasing. Which of the following is true for the function $f(x)$?

$\boxed{\textbf{(a)}}$ $f(x)$ is negative and decreasing for all positive values of x.

(b) $f(x)$ is positive and increasing for all positive values of x.

(c) $f(x)$ is negative and increasing for all positive values of x.

(d) $f(x)$ is positive and decreasing for all positive values of x.

(e) None of the above.

[27]. Suppose the derivative of a function $g(x)$ is given by $g'(x) = x^2 - 1$. Find all intervals on which $g(x)$ is increasing.

(a) $(-\infty, \infty)$ **(b)** $(-1, 1)$ $\boxed{\textbf{(c)}}$ $(-\infty, -1)$ and $(1, \infty)$

(d) $(0, \infty)$ **(e)** $(-\infty, 0)$

Extreme values problems using the first derivative

[28]. Suppose the derivative of the function $h(x)$ is given by $h'(x) = 1 - |x|$. Find the value of x in the interval $[-1, 1]$ where $h(x)$ takes on its minimum value.

(a) $-1/2$ $\boxed{\textbf{(b)}}$ -1 **(c)** 0 **(d)** $1/2$ **(e)** 1

[29]. Suppose the total cost, $C(q)$, of producing a quantity q of a product equals

$$C(q) = 1000 + q + \frac{1}{10}q^2.$$

The average cost, $A(q)$, equals the total cost divided by the quantity produced. What is the minimum average cost? (Assume $q > 0$)

(a) 20 $\boxed{\textbf{(b)}}$ 21 **(c)** 26 **(d)** 30 **(e)** 31

[30]. Suppose that a function $h(x)$ has derivative $h'(x) = x^2 + 4$. Find the x value in the interval $[-1, 3]$ where $h(x)$ takes its minimum.

$\boxed{\textbf{(a)}}$ -1 **(b)** 3 **(c)** 5 **(d)** 13 **(e)** 29

[31]. Suppose the cost, $C(q)$, of stocking a quantity q of a product equals $C(q) = \frac{100}{q} + q$. Which positive value of q gives the minimum cost?

$\boxed{\text{(a)}}$ 10 **(b)** 15 **(c)** 20 **(d)** 25 **(e)** 30

[32]. Find a local extreme point of $f(x) = \frac{\ln x}{x}$.

(a) $(1, 0)$ is a local maximum point. **(b)** $(1, 0)$ is a local minimum point.
(c) $(e, 1/e)$ is a local minimum point. $\boxed{\text{(d)}}$ $(e, 1/e)$ is a local maximum point.
(e) $f(x)$ has no local extreme points.

[33]. Suppose the derivative of $G(q)$ is given by $G'(q) = q^2(q+1)^2(q+2)^2$. Find the value of q in the interval $[-5, 5]$ where $G(q)$ takes on its maximum.

(a) -5 **(b)** -2 **(c)** -1 **(d)** 0 $\boxed{\text{(e)}}$ 5

[34]. Suppose the derivative of $H(s)$ is given by $H'(s) = s^2(s+1)$. Find the value of s in the interval $[-100, 100]$ where $H(s)$ takes on its minimum.

(a) -100 $\boxed{\text{(b)}}$ -1 **(c)** 0 **(d)** 1 **(e)** 100

Concavity problems

[35]. Find the intervals where $f(x) = x^4 - 12x^3 + 48x^2 + 10x - 8$ is concave downward.

(a) $(-\infty, \infty)$ **(b)** $(1, \infty)$ **(c)** $(-\infty, -4)$ and $(-2, \infty)$
(d) $(-\infty, 2)$ and $(4, \infty)$ $\boxed{\text{(e)}}$ $(2, 4)$

[36]. Let $f(x) = e^{-x^2}$. Find the intervals where $f(x)$ is concave upward.

(a) $(1, \infty)$ **(b)** $(-e, e)$ $\boxed{\text{(c)}}$ $(-\infty, -\sqrt{1/2})$ and $(\sqrt{1/2}, \infty)$
(d) $(-\sqrt{1/2}, \sqrt{1/2})$ **(e)** $(-\infty, -e)$ and (e, ∞)

[37]. Let $f(x) = x \ln x$. Find the intervals where $f(x)$ is concave downward.

(a) $(0, 1)$ **(b)** $(0, \infty)$ **(c)** $(0, 1/e)$
(d) $(1/e, \infty)$ $\boxed{\text{(e)}}$ $f(x)$ is not concave downward anywhere

[38]. Suppose that the derivative of $f(x)$ is given by $f'(x) = x^2 - 5x + 6$. Then the graph of $f(x)$ is concave downward on the following intervals(s).

(a) $(-\infty, 2)$ and $(3, \infty)$ **(b)** $(2, 3)$ $\boxed{\text{(c)}}$ $(-\infty, 2.5)$
(d) $(2.5, \infty)$ **(e)** $f(x)$ in not concave downward on any interval

[39]. Find the interval(s) where the graph of $f(x) = x^4 + 18x^3 + 120x^2 + 10x + 50$ is concave downward.

(a) $(-5, 4)$ **(b)** $(4, 5)$ **(c)** $(-\infty, 4)$ and $(5, \infty)$
$\boxed{\text{(d)}}$ $(-5, -4)$ **(e)** $(-\infty, -5)$ and $(-4, \infty)$