`Chapter 5: Practice/review problems`

The collection of problems listed below contains questions taken from previous MA123 exams.

Derivatives

[1]. If $f(x) = 6x^2 + 3x - 1$, find $f'(x)$.

(a) $6x + 1$ **(b)** $12x + 3$ **(c)** $12x - 1$ **(d)** $2x + 3$ **(e)** $2x + 5$

[2]. If $f(x) = x^3 + 4x^2 + 2x + 1$ then $f'(x) =$

(a) $3x^2 + 8x + 3$ **(b)** $x^2 + x + 1$ **(c)** $3x^2 + 8x + 2$ **(d)** $3x^2 + 8x + 1$ **(e)** $3x^2 + 4x + 1$

[3]. If $f(x) = x^3$ then

$$\lim_{h\to 0} \frac{f(1+h) - f(1)}{h}$$

(**Hint:** Relate the limit to the derivative of $f(x)$.)

(a) 0 **(b)** 1 **(c)** 2 **(d)** 3 **(e)** 4

[4]. Suppose $f(t) = t^3 - t^2 + t + 1$. Find the limit

$$\lim_{h\to 0} \frac{f(1+h) - f(1)}{h}$$

(**Hint:** Relate the limit to the derivative.)

(a) -1 **(b)** 0 **(c)** 1 **(d)** 2 **(e)** The limit does not exist

[5]. If $Q(s) = s^7 + 1$, find

$$\lim_{h\to 0} \frac{Q(1+h) - Q(1)}{h}$$

(a) 2 **(b)** 5 **(c)** 6 **(d)** 7 **(e)** 8

[6]. If $f(x) = |x - 1|$ find $\displaystyle\lim_{h\to 0} \frac{f(1+h) - f(1)}{h}$

(a) 3 **(b)** -3 **(c)** 1 **(d)** -1 **(e)** Does not exist

[7]. Let $f(x) = x\,|x| - x$. Find the derivative, $f'(0)$, by evaluating the limit

$$\lim_{h\to 0} \frac{f(h) - f(0)}{h}$$

(a) -2 **(b)** -1 **(c)** 0 **(d)** 1 **(e)** Does not exist

[8]. Let $[\![x]\!]$ denote the greatest integer function. Recall the definition:

$$[\![x]\!] \text{ equals the greatest integer less than or equal to } x \,.$$

How many points are there in the interval $(1/2, 9/2)$ where the derivative of $[\![x]\!]$ is not defined?

(a) 1 **(b)** 2 **(c)** 3 **(d)** 4 **(e)** 5

The product rule

[9]. Suppose that $h(x) = f(x)g(x)$. Assume that $f(2) = 3$, $f'(2) = -2$, $g(2) = 1$, and $g'(2) = 5$. Find $h'(2)$.

(a) -20 **(b)** -17 **(c)** 11 **(d)** 13 **(e)** Cannot be determined

[10]. If $h(t) = (t-1)(t+1)(t^2+1)$ then $h'(2)$ equals

(a) 0 **(b)** 4 **(c)** 8 **(d)** 16 **(e)** 32

[11]. Let $k(x) = (x+3)(x+4)(x+1)$. Find $k'(x)$.

(a) 12 **(b)** $3x^2 + 16x + 19$ **(c)** $3x^2 + 18x + 20$ **(d)** $3x^2 + 14x + 16$ **(e)** 1

[12]. If $R(x) = (x-2)(x^2-2)(x^3-2)$, find $R'(2)$

(a) 0 **(b)** 12 **(c)** 48 **(d)** -8 **(e)** -6

The quotient rule

[13]. If $f(x) = \dfrac{x-1}{x+1}$ then $f'(x) =$

(a) $\dfrac{2}{x^2+1}$ **(b)** $\dfrac{2}{(x+1)^2}$ **(c)** $\dfrac{-2}{(x+1)^2}$ **(d)** $\dfrac{-2}{x^2+1}$ **(e)** $\dfrac{-2}{(x-1)^2}$

[14]. Suppose that $f(x) = \dfrac{x^2+1}{x+4}$. Find $f'(-3)$.

(a) -8 **(b)** -9 **(c)** -10 **(d)** -14 **(e)** -16

[15]. Find $Y'(s)$ if $Y(s) = \dfrac{1}{4s^2} - \dfrac{5}{s}$.

(a) $\dfrac{5}{2}s^{-3} + s^{-2}$ **(b)** $-\dfrac{1}{2}s^{-3} + 5s^{-2}$ **(c)** $-\dfrac{2}{5}s^{-3} + s^{-2}$ **(d)** $\dfrac{1}{2}s^{-3} + 5s^{-2}$ **(e)** $-2s^{-3} - 3s^{-2}$

[16]. If $F(t) = \dfrac{3t+1}{t-1}$ then $F'(t) =$

(a) $-4/(t-1)^2$ **(b)** $-4/(3t+1)^2$ **(c)** $-2/(t-1)^2$ **(d)** $-3/(t-1)^2$ **(e)** $-2/(t-1)$

[17]. Let $T(x) = \dfrac{g(x)}{f(x)}$. If $f(2) = 3$, $f'(2) = 4$, $g(2) = 5$, and $g'(2) = 6$, find $T'(2)$.

(a) $\dfrac{38}{9}$ **(b)** $\dfrac{38}{25}$ **(c)** $\dfrac{2}{25}$ **(d)** $-\dfrac{2}{9}$ **(e)** 38

[18]. Evaluate the derivative, $H'(1)$ if

$$H(s) = \frac{2s}{s+1}$$

(a) 2/9 **(b)** 4/9 **(c)** 1/2 **(d)** 3/2 **(e)** 8/9

[19]. Suppose the cost, $C(q)$, of stocking a quantity q of a product equals

$$C(q) = 12 + 3q + \frac{8}{q}.$$

The rate of change of the cost with respect to q is called the marginal cost. What is the marginal cost when the cost equals 23 and the cost is decreasing?

(a) -5 **(b)** -1 **(c)** 0 **(d)** 1 **(e)** 5

[20]. Suppose the cost, $C(q)$, of stocking a quantity q of a product equals

$$C(q) = \frac{100}{q} + q$$

For which positive value of q is the tangent line to the graph of $C(q)$ a horizontal line?

(a) 1/100 **(b)** 1/10 **(c)** 1 **(d)** 10 **(e)** 100

[21]. Suppose $u(t)$ and $w(t)$ are differentiable for all t and the following values of the functions and derivatives are known: $u(7) = 2$, $u'(7) = -1$, $w(7) = 1$, and $w'(7) = 9$. Find the value of $h'(7)$ when

$$h(t) = \frac{w(t)+5}{u(t)}.$$

(a) 3 **(b)** 6 **(c)** -3 **(d)** 12 **(e)** -6

[22]. Suppose $f(t) = \dfrac{F(t)}{t}$ and $F(1) = 2$, $F'(1) = 6$. Find $f'(1)$.

(a) 2 **(b)** 4 **(c)** 1 **(d)** -4 **(e)** -1

[23]. If $f(x) = \dfrac{-x}{x^2-1}$ then $f'(x) =$

(a) $\dfrac{-x^2-1}{(x^2-1)^2}$ **(b)** $\dfrac{1}{2x}$ **(c)** $\dfrac{-x^2-1}{x^2-1}$ **(d)** $\dfrac{x^2+1}{x^2-1}$ **(e)** $\dfrac{x^2+1}{(x^2-1)^2}$

Tangent lines

[24]. Find the equation of the tangent line to the graph of $y = 2x^3 - 3x^2 + 4x + 2$ at $x = 1$.

(a) $y = x + 1$ **(b)** $y = 5x - 4$ **(c)** $y = 5x - 3$ **(d)** $y = 4x - 2$ **(e)** $y = 4x + 1$

[25]. Which horizontal line is tangent to the graph of $y = x^3 - x^2 - x + 2$?

(a) $y = 0$ **(b)** $y = 1$ **(c)** $y = 2$ **(d)** $y = 3$ **(e)** $y = 5$

[26]. If $g(t) = \dfrac{1}{t^2+1}$, then the slope of the tangent line to the graph of $g(t)$ at $t = 3$ is

(a) $-\dfrac{1}{25}$ **(b)** $-\dfrac{2}{25}$ **(c)** $-\dfrac{1}{50}$ **(d)** $-\dfrac{3}{50}$ **(e)** $-\dfrac{4}{25}$

[27]. The equation of the tangent line to the graph of $y = g(x)$ at $x = 3$ is $y = 2 + 4(x - 3)$.
What is the value of $g'(3)$?

(a) -6 **(b)** 4 **(c)** -12 **(d)** 0 **(e)** 2

[28]. If the line $y = 3 + 4(x - 2)$ is tangent to the graph of $g(x)$ at $x = 2$ and $g(x)$ is differentiable at $x = 2$, then $g(2) + g'(2) =$

(a) 2 **(b)** 3 **(c)** 4 **(d)** 6 **(e)** 7

[29]. If the line $y = 9 + 3(x - 4)$ is tangent to the graph of $G(x)$ at $x = 4$ and $G(x)$ is differentiable at $x = 4$, then $G(4) - G'(4)$ equals

(a) 3 **(b)** 4 **(c)** 5 **(d)** 6 **(e)** 9

[30]. The line $y = -1 + 4(x - 2)$ is tangent to the graph of $g(x)$ at $x = 2$. If $g(x)$ is differentiable at $x = 2$, and $h(x) = xg(x)$, then $h'(2)$ equals

(a) 2 **(b)** 3 **(c)** 4 **(d)** 6 **(e)** 7

[31]. Let

$$H(s) = \begin{cases} 3(s-1)^2 & \text{if} \quad s \le 1 \\ 5(s-1)^2 & \text{if} \quad s > 1 \end{cases}$$

Find the equation of the tangent line to the graph of $H(s)$ at $s = 2$ in the (s, t) plane.

(a) $t = 3 + 6s$ **(b)** $t = 3 - 6s$ **(c)** $t = 5 + 10(s - 2)$

(d) $t = 5 + 10(s - 1)$ **(e)** The tangent line does not exist

[32]. Let

$$H(s) = |s - 1|$$

Find the equation of the tangent line to the graph of $H(s)$ at $s = 0$ in the (s, t) plane.

(a) $t = 1 + s$ **(b)** $t = 1 - s$ **(c)** $t = 1$ **(d)** $t = s$ **(e)** The tangent line does not exist

The chain rule

[33]. If $f(s) = (s^2 + 5s + 4)^3$, find $f'(s)$.

(a) $3(s^2 + 5s + 4)^2$ **(b)** $3(s^2 + s + 4)^2$ **(c)** $2(s^2 + 5s + 4) \cdot (2s + 5)$
(d) $3(s^2 + s + 4)^2 \cdot (2s + 1)$ **(e)** $3(s^2 + 5s + 4)^2 \cdot (2s + 5)$

[34]. Find $f'(1)$ where $f(x) = \sqrt{x^4 + 3x^2 + 5}$.

(a) 1/3 **(b)** 2/3 **(c)** 1 **(d)** 4/3 **(e)** 5/3

[35]. Suppose that $h(x) = f(g(x))$. Assume that $f(3) = 6$, $f'(3) = 6$, $g(2) = 3$, and $g'(2) = 4$. Find $h'(2)$.

(a) -30 **(b)** 24 **(c)** 18 **(d)** -20 **(e)** -15

[36]. Suppose that $f(x) = (x^2 - 5)^{3/2}$. Find $f'(3)$.

(a) 9 **(b)** 18 **(c)** 27 **(d)** 12 **(e)** 36

[37]. Suppose that $g(x) = [f(x)]^3$ and the equation of the tangent line to the graph of $f(x)$ at $x = 2$ is $y = -1 + 4(x - 2)$. Find $g'(2)$.

(a) 15 **(b)** -15 **(c)** -1 **(d)** -12 **(e)** 12

[38]. Suppose that $f(t) = 12\sqrt{t + 7}$. Find the limit

$$\lim_{h \to 0} \frac{f(2 + h) - f(2)}{h}.$$

(**Hint:** Relate the limit to the derivative.)

(a) -1 **(b)** 0 **(c)** 1 **(d)** 2 **(e)** 3

[39]. Suppose $F(x) = g(h(x))$. If $g(2) = 3$, $g'(2) = 4$, $h(0) = 2$ and $h'(0) = 6$, find $F'(0)$.

(a) 12 **(b)** 4 **(c)** 24 **(d)** 6 **(e)** 3

[40]. Suppose $f(t) = H(G(t))$ and $H(3) = 5$, $H'(3) = 4$, $G(2) = 3$, and $G'(2) = 7$. Find $f'(2)$.

(a) 12 **(b)** 35 **(c)** 28 **(d)** 15 **(e)** 43

[41]. If $G(s) = u(s^2)$ and $u(1) = 10$, $u'(1) = 4$, $u(-1) = 7$, and $u'(-1) = 2$, then $G'(-1) =$

(a) -20 **(b)** 4 **(c)** 10 **(d)** 2 **(e)** -8

[42]. Suppose $f(t) = g(3t)$ and $F(t) = f(t)g(t)$. If $g(1) = 1$, $g(3) = 5$, $g'(1) = 2$ and $g'(3) = 7$, what is $F'(1)$?

(a) 14 **(b)** 17 **(c)** 24 **(d)** 31 **(e)** 42

[43]. Suppose $h(x) = [f(x)]^2$ and the equation of the tangent line to the graph of $f(x)$ at $x = 1$ is $y = 3+4(x-1)$. Find $h'(1)$.

(a) 28 **(b)** 40 **(c)** 14 **(d)** 24 **(e)** 20

[44]. If $F(x) = u(v(x))$ and

$$\begin{array}{lll} v(1) = 3 & u(1) = 2 & u(3) = 2 \\ v'(1) = 7 & u'(1) = 4 & u'(3) = 1 \end{array}$$

then $F'(1) =$

(a) 6 **(b)** 7 **(c)** 8 **(d)** 9 **(e)** 10

[45]. If $F(x) = u(x^2) + (v(x))^2$ and

$$\begin{array}{lll} v(1) = 3 & u(1) = 2 & u(3) = 2 \\ v'(1) = 7 & u'(1) = 4 & u'(3) = 1 \end{array}$$

then $F'(1) =$

(a) 20 **(b)** 30 **(c)** 40 **(d)** 50 **(e)** 60

[46]. If $u(t) = \sqrt{4t^2}$, then $u'(-1) =$

(a) -1 **(b)** -2 **(c)** 0 **(d)** 1 **(e)** 2

[47]. Let

$$f(t) = 1 + 10\sqrt{1+t} - t$$

For what nonnegative value of t is the tangent line to the graph of $f(t)$ horizontal?

(a) 0 **(b)** 6 **(c)** 12 **(d)** 18 **(e)** 24

[48]. Suppose $H'(4) = 9$. What is the value of $F'(2)$ if $F(s) = H(s^2)$?

(a) 24 **(b)** 30 **(c)** 36 **(d)** 42 **(e)** 48

Second derivative

[49]. Suppose that $f(x) = 64\sqrt{x}$. Find $f''(4)$.

(a) -2 **(b)** -1 **(c)** 1 **(d)** 2 **(e)** Does not exist