Fall 20_HW21 — Difference Equations: Stability

The handwritten homework assignment is **due** on Canvas on **Friday, November 6, by 11 pm**. The problems are similar to those from Section 5.7 of our textbook (problems 1 through 24 on pp. 277-278).

When submitting a written homework you will be required to follow these guidelines:

- the document must be in pdf format;
- you must either (a) have all problems in numerical order or (b) start every problem on the left side of the page so that the TA can easily find the problems that s/he chooses to grade.

You will be penalized two points for not following these guidelines on every handwritten assignment.

Problem 1. Assume the discrete-time population model

$$N_{t+1} = bN_t, \qquad t = 0, 1, 2...$$

Assume also that the population increases by 2% each generation.
(a) Determine b.
(b) Find the size of the population at generation 10 when $N_0 = 20$.
(c) After how many generations will the population size have doubled?

Problem 2. (a) Find all equilibria of

$$N_{t+1} = 0.9N_t, \qquad t = 0, 1, 2, \ldots$$

(b) Use cobwebbing to determine the stability of the equilibria you found in (a).

Problem 3. Use the stability criterion to characterize the stability of the equilibria of

$$x_{t+1} = \frac{2}{3} - \frac{2}{3}x_t^2, \qquad t = 0, 1, 2, \ldots$$

Problem 4. Use the stability criterion to characterize the stability of the equilibria of

$$x_{t+1} = \frac{x_t}{0.5 + x_t}, \qquad t = 0, 1, 2, \ldots$$

Problem 5. (a) Use the stability criterion to characterize the stability of the equilibria of

$$x_{t+1} = \frac{5x_t^2}{4 + x_t^2}, \qquad t = 0, 1, 2, \ldots$$

(b) Use cobwebbing to decide to which value x_t converges as $t \to \infty$ if (i) $x_0 = 0.5$ and (ii) $x_0 = 2$.

Problem 6. We investigate the canonical discrete-time logistic growth model

$$x_{t+1} = rx_t(1 - x_t)$$

for $t = 0, 1, 2, \ldots$

Show that for $r > 1$, there are two fixed points. For which values of r is the nonzero fixed point locally stable?

Problem 7. We consider density-dependent population growth models of the form

$$N_{t+1} = R(N_t)\, N_t$$

The function $R(N) = rN^{1-\gamma}$ describes the per capita growth.

Find all nontrivial fixed points $\widehat{N}$ (i.e., $\widehat{N} > 0$) and determine the stability as a function of the parameter values. We assume that the function parameters are $r > 0$ and $\gamma > 1$.

Problem 8. We consider density-dependent population growth models of the form

$$N_{t+1} = R(N_t)\, N_t$$

The function $R(N) = e^{r(1-N/K)}$ describes the per capita growth.

Find all nontrivial fixed points $\widehat{N}$ (i.e., $\widehat{N} > 0$) and determine the stability as a function of the parameter values. We assume that the function parameters are $r > 0$ and $K > 0$.