WORKSHEET 10

1. ⊛

$$\lim_{x\to\infty} \frac{(2+3x)^2}{4-x^2} = \lim_{x\to\infty} \frac{4+12x+9x^2}{4-x^2}$$

$$= \lim_{x\to\infty} \frac{(4+12x+9x^2)\frac{1}{x^2}}{(4-x^2)\frac{1}{x^2}} =$$

$$= \lim_{x\to\infty} \frac{4/x^2 + \frac{12}{x} + 9}{4/x^2 - 1} =$$

$$= \frac{\overbrace{\left(\lim_{x\to\infty} 4/x^2\right)}^{0} + \overbrace{\left(\lim_{x\to\infty} \frac{12}{x}\right)}^{0} + 9}{\underbrace{\left(\lim_{x\to\infty} 4/x^2\right)}_{\to 0} - 1} = \frac{9}{-1} = -9$$

⊛

$$\lim_{x\to-\infty} \frac{(2+3x)^2}{4-x^2} = \overset{\text{as before}}{\text{-----}} =$$

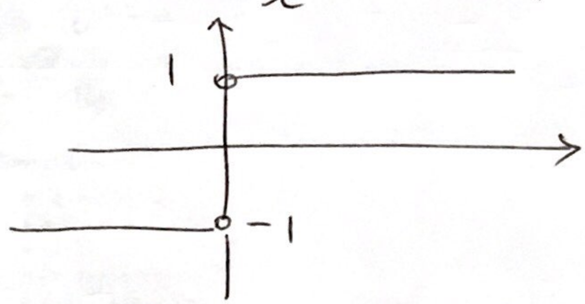
$$= \frac{\overbrace{\left(\lim_{x\to-\infty} 4/x^2\right)}^{\to 0} + \overbrace{\left(\lim_{x\to-\infty} \frac{12}{x}\right)}^{0} + 9}{\underbrace{\left(\lim_{x\to-\infty} 4/x^2\right)}_{\to 0} - 1} = -9$$

⊛ Consider the function

$$\frac{\sqrt{8-x+4x^2}}{5x+3} = \frac{\sqrt{x^2\left(\frac{8-x+4x^2}{x^2}\right)}}{x\left(\frac{5x+3}{x}\right)}$$

$$= \frac{\sqrt{x^2}\cdot\sqrt{\frac{8}{x^2}-\frac{1}{x}+4}}{x\cdot\left(5+\frac{3}{x}\right)} = \frac{|x|}{x}\cdot\frac{\sqrt{\frac{8}{x^2}-\frac{1}{x}+4}}{5+\frac{3}{x}}$$

Recall that $\frac{|x|}{x}$ has graph



So $\lim\limits_{x\to+\infty}\frac{|x|}{x}=1$ & $\lim\limits_{x\to-\infty}\frac{|x|}{x}=-1$

Thus $\lim\limits_{x\to+\infty}\frac{\sqrt{8-x+4x^2}}{5x+3} = \lim\limits_{x\to\infty}\frac{|x|}{x}\cdot\frac{\sqrt{\frac{8}{x^2}-\frac{1}{x}+4}}{5+3/x}$

$$= 1\cdot\frac{\sqrt{4}}{5} = \boxed{2/5}$$

$$\lim_{x\to-\infty}\frac{\sqrt{8-x+4x^2}}{5x+3} = \lim_{x\to-\infty}\frac{|x|}{x}\cdot\frac{\sqrt{8/x^2-\frac{1}{x}+4}}{5+\frac{3}{x}} = \boxed{(-1\cdot)\frac{2}{5}}$$

⊛ For the function $\dfrac{5x^5 + 4^{-x/2}}{(7x^3+1)x^2}$

$$= \frac{5x^5}{7x^5+x^2} + \frac{4^{-x/2}}{7x^5+x^2}$$

Notice $\displaystyle\lim_{x\to\pm\infty} \frac{5x^5}{7x^5+x^2} = 5/7$

however $\displaystyle\lim_{x\to+\infty} \frac{4^{-x/2}}{7x^5+x^2} = \frac{\lim_{x\to+\infty}\left(4^{-x/2}\right)}{\lim_{x\to+\infty}\left(7x^5+x^2\right)}$

$$= \frac{0}{+\infty} = 0$$

$\displaystyle\lim_{x\to-\infty} \frac{4^{-x/2}}{7x^5+x^2} = \frac{+\infty}{-\infty}$ Cannot really decide but $\boxed{-\infty}$

So $\displaystyle\lim_{x\to+\infty} \frac{5x^5+4^{-x/2}}{(7x^3+1)x^2} = \frac{5}{7} + 0$

$$\lim_{x\to-\infty} \frac{5x^5+4^{-x/2}}{(7x^3+1)x^2} = \text{DNE}$$

$\boxed{2.}$

$$\lim_{t \to -\infty} \left(A_1 + A_2 \cdot B^t \right) =$$

$$= A_1 + A_2 \cdot \underbrace{\lim_{t \to -\infty} B^t}_{0} = A_1 + A_2 \cdot 0 = \underline{\underline{A_1}}$$

$$\lim_{t \to 0^-} \left(A_1 + A_2 \cdot B^t \right) = A_1 + A_2 \cdot B^0 = \underline{\underline{A_1 + A_2}}$$

because the function is continuous

$\boxed{3.}$

$$f(x) = x^3 - 3^x + \log(x)$$

is a continuous function because it is made up by continuous functions on a closed interval $[2,3]$

We have that $f(2) = 2^3 - 3^2 + \log(2) = -0.69$

$$f(3) = \cancel{3^3} - \cancel{3^3} + \log(3) = 0.4771$$

By the Intermediate Value Theorem there exists a value $c \in (2,3)$ such that $f(c) = 0$

$\underline{\underline{-0.699}} < L = 0 < \underline{\underline{0.4771}}$

$\boxed{4.}$ $$f(x) = 72x^{15} - 37x^4 + 11 - e^x$$

is a continuous function on $[-1, 0]$ because it is made up of continuous functions on $[-1, 0]$.

$$f(-1) = 72(-1)^{15} - 37(-1)^4 + 11 - e^{-1}$$

$$= -72 - 37 + 11 - \frac{1}{e} = \underline{\underline{-98.3678}}$$

$$f(0) = 72\underbrace{\cancel{(0)^{15}}}_{0} - 37\underbrace{\cancel{(0)^4}}_{0} + 11 - \underbrace{e^0}_{1} = \underline{\underline{10}}$$

Thus there exists a $c \in (-1, 0)$ such that $f(c) = 0$ OR

$$72c^{15} - 37c^4 + 11 - e^c = 0$$

OR

$$\boxed{72c^{15} - 37c^4 + 11 = e^c}$$

$\underline{\underline{\text{i.e.}}}$ a root of the original equation