WORKSHEET #11

1. * $\lim_{x\to 0} x \cdot \cos\left(\frac{3}{x^2}\right) =$

$$= \left(\lim_{x\to 0} x\right)\left(\underbrace{\lim_{x\to 0} \cos\left(\frac{3}{x^2}\right)}\right)$$

this limit does not exist

Same reason why $\lim_{x\to\infty} \cos(x)$ DNE.

We want to use the <u>sandwich theorem</u> to "trap" $x \cos\left(\frac{3}{x^2}\right)$ to better behaved functions. We know

$$-1 \leq \cos\left(\frac{3}{x^2}\right) \leq 1$$

We can't multiply through by x since x could be positive or negative

But we have $-|x| \leq x \leq |x|$

So we can multiply the above set of inequalities by $|x|$.

And obtain $-|x| \leq x\cos\left(\frac{3}{x^2}\right) \leq |x|$

as $\lim_{x\to 0} -|x| = 0 = \lim_{x\to 0} |x|$ we obtain that $\boxed{\lim_{x\to 0} x \cos\left(\frac{3}{x^2}\right) = 0}$

by the sandwich theorem.

(*) $\lim_{x\to 0} \dfrac{\sin\left(\frac{\pi}{2}x\right)}{2x} = \dfrac{0}{0}$ but

we can rewrite it as

$$\lim_{x\to 0} \frac{\frac{\pi}{2}}{2} \cdot \frac{\sin\left(\frac{\pi}{2}x\right)}{\frac{\pi}{2}x} =$$

$$\frac{\pi}{4} \lim_{x\to 0} \frac{\sin\left(\pi/2\, x\right)}{\frac{\pi}{2}x}$$

as $x \to 0$ also $\frac{\pi}{2}x \to 0$

$$= \frac{\pi}{4} \underbrace{\lim_{\frac{\pi}{2}x\to 0} \frac{\sin\left(\pi/2\, x\right)}{\pi/2\, x}}_{1} = \frac{\pi}{4}$$

(*) $\lim_{x\to -1/2} \dfrac{\sin(2x+1)}{5+9x-2x^2} = \dfrac{\sin(0)}{5+9(-1/2)-2\left(\frac{1}{4}\right)}$

$$= \frac{0}{0}$$

Notice that $-2x^2+9x+5 =$
$(2x+1)(5-x)$ so

$$\lim_{x\to -1/2} \frac{\sin(2x+1)}{-2x^2+9x+5} = \lim_{x\to -1/2} \frac{\sin(2x+1)}{(2x+1)(5-x)}$$

$$= \underbrace{\lim_{x\to -1/2} \frac{\sin(2x+1)}{2x+1}}_{1} \cdot \underbrace{\lim_{x\to -1/2} \frac{1}{5-x}}_{\downarrow}$$

as $\lim_{\theta\to 0} \frac{\sin\theta}{\theta} = 1$ $\quad = 1 \cdot \frac{1}{5+\frac{1}{2}} = \boxed{\frac{2}{11}}$

(*) $$\lim_{x\to 0} \frac{\sin x\,(1-\cos x)}{x^3}$$

$$= \lim_{x\to 0} \frac{\sin x}{x} \cdot \lim_{x\to 0} \frac{1-\cos x}{x} \cdot \lim_{x\to 0} \frac{1}{x}$$

$$= 1 \cdot 0 \cdot \left(\underset{\pm\infty}{\text{DNE}}\right)$$ what do we do?

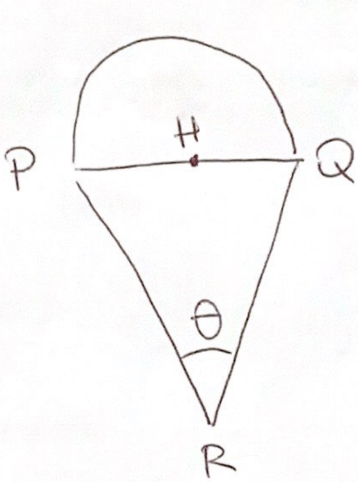
$$= \lim_{x\to 0} \frac{\sin x\,(1-\cos x)}{x^3} \cdot \frac{(1+\cos x)}{(1+\cos x)}$$

$$= \lim_{x\to 0} \frac{\sin x\,(1-\cos^2 x)}{x^3 \cdot (1+\cos x)} = \lim_{x\to 0} \left[\frac{\sin^3 x}{x^3} \cdot \frac{1}{1+\cos x}\right]$$

$$= \lim_{x \to 0} \frac{\sin^3 x}{x^3} \cdot \lim_{x \to 0} \left(\frac{1}{1 + \cos x} \right) =$$

$$= \left(\lim_{x \to 0} \frac{\sin x}{x} \right)^3 \cdot \lim_{x \to 0} \frac{1}{1 + \cos x} = 1^3 \cdot \frac{1}{2} = \boxed{\frac{1}{2}}$$

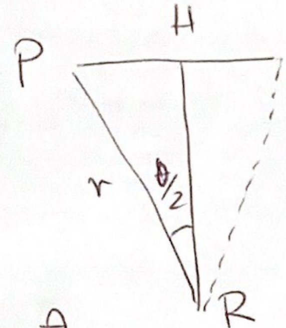
$\boxed{2}$



$A(\theta) =$ area of semicircle

$B(\theta) =$ area of $\widehat{PQR}$

Set $\overline{PR} = r = \overline{QR}$



$$\boxed{\lim_{\theta \to 0} \frac{1}{2} \pi \tan\left(\frac{\theta}{2}\right) = \boxed{0}}$$

We have $RH = r \cdot \cos \frac{\theta}{2}$

$PH =$ radius of circle $= r \cdot \sin \frac{\theta}{2}$

$$A(\theta) = \frac{1}{2}\left[\pi \cdot \left(r \cdot \sin\left(\frac{\theta}{2}\right)\right)^2\right] = \frac{1}{2} \pi r^2 \cdot \sin^2\left(\frac{\theta}{2}\right)$$

$$B(\theta) = \frac{1}{2}\left[r \cos\left(\frac{\theta}{2}\right)\right] \cdot \left[2r \sin\left(\frac{\theta}{2}\right)\right] = r^2 \sin\left(\frac{\theta}{2}\right) \cos\left(\frac{\theta}{2}\right)$$

$$\frac{A(\theta)}{B(\theta)} = \frac{\frac{1}{2} \pi r^2 \sin^2(\theta/2)}{r^2 \sin(\theta/2) \cos(\theta/2)} = \boxed{\frac{1}{2} \pi \tan(\theta/2)}$$