WORKSHEET #13

1. (a) the derivative of any constant is zero.

(b) the derivative of $cf(x)$ is $c f'(x)$

(c) the derivative of a sum is the sum of the derivatives

$$\left(f(x)+g(x)\right)' = f'(x) + g'(x)$$

(d) the derivative of x^n is nx^{n-1}

(e) the derivative of the product $f(x)g(x)$ is $f'(x)g(x) + f(x)g'(x)$

(f) the derivative of the quotient $\frac{f(x)}{g(x)}$ is $\frac{f'(x)g(x) - f(x)g'(x)}{[g(x)]^2}$

The first 4 rules allow us to compute the derivative of any polynomial

Up to (f) we can take the derivative of any rational function

$\boxed{2.}$

(*) $\frac{d}{dx}\left[x^3 - 5x^4 + 32\right] = \frac{d}{dx}\left[x^3\right] - 5\frac{d}{dx}\left[x^4\right] + $

$+ \frac{d}{dx}\left[32\right] = 3x^2 - 5(4x^3) = \underline{3x^2 - 20x^3}$

(*) $\frac{d}{dx}\left[\pi^4 - \frac{17}{x^{3/2}}\right] = \frac{d}{dx}\left[\pi^4\right] - 17\frac{d}{dx}\left[x^{-3/2}\right]$

Constant

$= 0 - 17\left(-\frac{3}{2}x^{-3/2-1}\right)$

power rule works for any real exponent

$= \frac{51}{2}x^{-5/2} = \underline{\frac{51}{2x^{5/2}}}$

* $\frac{d}{dx}\left[(x^7 + 6x - 1)(3x^{1/2} + 5x^3 + 2 - 12x)\right]$

$= \left[\frac{d}{dx}(x^7 + 6x - 1)\right] \cdot (3x^{1/2} + 5x^3 + 2 - 12x) +$

$(x^7 + 6x - 1) \cdot \left[\frac{d}{dx}(3x^{1/2} + 5x^3 + 2 - 12x)\right]$

$= (7x^6 + 6)(3x^{1/2} + 5x^3 + 2 - 12x) +$

$+ (x^7 + 6x - 1)\left(\frac{3}{2}x^{-1/2} + 15x^2 - 12\right)$

(*) $$\frac{d}{dx}\left[\frac{(2+3x)^2}{4-x^2}\right] = \frac{d}{dx}\left[\frac{(2+3x)\cdot(2+3x)}{4-x^2}\right]$$

$$= \frac{\overbrace{\left[3(2+3x)+(2+3x)\cdot 3\right]}^{\text{product rule}}(4-x^2) - (2+3x)^2\cdot(-2x)}{(4-x^2)^2}$$

$$= \frac{6(2+3x)(4-x^2) - (4+12x+9x^2)(-2x)}{(4-x^2)^2}$$

$$= \frac{48 - 12x^2 + 72x \cancel{-18x^3} + 8x + 24x^2 \cancel{+18x^3}}{(4-x^2)^2}$$

$$= \boxed{\frac{12x^2 + 80x + 48}{(4-x^2)^2}}$$

3. given $f(7) = a^2$, $f'(7) = ab^3$; $g(7) = a^2b$; $g'(7) = ab$

(*) $h'(7) = f'(7)g(7) + f(7)g'(7)$

$$\stackrel{!}{=} (ab^3)(a^2b) + (a^2)(ab) = \underline{a^3b^4 + a^3b}$$

(*) $$h'(7) = \frac{f'(7)g(7) - f(7)g'(7)}{[g(7)]^2} = \frac{(ab^3)(a^2b) - a^2(ab)}{(a^2b)^2}$$

$$= \frac{a^3b^4 - a^3b}{a^4b^2} = \frac{a^3b(b^3-1)}{a^4b^2} = \boxed{\frac{b^3-1}{ab}}$$

$$\circledast \quad h'(7) = [f'(7)g(7) + f(7)g'(7)] - 6\,\frac{0 \cdot f(7) - 1 \cdot f'(7)}{[f(7)]^2}$$

$$= f'(7)g(7) + f(7)g'(7) + \frac{6f'(7)}{[f(7)]^2}$$

$$= ab^3(a^2b) + a^2(ab) + \frac{6ab^3}{[a^2]^2}$$

$$= \boxed{a^3b^4 + a^3b + \frac{6b^3}{a^3}}$$