

WORK SHEET 14

1.

$$* f(x) = (5x^2 + 6)^{-1/2}$$

use the power chain rule

$$f'(x) = -\frac{1}{2} (5x^2 + 6)^{-3/2} \cdot (10x)$$

$$= \frac{-5x}{(5x^2 + 6)^{3/2}}$$

$$* g(x) = x \cdot (6x - 2)^5$$

$$g'(x) = 1 \cdot (6x - 2)^5 + x \cdot 5(6x - 2)^4 \cdot 6$$

$$= (6x - 2)^4 [(6x - 2) + 30x]$$

$$= (6x - 2)^4 (36x - 2)$$

$$* F(x) = x f(x) + g(f(x))$$

$$F'(x) = \underbrace{f(x) + x f'(x)}_{\text{product rule}} + \underbrace{g'(f(x)) \cdot f'(x)}_{\text{chain rule}}$$

2.

$$f(x) = \frac{x^2 - 4}{x^2 + 4}$$

$$f'(x) = \frac{2x(x^2 + 4) - (x^2 - 4)(2x)}{(x^2 + 4)^2}$$

$$= \frac{\cancel{2x^3} + 8x - \cancel{2x^3} + 8x}{(x^2 + 4)^2}$$

$$= \frac{16x}{(x^2 + 4)^2}$$

$$f''(x) = \frac{16 \cdot (x^2 + 4)^2 - 16x \cdot [2(x^2 + 4) \cdot 2x]}{[(x^2 + 4)^2]^2}$$

$$= \frac{16(x^2 + 4) \left[(x^2 + 4) - 4x^2 \right]}{(x^2 + 4)^4}$$

$$= \boxed{\frac{16(4 - 3x^2)}{(x^2 + 4)^3}}$$

3.

$$p(t) = 23 - 3t - 9.8t^2$$

$$v(t) = \text{velocity} = \frac{dp}{dt} = \underline{-3 - 9.8(2t)}$$

$$a(t) = \text{acceleration} = \frac{dv}{dt} = \frac{d^2p}{dt^2}$$

$$= -9.8 \cdot 2 = \underline{\underline{-19.6}}$$

4.

$$f(x) = x^n$$

$$f'(x) = n \cdot x^{n-1}$$

$$f''(x) = n(n-1)x^{n-2}$$

$$f'''(x) = n(n-1)(n-2) \cdot x^{n-3}$$

⋮

$$f^{(n)}(x) = n(n-1)(n-2) \cdots 3 \cdot 2 \cdot 1$$

$$= \underline{\underline{n!}}$$