WORKSHEET #15

1. Sometimes it is difficult/impossible to solve explicitly for y as a function of x. At other times solving for y can lead to a piecewise function which takes extra steps to differentiate. In either case, it may be easier to use implicit differentiation, especially if you don't need to know an explicit equation for y.

2. ⊛ Suppose $x^2 + y^3 = 5xy - 5$ and $P(2,1)$ is a point on that curve. Indeed

$$2^2 + 1^3 \stackrel{?}{=} 5(2)(1) - 5$$

$$\Longrightarrow \quad 5 \stackrel{?}{=} 5 \quad \underline{\underline{\text{Yes}}}$$

Take the derivative of both sides wrt x

$$\frac{d}{dx}\left(x^2+y^3\right)=\frac{d}{dx}\left(5xy-5\right)$$

$$2x+\underbrace{3\cdot y^2\cdot\frac{dy}{dx}}_{\text{chain rule}}=\underbrace{5(1)y+5x\cdot\frac{dy}{dx}}_{\text{product rule}}$$

$$3y^2\frac{dy}{dx}-5x\frac{dy}{dx}=5y-2x$$

$$\therefore\quad\frac{dy}{dx}=\frac{(5y-2x)}{(3y^2-5x)}$$

$$\left.\frac{dy}{dx}\right|_{(2,1)}=\frac{5(1)-2(2)}{3(1)^2-5(2)}=\frac{1}{-7}=-1/7$$

Hence the equation of the tg line is

$$\underline{(y-1)=-1/7(x-2)}$$

$$7y-7=-x+2$$

OR $\boxed{7y+x-9=0}$

(*) $$\sqrt{2x + \frac{32}{y^4}} = 6$$

Notice that $P(2,1)$ satisfies the equation as $\sqrt{2(2) + \frac{32}{(1)^4}} = 6$ ✓

Before we take the derivative we can rewrite the equation as

$$\frac{2xy^4 + 32}{y^4} = 6^2$$

OR $$2xy^4 + 32 - 36y^4 = 0$$

Now it is easier to take the derivative

$$\frac{d}{dx}\left[2xy^4 + 32 - 36y^4\right] = 0$$

$$\cancel{2}y^4 + \cancel{2}x\left(4y^3\frac{dy}{dx}\right) - 36\cdot\cancelto{2}{4}y^3\cdot\frac{dy}{dx} = 0$$

$$4xy^3\frac{dy}{dx} - 72y^3\frac{dy}{dx} = -y^4$$

$$\frac{dy}{dx}(4x - 72)\cdot y^3 = -y^4 \quad \therefore \boxed{\frac{dy}{dx} = \frac{-y}{4x-72}}$$

Hence $\left.\frac{dy}{dx}\right|_{(2,1)} = \frac{-1}{4(2)-72} = \frac{1}{64}$

and the equation of the tg line is

$$\boxed{y-1 = \frac{1}{64}(x-2)}$$ OR

$$64y - 64 = x - 2$$

$$\boxed{64y - x - 62 = 0}$$

Notice $\frac{d}{dx}\left[\sqrt{2x + 32y^{-4}}\right] = \frac{d}{dx}(6)$

gives $\frac{1}{2\sqrt{2x + 32y^{-4}}} \cdot \left(2 + 32(-4)y^{-5}\frac{dy}{dx}\right) = 0$

after simplifing the equation

$$1 - 64y^{-5} \cdot \frac{dy}{dx} = 0$$ So $$\boxed{\frac{dy}{dx} = \frac{y^5}{64}}$$

$$\left.\frac{dy}{dx}\right|_{(2,1)} = \frac{1^5}{64} = \frac{1}{64}$$

These are two equivalent expressions for dy/dx

$\boxed{3.}$

$$\frac{dV}{dt} = 0.06 \ m^3/sec$$

$$\frac{dr}{dt} = ? \quad \text{when } r = 0.1$$

We assume the lump can be represented by a sphere so

$$\boxed{V = \frac{4}{3}\pi r^3}$$

Take the derivative wrt time of the equation

$$\frac{dV}{dt} = \frac{4}{3}\pi \underbrace{\frac{d}{dt}(r^3)}_{\text{chain rule}}$$

$$\frac{dV}{dt} = \frac{4}{\cancel{3}}\pi\left(\cancel{3}r^2\frac{dr}{dt}\right)$$

So $\boxed{\frac{dr}{dt} = \frac{1}{4\pi r^2}\cdot\frac{dV}{dt}}$

with our data $\frac{dr}{dt} = \frac{1}{4\pi(0.1)^2}\ 0.06\ m/sec = 0.4775\ m/sec$

$\boxed{4.}$

$$\frac{1}{T} = \frac{1}{R} + \frac{1}{S}$$

$T = 20$ ohms constant ; $\frac{dT}{dt} = 0$

$\frac{dS}{dt} = -2$ ohms per minute

$\frac{dR}{dt} = ?$ when $S = 30$ ohms

$$\underbrace{\frac{d}{dt}\left(\frac{1}{T}\right)}_{=0} = \frac{d}{dt}\left(\frac{1}{R} + \frac{1}{S}\right)$$

as T is constant at 20 ohms

Thus

$$0 = \frac{d}{dt}(R^{-1}) + \frac{d}{dt}(S^{-1})$$

$$0 = \frac{d}{dR}(R^{-1}) \cdot \frac{dR}{dt} + \frac{d}{ds}(S^{-1}) \cdot \frac{ds}{dt}$$

$$0 = -\frac{1}{R^2}\frac{dR}{dt} - \frac{1}{S^2}\frac{dS}{dt}$$, Thus

$$\boxed{\frac{dR}{dt} = -\frac{R^2}{S^2} \cdot \frac{dS}{dt}}$$

with our data

$\frac{dS}{dt} = -2$ $S = 30$

and $\boxed{\frac{1}{20} = \frac{1}{R} + \frac{1}{30}}$

$R = 60$

$\frac{dR}{dt} = +8$ ohm/min