WORKSHEET 18

1. $f(x) \approx \underbrace{L(x) = f(x_0) + f'(x_0)(x - x_0)}$

Geometrically, the linearization $L(x)$ of f at x_0 is the equation of the tangent line to the graph of f at the point $(x_0, f(x_0))$

If $|x - x_0|$ is sufficiently small $f(x)$ can be "linearly" approximated by $L(x)$.

2. Find $L(x)$ to $f(x) = \sqrt{x^2+3}$ at $x_0 = 1$

$$f(1) = \sqrt{1^2+3} = 2$$

$$f'(x) = \frac{1}{\cancel{2}}(x^2+3)^{-1/2} \cdot (\cancel{2}x) = \frac{x}{\sqrt{x^2+3}}$$

$$f'(1) = \frac{1}{\sqrt{1^2+3}} = \frac{1}{2}$$

Thus $L(x) = \underset{f(1)}{2} + \underset{f'(1)}{\frac{1}{2}}(x-1)$

$L(x) = 2 + \frac{1}{2}x - \frac{1}{2} \rightsquigarrow \boxed{L(x) = \frac{1}{2}x + \frac{3}{2}}$

$\boxed{3.}$ $f(x) = \sqrt{5x^2 - 9}$ at $x_0 = 3$

$f(3) = \sqrt{5 \cdot 9 - 9} = \sqrt{36} = 6$

$f'(x) = \dfrac{1}{2\sqrt{5x^2-9}} \cdot (10x) = \dfrac{5x}{\sqrt{5x^2-9}}$

$f'(3) = \dfrac{15}{\sqrt{36}} = \dfrac{15}{6} = \dfrac{5}{2} = 2.5$

Thus the linearization is

$$L(x) = 6 + 2.5(x-3)$$

$f(3.14) \approx L(3.14) = 6 + 2.5(3.14 - 3)$

$= 6 + 2.5(0.14) = \underline{\underline{6.35}}$

$\boxed{4.}$ We know that $B(1) = 10$ grams

Also $\dfrac{1}{B(t)} \dfrac{dB}{dt} = 0.01$ is given in the text of the problem.

So $\dfrac{dB}{dt} = 0.01 \cdot B(t)$. At $t = 1$

we get $\left.\dfrac{dB}{dt}\right|_{t=1} = 0.01 \cdot B(1) = 0.01 \cdot 10$
$= \boxed{0.1}$

Thus the linearization of the biomass at $t = 1$ is:

$$L(t) = B(1) + \left.\frac{dB}{dt}\right|_{t=1} \cdot (t-1)$$

$$= 10 + 0.1(t-1)$$

Hence

$$B(1.1) \approx L(1.1) = 10 + 0.1\,(1.1 - 1)$$
$$= 10 + 0.1\,(0.1) = 10 + 0.01 = \boxed{10.01}$$