1. \( f(x) = \frac{x^2 - 9}{x^2 + 9} \) First find critical values

\[
f'(x) = \frac{2x(x^2 + 9) - (2x(x^2 - 9))}{(x^2 + 9)^2} = 0
\]

\( x^2 + 9 \neq 0 \) for any \( x \)

\[
2x(x^2 + 9) - 2x(x^2 - 9) = 0
\]

Factor a 2x

\[
2x(x^2 + 9 - x^2 + 9) = 0
\]

\( 2x = 0 \) \( x^2 + 9 \neq 0 \)

\( x = 0 \) \( 13 = 0 \) (can't happen)

So \( x = 0 \) is our only critical value

We need to test

\(-9, 0, \text{ and } 9 \) into \( f(x) \)

\( f(9) = f(-9) = \frac{(-9)^2 - 9}{(-9)^2 + 9} = \frac{72}{81} = \frac{4}{9} = \frac{8}{18} = 0.8 \)

\( f(0) = \frac{-9}{9} = -1 \)

\( \max \text{ at } (-9, 0.8) \)

\( \min \text{ at } (0, -1) \)
2. \( f(x) = 2x - 3 \ln(x) \quad 0 \leq x \leq 3 \)

\[ f'(x) = 2 - \frac{3}{x} = 0 \]

\[ x = \frac{3}{2} \]

So we need to test:

\( f\left(\frac{3}{2}\right) = 1.78 \)  

\( f(5) = 5.172 \)

3. \( f(x) \) is continuous on \([0, 4]\)

\( f(x) \) is differentiable on \((0, 4)\)

So there is a \( c \) in \([0, 4]\) such that

\[ f(c) = \frac{\frac{2}{3}c^3 - 6c}{4 - 0} = \frac{\frac{2}{3}c^3 - 6c}{4} \]

\[ f'(c) = \frac{2c^2}{3} = \frac{14}{3} \]

Thus

\[ c = \frac{14}{3} \]

Not in \([0, 4]\)
4. \( f'(x) = \frac{2}{3} x^{-\frac{1}{3}} = \frac{2}{3\sqrt[3]{x}} \) which is undefined at \( x=0 \)

So \( f \) is not differentiable at \( x=0 \)

(It has a corner point) so it does not satisfy MVT