1. Find an equation for the line through the points $(-4,-3)$ and $(1,7)$

Use the formula for slope: $\dfrac{y_2 - y_1}{x_2 - x_1}$

$$\rightarrow \quad \frac{7-(-3)}{1-(-4)} = \frac{10}{5} = 2$$

So, $m = 2$. Then either use point slope or, $y = mx + b$. I will use $y = mx + b$:

$$7 = (2)(1) + b$$
$$7 = 2 + b$$
$$b = 5$$

so the equation is $\boxed{y = 2x + 5}$

$\boxed{2}$ Find a simplification of the expression:

$\dfrac{f(x+h) - f(x)}{h}$ where $f(x) = x^2 + 5$

$f(x+h)$ means take every x in $f(x)$ and plug in $(x+h)$

$\Rightarrow$ So $f(x+h) = (x+h)^2 + 5$. Let's use the expression:

$$\frac{f(x+h) - f(x)}{h} = \frac{(x+h)^2 + 5 - (x^2+5)}{h}$$

Foil/Expand $= \dfrac{(x^2 + 2xh + h^2) + 5 - x^2 - 5}{h}$

Combine terms $= \dfrac{2xh + h^2}{h}$

Factor $= \dfrac{h(2x+h)}{h} = \boxed{2x+h}$

3. From left to right,

$f(x)+3$, $3f(x)$, $f(x-3)$

Moves up 3 — Stretches by a factor of 3 — Moves right 3.

4. Consider $f(x)=2x-4$ and $g(x)\ \frac{1}{x+2}$

Find $(g\circ f)(x)$.

We will want to take g and plug in all of $f(x)$ into g. So $g(f(x)) = \frac{1}{(2x-4)+2}$.

★ Note if $x=-2$ we have a problem as $g(-2) = \frac{1}{0}$.

Then simplify: $\frac{1}{2x-2} = g\circ f(x)$

Here, note that $x \neq 1$. As $g\circ f(1) = \frac{1}{0}$.

So, the restrictions on x are $x \neq 1$ and $x \neq -2$

$\boxed{5}$ Let $f(x) = \frac{2x+7}{3-4x}$. Find the inverse, $f^{-1}(x)$ of f.

What are the domains of $f(x)$ and $f^{-1}(x)$.

- To find an inverse,
  1. Replace $f(x)$ with y.
  2. Replace x with y and y with x.
  3. Solve back for y.

1. $y = \frac{2x+7}{3-4x}$ 2. $x = \frac{2y+7}{3-4y}$

3. $x = \frac{2y+7}{3-4y}$

$(3-4y)(x) = 2y+7$

$\to 3x-4yx = 2y+7$ Move all "y" to same side

$\to 3x-7 = 2y+4yx$ Factor out y

$\to 3x-7 = (2+4x)(y)$ Divide

$\to \frac{3x-7}{2+4x} = y = f^{-1}(x)$.

5 cont. So $f(x) = \dfrac{2x+7}{3-4x}$ and $f^{-1}(x) = \dfrac{3x-7}{2+4x}$

Domain: We always look for dividing by zeros and power/root complications. Here, we have division so we look at dividing by zero

$f(x)$ divides by zero if the denominator equals 0.

So, we set $3-4x=0$ and solve.

$$3 = 4x$$

$$x = \frac{3}{4}$$ So $f(x)$ has domain : $(-\infty, 3/4) \cup (3/4, \infty)$

$f^{-1}(x)$ also has a divide by zero problem. We set its denominator to 0 and solve,

$$2+4x=0$$

$$2 = -4x$$

$$x = -\frac{1}{2}$$ So $f^{-1}(x)$ has domain : $(-\infty, -1/2) \cup (-1/2, \infty)$

★Note The domain of $f(x)$ $\longleftrightarrow$ range of $f^{-1}(x)$

The domain becomes the range of the inverse and vice versa. If you check the range of $f(x)$, what do you notice?

6. Find the center and radius of the circle with the following equation:

$$x^2 + y^2 + 10x - 6y + 30 = 0.$$

This equation fully defines the circle, but gives no info about the center and radius. We want to convert the equation into center-radius form:

$$(x-h)^2 + (y-k)^2 = r^2$$

where (h,k) = center and r = radius.

To do this, we must complete the square:

First, let's group our like terms:

$$(x^2 + 10x) + (y^2 - 6y) = -30.$$

For "x", let's complete the square. We need to add the following things to both sides,

$\left(\frac{b}{2}\right)^2$ where b = the number next to x

in this case $b = 10$.

So, $\left(\frac{10}{2}\right)^2 = 25$, we add 25 to both sides like so,

$$(x^2 + 10x + \underline{25}) + y^2 - 6y = -30 + \underline{25}.$$

Now, we do this with y: Note the "b" = -6.

$$\left(\frac{-6}{2}\right)^2 = 9 \implies$$

$$(x^2 + 10x + 25) + (y^2 - 6y + \underline{9}) = -30 + 25 + \underline{9}$$

$$\downarrow \qquad \downarrow \qquad \downarrow$$

$$(x+5)^2 + (y-3)^2 = 4$$

We are now in the right form! The center is the negative of the numbers in the parenthesis, radius is square root of right side

$\boxed{\text{Center} = (-5, 3) \quad \text{radius} = \sqrt{4} = 2.}$

☆ Note to complete the square if x^2 has a coefficient, like $2x^2 + 20x + \ldots = -30$ first, factor out x^2's coefficient,

$2(x^2 + 10x)$ then complete the square as normal,

$$2(x^2 + 10x + 25) + \ldots = -30 + 2(25)$$

but add the outside factor times $\left(\frac{b}{2}\right)^2$