If $f$ satisfies conditions for MVT then

1. $f'(c) = \frac{f(b) - f(a)}{b - a}$ for some $c \in [a, b]$.

So in our case

$$f'(c) = \frac{f(6) - f(-1)}{6 - (-1)}$$

Since $3 \leq f'(x) \leq 10$ for all $x$ in $[-1, 6]$.

Satisfying

$$3 \leq f'(c) \leq 10$$

Since $c \in [-1, 6]$ and $f(-1) = 3$

$$3 \leq \frac{f(6) - f(-1)}{6 - (-1)} \leq 10$$

$$3 \leq f(6) - 3 \leq 10$$

Multiply by 7

$$21 \leq f(6) - 3 \leq 70$$

Add 3

$$24 \leq f(6) \leq 73$$

Thus $73$ is the largest possible value for $f(6)$.

2. Like in the first problem.

Thus we can say that $f(3)$ will be in

$$-3 \leq \frac{f(3) - f(-1)}{3 - (-1)} \leq 2$$

$$-3 \leq \frac{f(3) - 75}{4} \leq 2$$

$$-12 \leq f(3) - 75 \leq 8$$

thus $63 \leq f(3) \leq 83$. 

3. \( f(x) = 2x^3 - 14x^2 + 48x + 21 \)

\[
f'(x) = 6x^2 - 28x + 48
\]

Set equal to 0

\[
6x^2 - 28x + 48 = 0
\]

Under \( \sqrt{ } \) is negative

\[
\frac{28 \pm \sqrt{(28)^2 - 4 \cdot 6 \cdot 48}}{12} = x
\]

So this does not equal zero

Also notice

\[
6(0)^2 - 28(0) + 48 = 48
\]

So \( f'(x) \) is always positive

6 So \( f(x) \) is always increasing

for all real numbers

4. If \( f'(x) = x(x+2)(x-3) \)

Our critical values will be

0, -2, 3.

We will test -3, -1, 1, 4

\[
f(-3) = -3(-1)(-6) = -18
\]

\[
f(-1) = -(1)(-4) = 4
\]

\[
f(1) = 1(3)(-2) = 6
\]

\[
f(4) = 4(6)(1) = 24
\]

So \( f \) is increasing \(( -2, 0 ) \cup ( 3, \infty )\)