WS 20

If f satisfies conditions for MVT then

1. $f'(c) = \frac{f(b)-f(a)}{b-a}$ for some $c \in [a,b]$ (in)

So in our case

$$f'(c) = \frac{f(6)-f(-1)}{6-(-1)}$$

Since $3 \leq f'(x) \leq 10$ for all x in $[-1,6]$

Substituting

$3 \leq f'(c) \leq 10$ since $c \in [-1,6]$

& $[f(-1)=3]$

$$3 \leq \frac{f(6)-3}{6-(-1)} \leq 10$$

$$3 \leq \frac{f(6)-3}{7} \leq 10$$

Mult. by 7

$$21 \leq f(6)-3 \leq 70$$

add 3

$$24 \leq f(6) \leq 73$$

Thus 73 is the largest possible value for f(6)

2. Like in the first problem

$$-3 \leq \frac{f(3)-f(-1)}{3-(-1)} \leq 2$$

$$-3 \leq \frac{f(3)-75}{4} \leq 2$$

$$-12 \leq f(3)-75 \leq 8$$

$$63 \leq f(3) \leq 83$$

Thus we can say that f(3) will be in between 63 and 83

3. $f(x) = 2x^3 - 14x^2 + 48x + 21$

$f'(x) = 6x^2 - 28x + 48$

set equal to zero

$$6x^2 - 28x + 48 = 0$$

$$\frac{28 \pm \sqrt{(-28)^2 - 4 \cdot 6 \cdot 48}}{12} = x$$

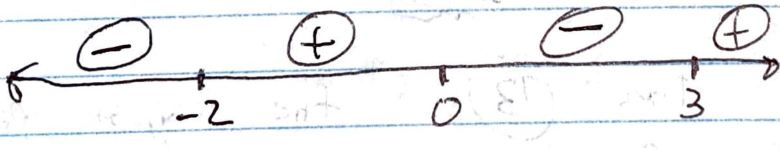
under $\sqrt{\ }$ is negative so this does not equal zero

also notice

$$6(0)^2 - 28(0) + 48 = 48$$

so $f'(x)$ is always positive
so $f(x)$ is always increasing for all real numbers

4. If $f'(x) = x(x+2)(x-3)$
our critical values will be
$0, -2, 3$.



We will test $-3, -1, 1, 4$

$f'(-3) = -3(-1)(-6) = -18$
$f'(-1) = -1(1)(-4) = +4$
$f'(1) = 1(3)(-2) = -6$
$f'(4) = 4(6)(1) = +24$

so f is increasing $(-2, 0) \cup (3, \infty)$