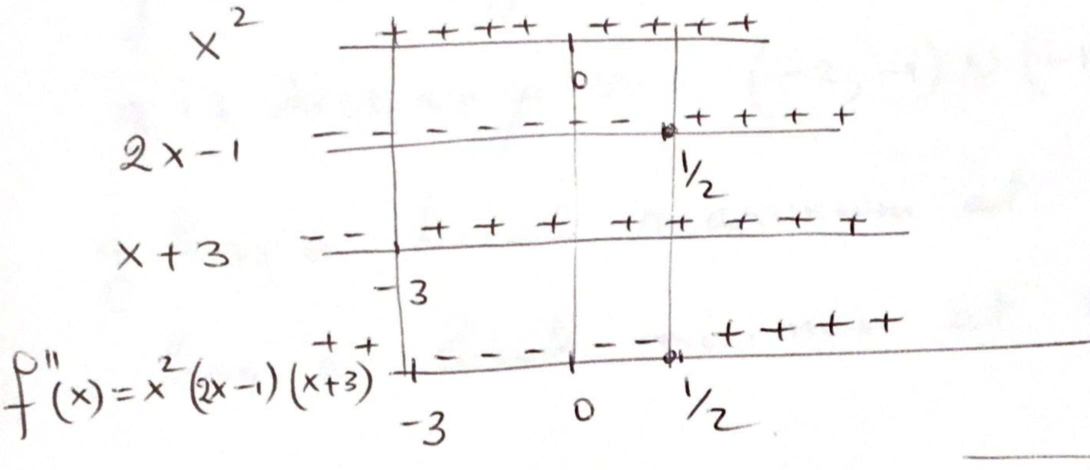
WORKSHEET 21

1. $f''(x) = x^2(2x-1)(x+3)$

The points where $f''(x) = 0$ are

$x = 0, \quad x = \frac{1}{2}, \quad x = -3$

To have an inflection point we need to have a change of sign in f''.

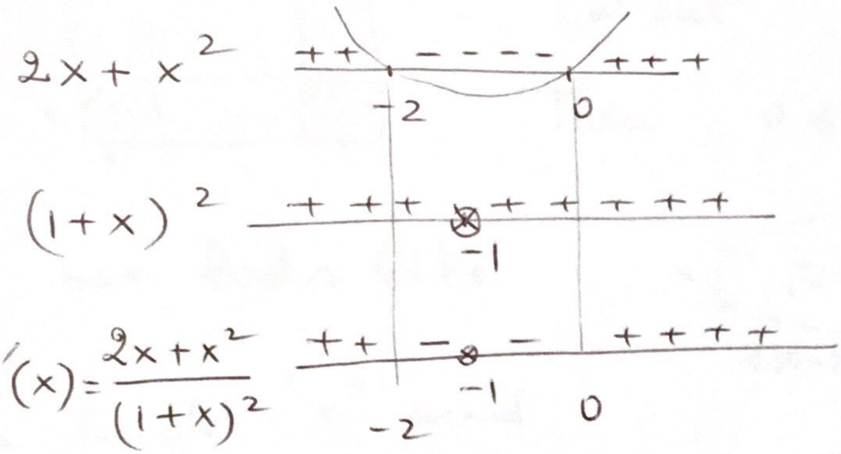


hence only $\boxed{x = -3}$ and $\boxed{x = \frac{1}{2}}$ are inflection points

2. $g'(x) = \dfrac{2x + x^2}{(1+x)^2} \qquad x \neq -1$

$g'(x) = 0 \iff 2x + x^2 = 0$

$x(2+x)=0 \iff x=0$ or $x=-2$



g is increasing on $(-\infty,-2)\cup(0,+\infty)$

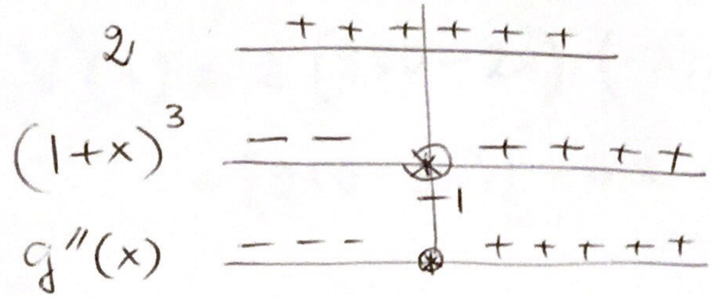
g is decreasing on $(-2,-1)\cup(-1,0)$

g has a local maximum at $x=-2$

g has a local minimum at $x=0$

(b) $g''=\frac{2}{(1+x)^3}$ $\quad x=-1$

$g''(x)=0$ never



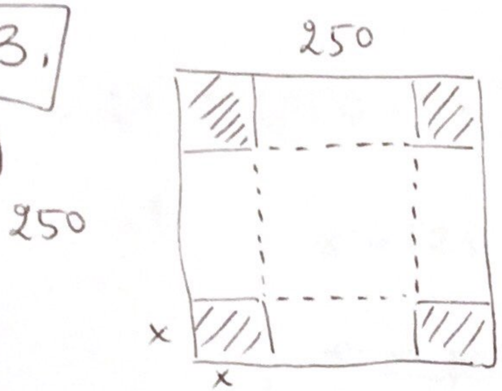
hence g is concave up on $(-1,+\infty)$

g is concave down on $(-\infty,-1)$

There is no inflection point

$\boxed{3.}$

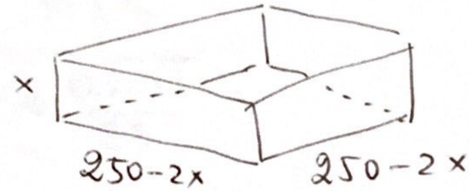
(a)



Let x be the length of the site of the corner cut out.

Then $\quad 0 \le x \le 125 = \frac{250}{2}$

The box looks like



has height "x" and sides of length "$250-2x$"

the volume is $V(x) = \underbrace{(250-2x)^2}_{\text{area base}} \cdot \underbrace{x}_{\text{height}}$

We need to <u>maximize</u>

$$\boxed{V(x) = (250-2x)^2 \cdot x \quad \text{on} \quad 0 \le x \le 125}$$

By the Corollary to the EVT we need to check the value of $V(x)$ at the end points and at the critical numbers

$$V'(x) = 2(250-2x)\cdot(-2)x + (250-2x)^2 \cdot 1$$
$$= (250-2x)\left[-4x + 250 - 2x\right]$$

$$V'(x) = (250-2x)\left(250-6x\right) = 0$$

at $\quad x = 125 \quad , \quad x = \dfrac{250}{6} = \dfrac{125}{3}$

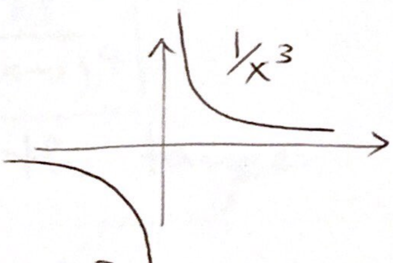
	end points		critical numbers
x	0	125	$125/3$
$V(x)$	0	0	$2\dfrac{(250)^3}{27}$

$$V\left(\frac{125}{3}\right) = \left(250 - 2\cdot\frac{125}{3}\right)^2 \cdot \frac{125}{3} = (250)^2\left(1-\frac{1}{3}\right)^2 \cdot \frac{125}{3}$$

$$= (250)^2 \cdot \left(\frac{2}{3}\right)^2 \frac{125}{3} = \frac{2\,(250)^3}{27}$$

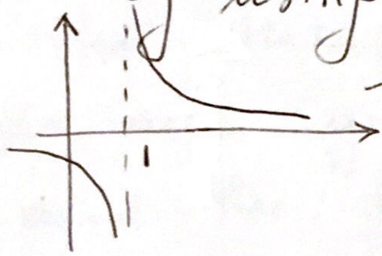
max volume attained at $\boxed{x = \dfrac{125}{3}}$

(b) graph of $y = \dfrac{1}{x^3}$

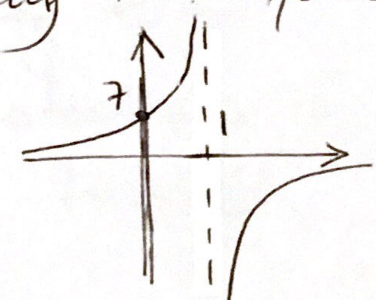


Our function is $y = -\dfrac{7}{(x-1)^3} - 1$ so we can obtain it by using elementary transformation

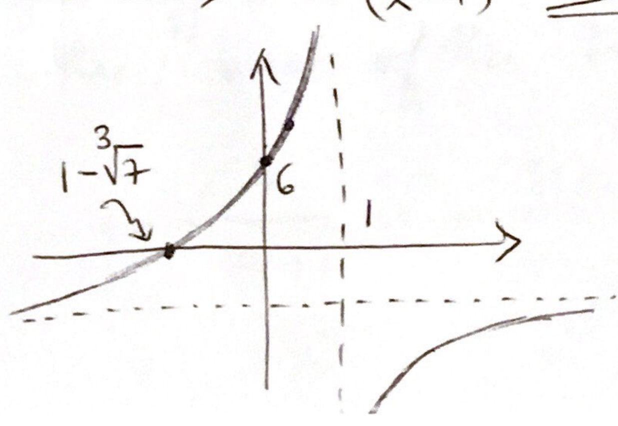
$\dfrac{1}{(x-1)^3}$



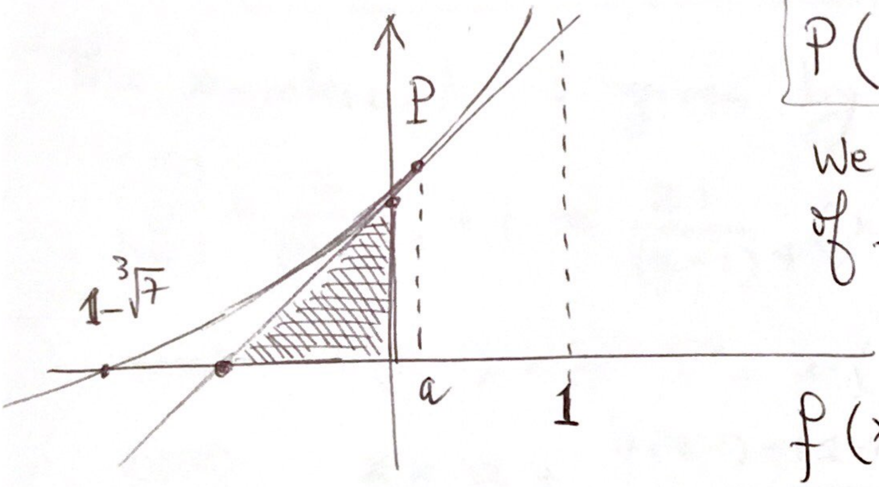
$\rightsquigarrow \quad -\dfrac{7}{(x-1)^3}$



$y = -\dfrac{7}{(x-1)^3} - 1$



Pick now an arbitrary point P on the curve with coordinate a such that $\boxed{1-\sqrt[3]{7} \le a < 1}$



$$\boxed{P\left(a, -\frac{7}{(a-1)^3} - 1\right)}$$

We need the derivative of $f(x)$ to write the tg line at P

$$f(x) = -7(x-1)^{-3} - 1$$

So $\boxed{f'(x) = 21(x-1)^{-4} = \frac{21}{(x-1)^4}}$

Hence the equation of the tangent line at P is:

$$\boxed{y + \frac{7}{(a-1)^3} + 1 = \frac{21}{(a-1)^4}(x-a)}$$

We need the intercepts of the tg. line with the x-axis and y-axis to determine the height and the base of the triangle.

the y-intercept is: $y = -\frac{7}{(a-1)^3} - 1 - \frac{21a}{(a-1)^4} =$

$$= \frac{-7(a-1) - (a-1)^4 - 21a}{(a-1)^4}$$

$$y = \frac{-7a+7 - (a^4 - 4a^3 + 6a^2 - 4a + 1) - 21a}{(a-1)^4}$$

$$= \boxed{\frac{-a^4 + 4a^3 - 6a^2 - 24a + 6}{(a-1)^4}}$$

this must be a positive number!

The x-intercept is given by solving the equation

$$\boxed{0} + \frac{7}{(a-1)^3} + 1 = \frac{21}{(a-1)^4}(x-a)$$

$$\Longleftrightarrow \quad 7(a-1) + (a-1)^4 = 21(x-a)$$

$$\Longleftrightarrow \quad x = a + \frac{7(a-1) + (a-1)^4}{21}$$

$$= \frac{21a + 7a - 7 + a^4 - 4a^3 + 6a^2 - 4a + 1}{21}$$

$$= \boxed{\frac{a^4 - 4a^3 + 6a^2 + 24a - 6}{21}}$$

notice that this number must be negative (it is in the second quadrant)

Hence the area of the triangle is:

$$\boxed{\text{Area} = \frac{(a^4 - 4a^3 + 6a^2 + 24a - 6)^2}{42\,(a-1)^4} \quad \text{over} \quad 1 - \sqrt[3]{7} \le a < 1}$$

We would need to study the sign of the derivative of the function

$A = \text{Area}(a)$

$$\frac{dA}{da} = \frac{1}{42} \cdot \frac{2(a^4-4a^3+6a^2+24a-6)' \cdot (4a^3-12a^2+12a+24)(a-1)^4 - (a^4-4a^3+6a^2+24a-6)^2 (4(a-1)^3)}{[(a-1)^4]^2}$$

$$= \frac{2(a^4-4a^3+6a^2+24a-6)(a-1)^3 \cdot \left[(4a^3-12a^2+12a+24)(a-1) - 2(a^4-4a^3+6a^2+24a-6) \right]}{42(a-1)^8}$$

$$= \frac{\cancel{2}(a^4-4a^3+6a^2+24a-6)\cancel{(a-1)^3} \left[\begin{array}{l} 4a^4-12a^3+12a^2+24a \\ -4a^3+12a^2-12a-24 \\ -2a^4+8a^3-12a^2-48a+12 \end{array} \right]}{\underset{21}{\cancel{42}}(a-1)^{\cancel{8}\,5}}$$

$$= \frac{(a^4-4a^3+6a^2+24a-6)(2a^4-8a^3+12a^2-36a-12)}{21(a-1)^5}$$

now we would need to find the roots of that derivative ...