1. \[ f''(x) = x^2 (2x-1)(x+3) \]
   
   The points where \( f''(x) = 0 \) are
   
   \[ x = 0, \ x = \frac{1}{2}, \ x = -3 \]

   To have an inflection point we need to have a change of sign in \( f'' \).

   \[
   \begin{array}{c|cccc}
   x^2 & + & + & + & + \\
   2x-1 & - & - & - & + \\
   x+3 & - & + & + & + \\
   \hline
   f''(x) = x^2(2x-1)(x+3) & + & + & - & + \\
   \end{array}
   \]

   hence only \( [x = -3] \) and \( [x = \frac{1}{2}] \) are inflection points

2. \[ g'(x) = \frac{2x+x^2}{(1+x)^2} \quad x \neq -1 \]

   \[ g'(x) = 0 \quad \Rightarrow \quad 2x + x^2 = 0 \]
\( x(2 + x) = 0 \) \( \iff \) \( x = 0 \) or \( x = -2 \)

\[
\begin{array}{cccc}
2x + x^2 & + & - & + \\
\hline
-2 & 0 & \times & + & + & + \\
\end{array}
\]

\[
(1 + x)^2 & + & + & + & + & + & + \\
\hline
-1 & - & - & + & + & + & + \\
\end{array}
\]

\[
g'(x) = \frac{2x + x^2}{(1 + x)^2} & + & - & - & + & + & + \\
\hline
-2 & 1 & 0 & + & + & + & + \\
\end{array}
\]

\( g \) is increasing on \( (-\infty, -2) \cup (0, +\infty) \)

\( g \) is decreasing on \( (-2, -1) \cup (-1, 0) \)

\( g \) has a local maximum at \( x = -2 \)

\( g \) has a local minimum at \( x = 0 \)

(b) \( g'' = \frac{2}{(1 + x)^3} \quad x = -1 \)

\[
g''(x) = 0 \quad \text{never} \quad \text{hence} \quad g \text{ is concave up on } (-1, +\infty)
\]

\[
\begin{array}{cccc}
2 & + & + & + & + & + & + \\
\hline
(1 + x)^3 & - & - & + & + & + & + \\
\end{array}
\]

\[
g''(x) & - & - & - & + & + & + \\
\hline
1 & 1 & 0 & + & + & + & + \\
\end{array}
\]

\( g \) is concave down on \( (-\infty, -1) \)

There's no inflection point
Let \( x \) be the length of the side of the corner cut out.

Then \( 0 \leq x \leq 125 = \frac{250}{2} \)

The box looks like

has height "\( x \)" and

sides of length "\( 250 - 2x \)"

The volume is \( V(x) = (250 - 2x)^2 \cdot x \)

We need to maximize

\[
V(x) = (250 - 2x)^2 \cdot x \quad \text{on} \quad 0 \leq x \leq 125
\]

By the Corollary to the EVT we need to check the value of \( V(x) \) at the end points and at the critical numbers.

\[
V'(x) = 2(250 - 2x) \cdot (-2)x + (250 - 2x)^2 \cdot 1
\]

\[
= (250 - 2x) \left[ -4x + 250 - 2x \right]
\]
\[ V'''(x) = (250 - 2x)(250 - 6x) = 0 \]

at \[ x = 125 \quad , \quad x = \frac{250}{6} = \frac{125}{3} \]

end points \[ x \quad \text{critical numbers} \quad \frac{125}{3} \]

\[
\begin{array}{c|c|c|c}
\text{end points} & 0 & 125 & \frac{125}{3} \\
\text{critical numbers} & \frac{2}{27} (\frac{250}{3})^3 & \end{array}
\]

\[ V(\frac{125}{3}) = (250 - 2\cdot\frac{125}{3})^2, \quad \frac{125}{3} = (250)^2 \left(1 - \frac{1}{3}\right)^2, \quad \frac{125}{3} = (250)^2 \cdot \left(\frac{2}{3}\right)^2 \frac{125}{3} = \frac{2}{27} (250)^3 \]

max volume attained at \[ x = \frac{125}{3} \]

(b) graph of \[ y = \frac{1}{x^3} \]

Our function is \[ y = -\frac{7}{(x-1)^3} - 1 \] so we can obtain it by using elementary transformations.
Pick now an arbitrary point $P$ on the curve with coordinates $a$ such that $1 - \sqrt[3]{7} \leq a < 1$.

We need the derivative $f'(x)$ to write the tangent line at $P$.

So $f'(x) = 21(x-1)^{-4} = \frac{21}{(x-1)^4}$.

Hence the equation of the tangent line at $P$ is:

$$y + \frac{7}{(a-1)^3} + 1 = \frac{21}{(a-1)^4} (x - a)$$

We need the intercepts of the tangent line with the $x$-axis and $y$-axis to determine the height and the base of the triangle.

The $y$-intercept is:

$$y = -\frac{7}{(a-1)^3} - \frac{21a}{(a-1)^4} = -\frac{7(a-1) - (a-1)^4 - 21a}{(a-1)^4}$$
\[ y = \frac{-7a + 7 - (a^4 - 4a^3 + 6a^2 - 4a + 1) - 21a}{(a-1)^4} \]

\[ = \frac{-a^4 + 4a^3 - 6a^2 - 24a + 6}{(a-1)^4} \]

This must be a positive number!

The x-intercept is given by solving the equation

\[ \frac{7}{(a-1)^3} + 1 = \frac{21}{(a-1)^4} (x-a) \]

\[ \Rightarrow \quad 7(a-1) + (a-1)^4 = 21(x-a) \]

\[ \Rightarrow \quad x = a + \frac{7(a-1) + (a-1)^4}{21} \]

\[ = \frac{21a + 7a - 7 + a^4 - 4a^3 + 6a^2 - 4a + 1}{21} \]

\[ = \frac{a^4 - 4a^3 + 6a^2 + 24a - 6}{21} \]

Notice that this number must be negative (it is in the second quadrant).

Hence the area of the triangle is:

\[ \text{Area} = \frac{(a^4 - 4a^3 + 6a^2 + 24a - 6)^2}{42(a-1)^4} \quad 1 - \sqrt[3]{7} \leq a < 1 \]
We would need to study the sign of the derivative of the function

\[ A = \text{Area}(a) \]

\[
\frac{dA}{da} = \frac{1}{42} \cdot \frac{2(a^4-4a^3+6a^2+24a-6)^1 \cdot (4a^3-12a^2+12a+24)(a-1)^4}{(a-1)^4} 
\]

\[
= \frac{2(a^4-4a^3+6a^2+24a-6)(a-1)^3 \cdot \left[ (4a^3-12a^2+12a+24)(a-1) \right]}{42 (a-1)^8} 
\]

\[
= \frac{\left[ (4a^4-12a^3+12a^2+24a) \right] - \left[ -4a^3+12a^2-12a-24 \right]}{2(a-1)^5} 
\]

\[
= \frac{4a^4-8a^3+12a^2-36a-12}{21 (a-1)^5} 
\]

Now we would need to find the roots of that derivative. . .