WORKSHEET #22

(a) Let x be y be two positive numbers

We know $xy = 180$. So

$$y = \frac{180}{x}$$

We want to minimize

$$S(x) = \text{sum} = x + \underbrace{\frac{180}{x}}_{y} = x + 180 \cdot x^{-1}$$

on the interval $0 < x < +\infty$

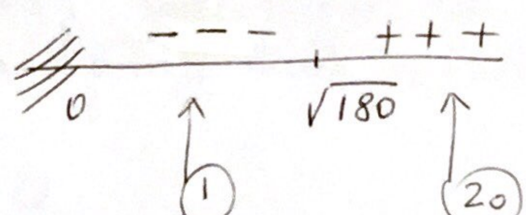
We need to study the interval of increase and decrease for $S(x)$.

$$S'(x) = 1 - \frac{180}{x^2} = 0 \iff 1 = \frac{180}{x^2}$$

$$\iff x^2 = 180 \iff x = \pm\sqrt{180}$$

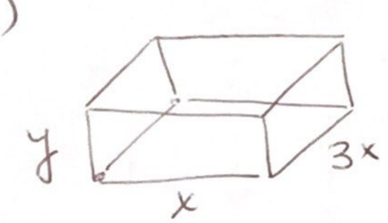
But $x > 0$ so $x = +\sqrt{180}$

Sign of $S'(x)$:



So the minimum (absolute) occurs when $x = y = \sqrt{180} \approx 13.42$

(b)



Let x be one dimension (length) of the base. the width is $3x$

Let y be the height of the box.

Thus $4x + 4(3x) + 4y = 450$ as we have 450 inches of wire.

$16x + 4y = 450$ so $\boxed{y = 112.5 - 4x}$

$$\boxed{0 \le x \le \frac{450}{16} = 28.125}$$

We need to maximize the volume on the closed interval.

$$V(x) = x \cdot (3x) \cdot (\underbrace{112.5 - 4x}_{y}) = 3x^2(112.5 - 4x)$$

$$= \boxed{337.5x^2 - 12x^3}$$

Since $V(x)$ is continuous on a closed interval the EVT applies. We need to check $V(x)$ at the endpoints and critical #.

$$V'(x) = 675x - 36x^2 = 0$$

so $\underline{x = 0}$ or $x = \frac{675}{36} = \underline{18.75}$

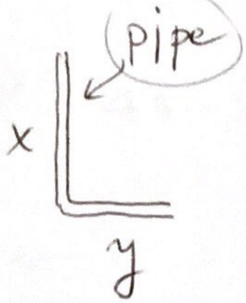
	endpoints		critical numbers	
x	0	28.125	0	18.75
$V(x)$	0	0	0	$\underline{39{,}550.78125}$

Hence the global max occurs at $x = 18.75$

So the dimensions are

$\underset{\text{length}}{18.75}$, $\underset{\text{width}}{56.25}$, $\underset{\text{height}}{37.5}$

(c)



Let x and y be the 2 lengths of the pipe

$$x + y = 4$$

So $y = 4 - x$

we need to minimize the length of the hypotenuse

$$h(x) = h = \sqrt{x^2 + (4-x)^2} = \sqrt{x^2 + 16 - 8x + x^2}$$

$$= \sqrt{2x^2 - 8x + 16}$$

on the interval $\boxed{0 \le x \le 4}$

Because $h(x)$ is a continuous function on a closed interval we can apply the EVT. The minimum can be obtained at either the end points or at the critical numbers.

$$h'(x) = \underbrace{\frac{1}{2\sqrt{2x^2-8x+16}} \cdot (4x-8)}_{\text{chain rule}} = 0$$

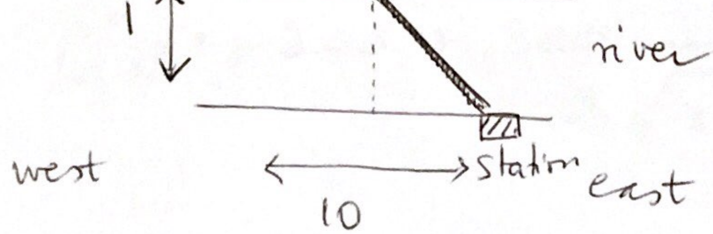
$$\Longleftrightarrow \quad 4x-8=0 \qquad \boxed{x=2}$$

Hence

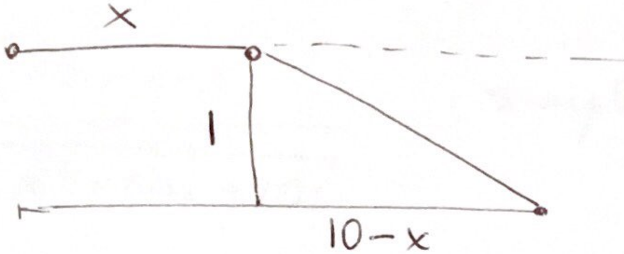
x	0	4	2
$h(x)$	4	4	$\sqrt{8} \approx 2.83$

the minimum (absolute) occurs when $x = y = 2$.

(d)



Let x be the length of the pipe on land.



the length of the pipe under water is the hypotenuse of the Triangle

$$\sqrt{(10-x)^2+1^2} = \sqrt{100-20x+x^2+1}$$

the cost function is

$$C(x) = 200x + 300\sqrt{x^2-20x+101}$$

in thousand of dollars

$$0 \le x \le 10$$

The function is continuous on the closed interval so the EVT applies. We need the critical numbers.

$$C'(x) = 200 + 300\,\frac{1}{2\sqrt{x^2-20x+101}}\cdot(2x-20)$$

$$C'(x)=0 \iff 200+\frac{150(2x-20)}{\sqrt{x^2-20x+101}}=0$$

$$200=\frac{150(20-2x)}{\sqrt{x^2-20x+101}} \quad \text{simplify "2"}$$

$$\overset{2}{\cancel{100}}\sqrt{x^2-20x+101}=\overset{3}{\cancel{150}}(10-x)$$

$$4(x^2-20x+101)=9(10-x)^2$$

$$4x^2-80x+404=900-180x+9x^2$$

$$\iff 5x^2-100x+496=0$$

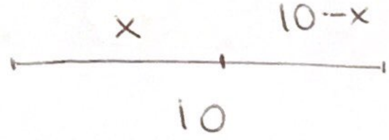
$$x=\frac{100\pm\sqrt{100^2-4\cdot5\cdot496}}{10}= \begin{cases}\frac{100+\sqrt{80}}{10}=10{,}89\\ \frac{100-\sqrt{80}}{10}=9{,}11\end{cases}$$

x	0	10	9.11
$C(x)$	3,014.96	2,300	2,223.61 thousand of dollars

Hence the global minimum occurs when 9.11 kilometers are on land.

The cost is 2.223 million.

(e)



Let x the point where we cut the rope.

With x we make the square

with 10−x we make the circle

The side of the square is $\frac{x}{4}$

The radius of the circle is $2\pi r = 10-x$

so $r = \frac{10-x}{2\pi}$

We need to maximize

$$A(x) = \left(\frac{x}{4}\right)^2 + \pi \cdot \left(\frac{10-x}{2\pi}\right)^2$$

$$= \boxed{\frac{1}{16}x^2 + \frac{1}{4\pi}(10-x)^2}$$

on the interval $\boxed{0 \leq x \leq 10}$

The EVT applies as A(x) is continuous on a closed interval.

$$A'(x) = \frac{2}{16}x + \frac{2}{4\pi}(10-x)\cdot(-1)$$

$$A'(x) = 0 \iff$$

$$\frac{x}{8} - \frac{20}{4\pi} + \frac{x}{2\pi} = 0$$

$$x\left(\frac{1}{8} + \frac{1}{2\pi}\right) = \frac{5}{\pi}$$

$$x = \frac{5/\pi}{\frac{\pi + 4}{8\pi}} = \frac{5}{\cancel{\pi}} \cdot \frac{8\cancel{\pi}}{\pi+4} = \frac{40}{\pi+4}$$

$$\approx 5.6$$

	end points		critical #
x	0	10	5.6
$A(x)$	$\underbrace{\frac{25}{\pi}}_{7.95}$	$\underbrace{25/4}_{6.25}$	3.506

Hence the maximum area is obtained by making a circle with 10 feet of rope.