WORKSHEET #24

1. (a) $\int \left(x^3 - 3x + 2 - \frac{5}{x} \right) dx$

$= \int x^3 dx - 3\int x dx + 2\int 1 \cdot dx - 5\int \frac{1}{x} dx$

$= \frac{1}{4} x^4 - 3 \cdot \left(\frac{1}{2} x^2 \right) + 2(x) - 5 \ln|x| + C$

$= \boxed{\frac{x^4}{4} - \frac{3}{2} x^2 - 2x - 5 \ln|x| + C}$

(b) $\int \left(\sqrt[3]{x^2} + x\sqrt{x} \right) dx =$

$= \int x^{2/3} dx + \int x^{1+\frac{1}{2}} dx$

$= \int x^{2/3} dx + \int x^{3/2} dx$

$= \frac{1}{2/3+1} x^{2/3+1} + \frac{1}{3/2+1} x^{3/2+1} + C$

$$= \boxed{\frac{3}{5}x^{5/3} + \frac{2}{5}x^{5/2} + c}$$

(c) Recall that

$$\frac{d}{dx}(\underbrace{sec(x)}_{\frac{1}{\cos x}}) = sec(x) \cdot \tan(x)$$

Hence :

$$\int (Sec(x) \cdot \tan(x) - 2e^{x})\, dx$$

$$= \int sec(x)\tan(x)\, dx - 2\int e^{x} dx$$

$$= \boxed{sec(x) - 2e^{x} + c}$$

(d) Recall that

$$\frac{d}{dx}(\tan x) = \sec^2(x)$$

$$\frac{d}{dx}(-\cos(x)) = \sin(x)$$

hence

$$\int (2\sin(x) - \sec^2(x))\,dx$$

$$= 2\int \sin(x)\,dx - \int \sec^2(x)\,dx$$

$$= 2(-\cos(x)) - \tan(x) + C$$

$$= \boxed{-2\cos(x) - \tan(x) + C}$$

$\boxed{2.}$ (a) $f''(t) = 2e^t + 3\sin(t)$

hence $f'(t) = \int \left(2e^t + 3\sin(t)\right)dt$

$= 2e^t - 3\cos(t) + C$

Since $f'(0) = 6$ we have

$6 = f'(0) = 2\underbrace{e^0}_{1} - 3\underbrace{\cos(0)}_{1} + C$

$6 = 2 - 3 + C$ $\boxed{C = 7}$

$f'(t) = 2e^t - 3\cos(t) + 7$

$f(t) = \int \left(2e^t - 3\cos(t) + 7\right) dt$

$= 2e^t - 3\sin(t) + 7t + \underbrace{K}_{\text{another constant}}$

$f(0) = -6$ so

$-6 = f(0) = 2\underbrace{e^0}_{1} - 3\underbrace{\sin(0)}_{0} + \underbrace{7.0}_{0} + K$

hence $K = -8$

and $\boxed{f(t) = 2e^t - 3\sin(t) + 7t - 8}$

(b) $f''(x) = -9.8$

So $f'(x) = \int -9.8\,dx = -9.8x + C$

Since $f'(0) = 1$ we have

$1 = f'(0) = -\cancel{9.8 \cdot 0} + C \qquad C = 1$

$\quad 0$

$f'(x) = -9.8x + 1$

$f(x) = \int (-9.8x + 1)\,dx = -9.8\frac{1}{2}x^2 + x + K$

Since $f(0) = 2$

$2 = f(0) = -\cancel{9.8\frac{1}{2}0^2} + \cancel{0} + K$

$\quad 0 \qquad 0$

$K = 2$

So $\boxed{f(x) = -4.9x^2 + x + 2}$