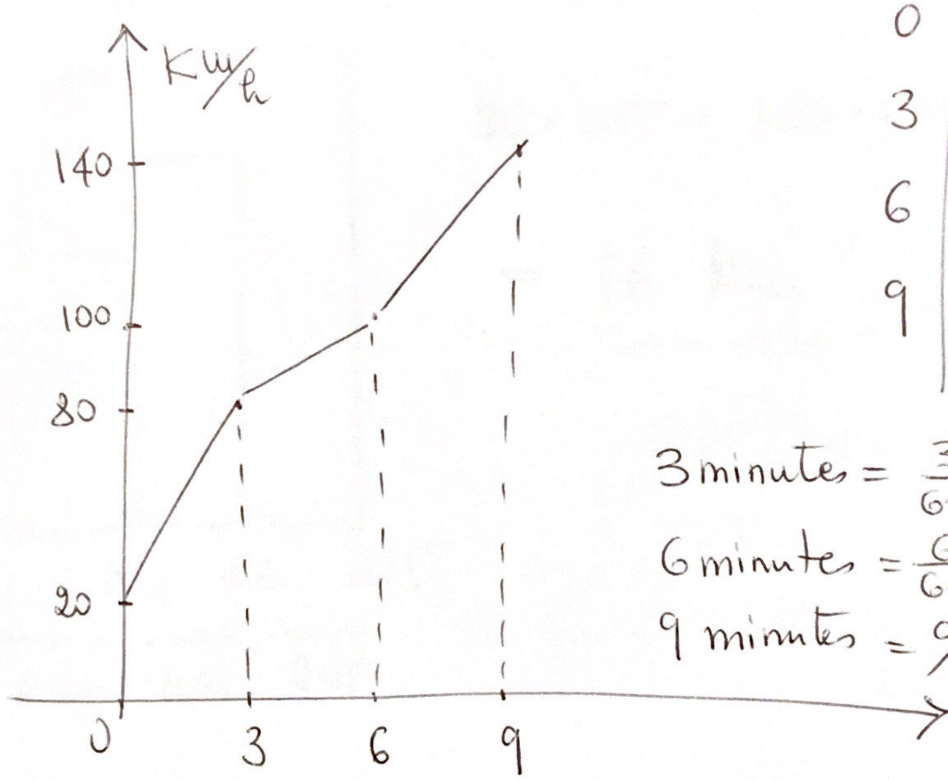
WORKSHEET 25

1.



t	$v(t)$
0	20
3	80
6	100
9	140

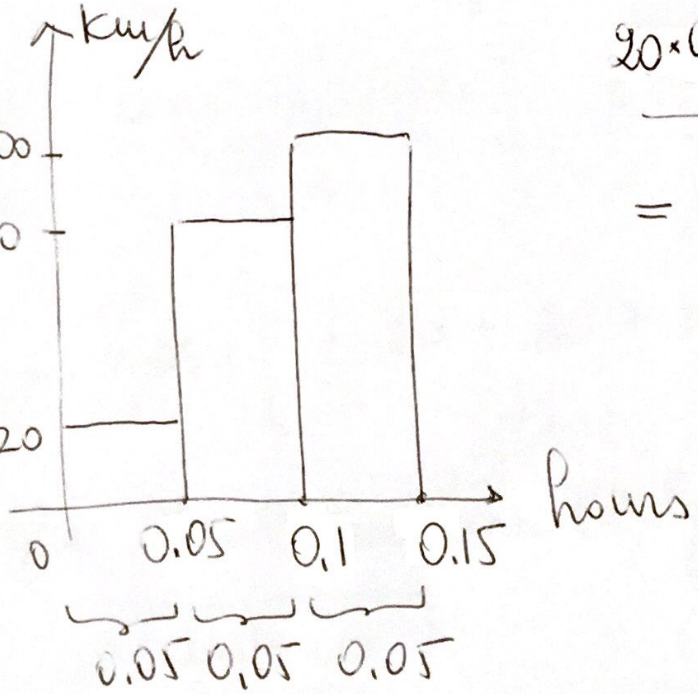
3 minutes $= \frac{3}{60} = 0{,}05$ hours

6 minutes $= \frac{6}{60} = 0.1$ hours

9 minutes $= 9/60 = 0{,}15$ hours

minutes

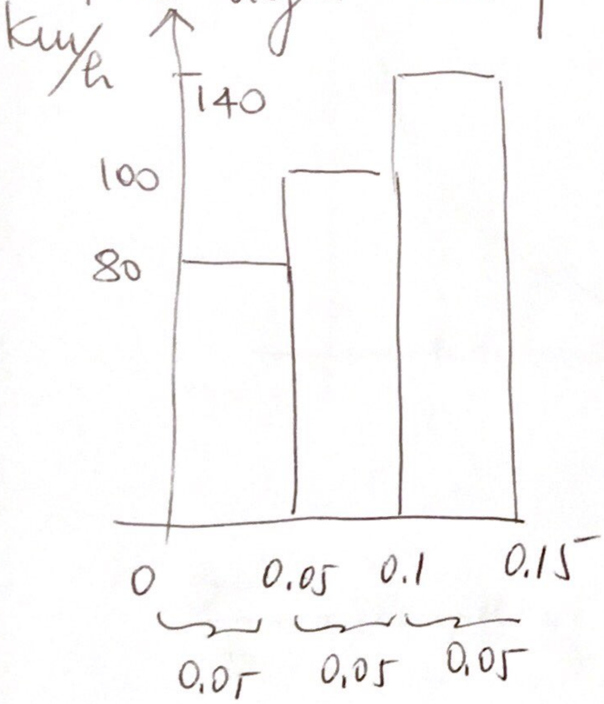
The left end-point approximation to that area is



$20 \times 0{,}05 + 80 \times 0{,}05 + 100 \times 0{,}05$

$= 10$ Km

The right end-point approximation is



$80 \times 0.05 + 100 \times 0.05 + 140 \times 0.05$

$= \underline{16 \text{ km}}$

The actual distance traveled is the area of those 3 trapezoid

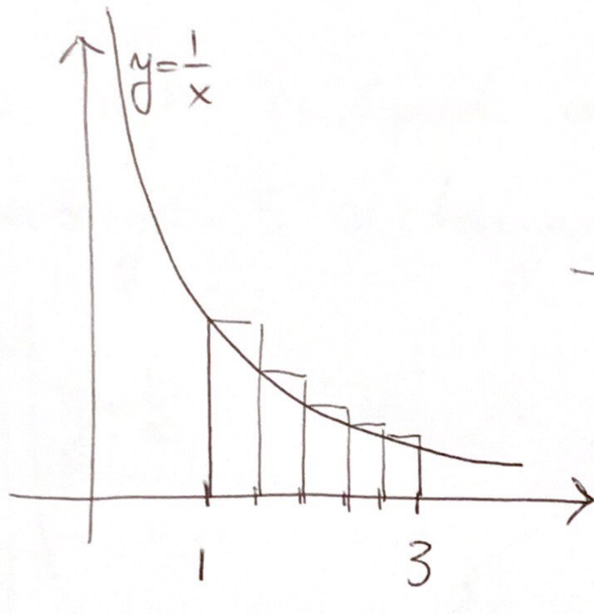
$$= \frac{0.05 \times (20+80)}{2} + \frac{0.05 \times (80+100)}{2} + \frac{0.05 (100+140)}{2}$$

$$= 0.05 \times \left[\frac{1}{2} \cdot 20 + 80 + 100 + \frac{1}{2} 140 \right] = \underline{\underline{13 \text{ km}}}$$

note that we multiply $\underline{\text{times} \times \text{velocity}}$

$= \boxed{\text{distance}}$

2.



this is the left-endpoint approximation using 5 rectangles

$$\frac{3-1}{5} = \text{length of each subinterval} = \frac{2}{5} = \underline{\underline{0.4}}$$

So the point in the subdivision are

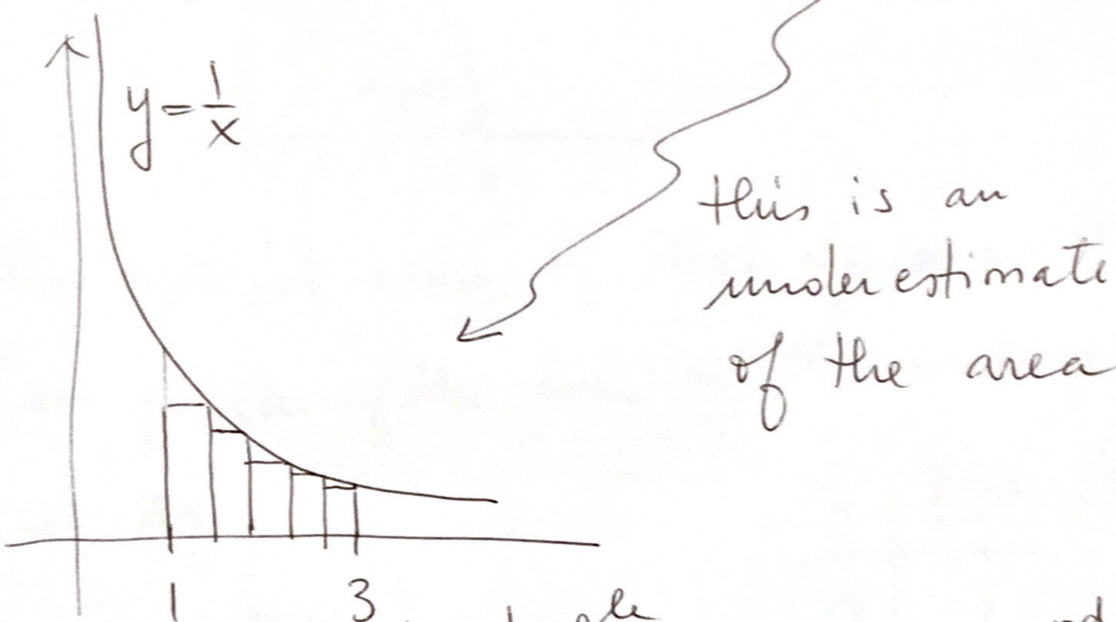
1, 1.4, 1.8, 2.2, 2.6, 3

x	$f(x) = 1/x$
1	1
1.4	$\frac{1}{1.4} = 0.7143$
1.8	$\frac{1}{1.8} = 0.5555$
2.2	$1/2.2 = 0.4545$
2.6	$1/2.6 = 0.3846$
3	$1/3 = 0.3333$

$$L_5 = \underbrace{0.4 \cdot 1}_{1^{st}\text{ rectangle}} + \underbrace{0.4 \cdot \frac{1}{1.4}}_{2^{nd}} + \underbrace{0.4 \cdot \frac{1}{1.8}}_{3^{rd}} + \underbrace{0.4 \cdot \frac{1}{2.2}}_{4^{th}} + \underbrace{0.4 \cdot \frac{1}{2.6}}_{5^{th}}$$

$$= \boxed{1.2436}$$

The right endpoint approximation using 5 rectangles is



1st rectangle, 2nd, 3rd

$$R_5 = 0.4 \cdot \frac{1}{1.4} + 0.4 \cdot \frac{1}{1.8} + 0.4 \cdot \frac{1}{2.2} +$$

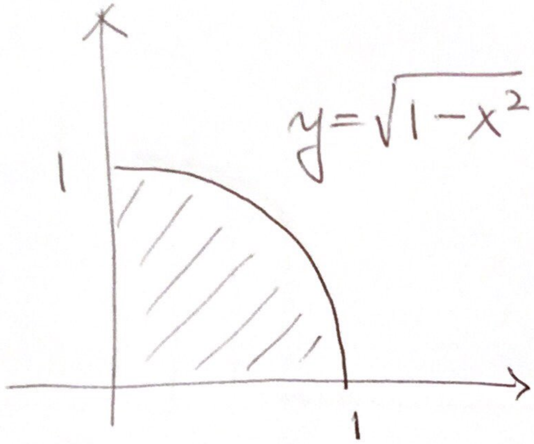
$$+ 0.4 \frac{1}{2.6} + 0.4 \frac{1}{3}$$

4th, 5th

$$= 0.4 \left[\frac{1}{1.4} + \frac{1}{1.8} + \frac{1}{2.2} + \frac{1}{2.6} + \frac{1}{3} \right]$$

$$= \boxed{0.9769}$$

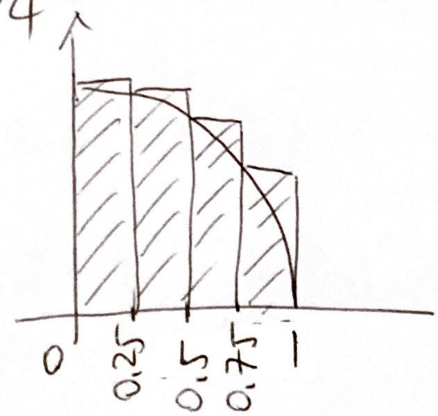
3.



the actual area of that region is $\frac{\pi(1)^2}{4}$ (one quarter of the area of the circle of radius 1)

$= \pi/4$

L_4:



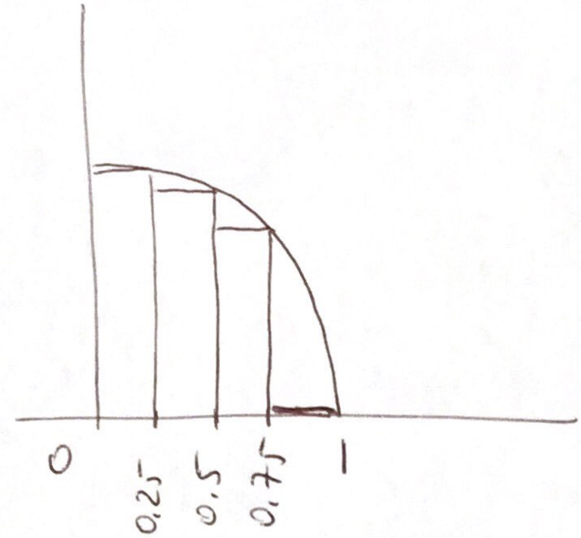
x	$f(x)=\sqrt{1-x^2}$
0	1
0.25	0.9682
0.5	0.8660
0.75	0.6614

hence L_4 is the area of those 4 rectangles whose base is $\underline{\underline{0.25}} = \frac{1-0}{4}$

$$0.25(1) + 0.25(0.9682) + 0.25(0.8660) + 0.25(0.6614)$$

$= 0.8739$ this is an <u>over</u> <u>estimate</u>

R_4



x	$f(x) = \sqrt{1-x^2}$
0.25	0.9682
0.5	0.8660
0.75	0.6614
1	0

hence the area R_4 of those 4 rectangles whose base is $0.25 = \frac{1-0}{4} = \frac{1}{4}$ is:

$$0.25(0.9682) + 0.25(0.8660) + 0.25(0.6614) + 0.25(0)$$

$= 0.6239$ this is an under estimate

So:

$$0.6239 \leq \text{Area} \leq 0.8739$$

$$\updownarrow$$

$$0.6239 \leq \frac{\pi}{4} \leq 0.8739$$

$$2.4956 = 4 \cdot 06239 \leq \pi \leq 4 \cdot 0.8739 = 3.4956$$

$\uparrow$
3.14159...