WORKSHEET 26

1.
$$\sum_{k=11}^{20} (3k+2) = [3(11)+2] + [3(12)+2]$$
$$+ [3(13)+2] + \dots + [3(20)+2]$$

is boring an tedious. What if there were many more terms.

A more computational friendly approach is to use the fact that $\sum_{k=1}^{n} k = \frac{n(n+1)}{2}$

in this way

$$\sum_{k=11}^{20} (3k+2) = \sum_{k=1}^{20} (3k+2) - \sum_{k=1}^{10} (3k+2)$$

$$= \left[3\sum_{k=1}^{20} k + \sum_{k=1}^{20} 2 \right] - \left[3\sum_{k=1}^{10} k + \sum_{k=1}^{10} 2 \right]$$

$$= 3\,\frac{20(21)}{2} + 2\cdot 20 - 3\,\frac{10\cdot 11}{2} - 2\cdot 10 = \underline{\underline{485}}$$

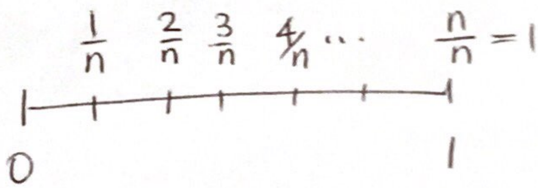
$\boxed{2.}$

$$\lim_{n\to\infty} \sum_{i=1}^{n} \frac{i^3}{n^4} =$$

$$= \lim_{n\to\infty} \sum_{i=1}^{n} \frac{1}{n} \cdot \frac{i^3}{n^3} =$$

$$= \lim_{n\to\infty} \sum_{i=1}^{n} \underbrace{\frac{1}{n}}_{\substack{\text{base of} \\ \text{rectangle} \\ \Delta x}} \cdot \underbrace{\left(\frac{i}{n}\right)^3}_{f(\text{sample point})}$$

$$= \int_0^1 x^3\, dx$$

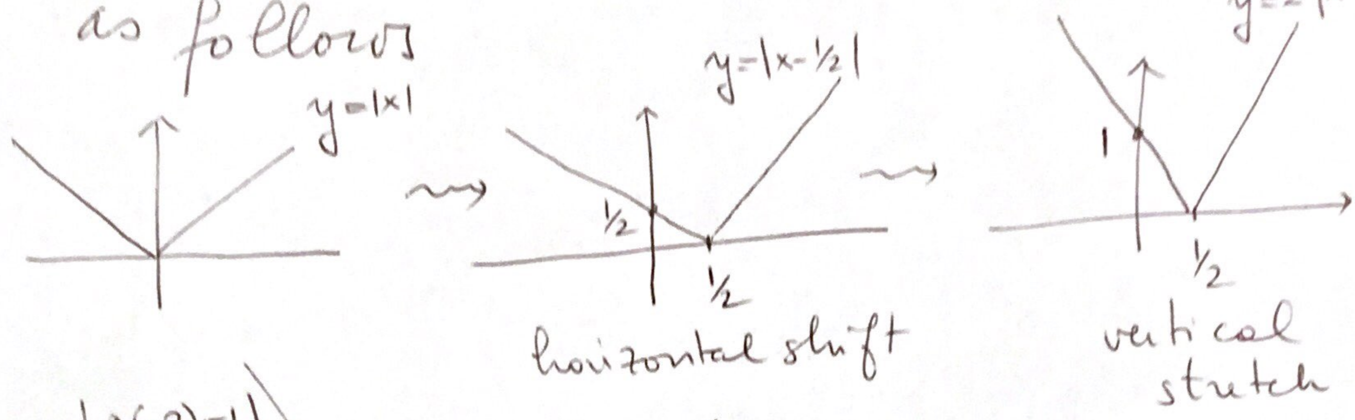


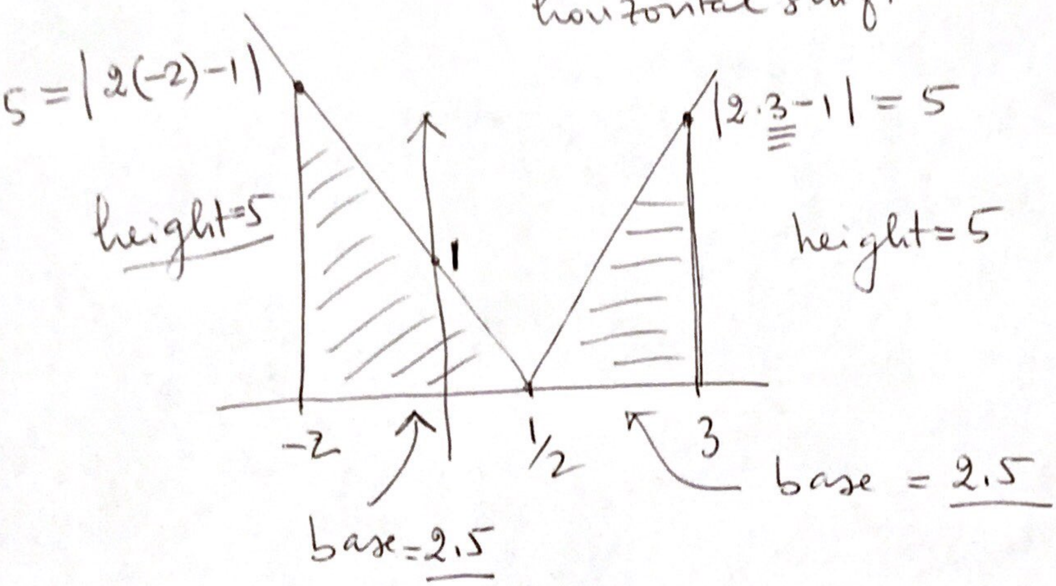
each subinterval has length $\frac{1-0}{n} = \frac{1}{n}$

the i-point in the partition is $\frac{i}{n}$

$\boxed{3.}$ (a) $\displaystyle\int_{-2}^{3} |2x-1|\,dx$

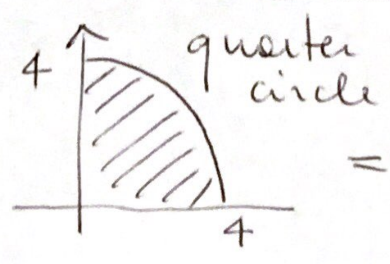
$|2x-1| = 2\left|x-\frac{1}{2}\right|$ so the graph is obtained as follows





$$\text{Area} = \frac{5\cdot 2.5}{2} + \frac{5\cdot 2.5}{2} = \boxed{12.5}$$

(b) $\displaystyle\int_0^4 \sqrt{16-x^2}\,dx =$



$= \frac{1}{4}\pi(4)^2 = 4\pi$