WORSHEET #27

1. Suppose $\int_0^x f(t)\,dt = \sin(x)$

(a) $\int_0^\pi f(t)\,dt = \sin(\pi) = \boxed{0}$

(b) $\int_{\pi/2}^{\pi} f(t)\,dt = \int_0^\pi f(t)\,dt - \int_0^{\pi/2} f(t)\,dt$

$= \underline{\sin(\pi)} - \underline{\sin(\pi/2)}$

$= 0 - 1 = \boxed{-1}$

(c) $\int_\pi^{-\pi} f(t)\,dt = \int_\pi^0 f(t)\,dt + \int_0^{-\pi} f(t)\,dt$

$= -\int_0^\pi f(t)\,dt + \int_0^{-\pi} f(t)\,dt =$

$= \underline{-\sin(\pi)} + \underline{\sin(-\pi)} = -0 + 0 = \boxed{0}$

$\boxed{2.}$ (a) $\dfrac{d}{dx}\displaystyle\int_x^1 (2+t^4)^5\,dt =$

$$= \frac{d}{dx}\left[-\int_1^x (2+t^4)^5\,dt\right] =$$

$$= -\frac{d}{dx}\int_1^x (2+t^4)^5\,dt = \underline{\underline{-(2+x^4)^5}}$$

(b) $\dfrac{d}{dx}\displaystyle\int_0^{x^2} \sqrt{1+r^3}\,dr =$

$$= \underbrace{\sqrt{1+(x^2)^3}}\cdot\underbrace{2x}$$

by Leibniz's Rule (chain rule + FTC)

(c) $\dfrac{d}{dx}\displaystyle\int_{\sqrt{x}}^{x^2} \sqrt{t}\cdot\sin(t)\,dt =$

$$= \frac{d}{dx}\left[\int_0^{x^2}\sqrt{t}\cdot\sin(t)\,dt - \int_0^{\sqrt{x}}\sqrt{t}\,\sin(t)\,dt\right]$$

$$= \sqrt{x^2} \cdot \sin(x^2) \cdot 2x - \sqrt{\sqrt{x}} \sin(\sqrt{x}) \cdot \frac{1}{2\sqrt{x}}$$

by Leibniz's Rule

$$\boxed{= 2x|x| \sin(x^2) - \frac{\sqrt[4]{x} \cdot \sin\sqrt{x}}{2\sqrt{x}}}$$

$\boxed{3.}$ Find "a" and "f(t)" such that

$$6 + \int_a^x \frac{f(t)}{t^2}\, dt = 2\sqrt{x}$$

Substitute $x = a$ in the equation

$$6 + \underbrace{\int_a^{\boxed{a}} \frac{f(t)}{t^2}\, dt}_{0} = 2\sqrt{a}$$

So $\cancel{6}^{3} = \cancel{2}\sqrt{a}$ $\qquad \sqrt{a} = 3$ or $\boxed{a = 9}$

Thus $\quad 6 + \int_9^x \frac{f(t)}{t^2}\, dt = 2\sqrt{x}$

Take the derivative of both sides and use FTC :

$$\frac{d}{dx}\left[6+\int_9^x \frac{f(t)}{t^2}\,dt\right]=\frac{d}{dx}\left(2\sqrt{x}\right)$$

$$0+\frac{f(x)}{x^2}=\cancel{2}\cdot\left(\frac{1}{\cancel{2}\sqrt{x}}\right)$$

So $f(x)=x^2\cdot\left(\frac{1}{\sqrt{x}}\right)=\underline{\underline{x\sqrt{x}=x^{3/2}}}$

$\boxed{4.}$ $$\lim_{x\to 0}\frac{\int_0^{\boxed{x}} e^t\,dt}{3\boxed{x}}=\frac{\int_0^0 e^t\,dt}{3\cdot 0}=\frac{0}{0}$$

By l'Hôpital's Rule

$$=\lim_{x\to 0}\frac{\frac{d}{dx}\left(\int_0^x e^t\,dt\right)}{\frac{d}{dx}(3x)}=\lim_{x\to 0}\frac{e^x}{3}=\boxed{\frac{1}{3}}$$

$\boxed{5.}$ (a) $$\int_{-2}^{7}\frac{1}{x^5}\,dx=\int_{-2}^{7}x^{-5}\,dx=$$

$$= \frac{1}{-4} x^{-4} \Big|_{-2}^{7} = -\frac{1}{4x^4} \Big|_{-2}^{7} =$$

$$= -\frac{1}{4 \cdot 7^4} - \left(-\frac{1}{4 \cdot (2)^4} \right) = \frac{1}{4 \cdot 2^4} - \frac{1}{4 \cdot 7^4}$$

$$= \frac{1}{4} \left(\frac{7^4 - 2^4}{2^4 \cdot 7^4} \right) = \boxed{\frac{2385}{153664}} \approx \boxed{0.01552}$$

(b) $$\int_{-1}^{1} e^{x+1} \, dx = \int_{-1}^{1} e^x \cdot e \, dx =$$

$$= e \int_{-1}^{1} e^x \, dx = e \left[e^x \Big|_{-1}^{1} \right] =$$

$$= e \left(e - e^{-1} \right) = \boxed{e^2 - 1} \approx \boxed{6.389}$$

(c) $$\int_{\pi/2}^{\pi} \left(1 + 2 \sin(x) + \frac{3}{\sqrt{x}} \right) dx$$

$$\int \left(1 + 2\sin(x) + 6 \cdot \frac{1}{2\sqrt{x}}\right) dx$$

$$= \int 1 \cdot dx + 2\int \sin(x)\, dx + 6\int \frac{1}{2\sqrt{x}}\, dx$$

$$= x + 2(-\cos(x)) + 6\sqrt{x} + C$$

hence

$$\int_{\pi/2}^{\pi} \left(1 + 2\sin(x) + \frac{3}{\sqrt{x}}\right) dx = \left(x - 2\cos(x) + 6\sqrt{x}\right)\Big|_{\pi/2}^{\pi}$$

$$= \Big[\pi - 2\underbrace{\cos(\pi)}_{(-1)} + 6\sqrt{\pi}\Big] - \Big[\pi/2 - 2\underbrace{\cos\left(\tfrac{\pi}{2}\right)}_{0} + 6\sqrt{\tfrac{\pi}{2}}\Big]$$

$$= \pi + 2 + 6\sqrt{\pi} - \pi/2 - \frac{6}{\sqrt{2}}\sqrt{\pi}$$

$$= \pi/2 + 2 + 6\sqrt{\pi}\left(1 - \frac{1}{\sqrt{2}}\right) \approx \boxed{6.6856}$$