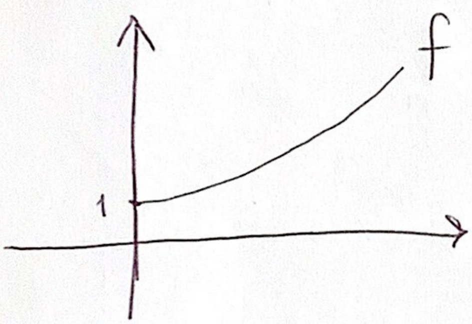
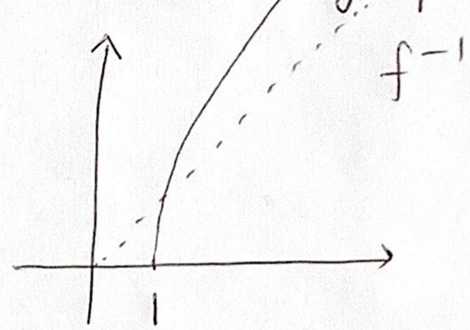
WORK SHEET 2

1. The function $f(x)=\sqrt[3]{x^2+1}$ is defined according to the problem for $x \geqslant 0$. Thus the domain is $[0,+\infty)$. For the inverse set $y=\sqrt[3]{x^2+1}$ and solve for x.

$$y^3 = x^2+1 \Rightarrow x^2 = y^3-1 \Rightarrow x=\sqrt{y^3-1}.$$

Swith x and y : $f^{-1}(x)=\sqrt{x^3-1}$. Its domain is $[1,\infty)$. About graphs

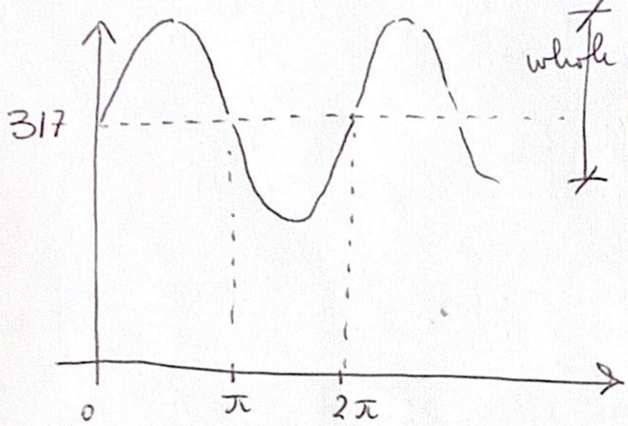




notice that f is defined for every value of x. However when defined on $(-\infty,\infty)$ it is not one-one. Thus to make it one-one we need to consider it for $x \in [0,+\infty)$

2. $f(x)=317+22\sin(wx)$. Find ω so that it represents the flow of blood in the heart at 72 beats per second

If we had $\omega=1$ the graph of f looks like



whole amplitude of a full oscillation is actually $22 \cdot 2 = 44$

We need to compress that sine wave by changing "ω" so that a full wave repeats not after 2π but after $\frac{1}{72}$

In particular $f(x) = f\left(x+\frac{1}{72}\right) = f\left(x+\frac{2}{72}\right) = \text{etc...}$

From the first 2 identities we have

$$317 + 22\sin(\omega x) = 317 + 22\sin\left(\omega\left(x+\frac{1}{72}\right)\right)$$

$\Rightarrow$ (after simplifying) $\sin(\omega x) = \sin\left(\omega x + \frac{\omega}{72}\right)$

so $\frac{\omega}{72} =$ must be 2π because that is the smallest period for sin. So $\boxed{\omega = 144\pi}$

3.

$$17\log_b(q) - 4\log_b(qy) =$$

$$\log_b\left(q^{17}\right) - \log_b\left((qy)^4\right) = \log_b\left(\frac{q^{17}}{q^4 y^4}\right) =$$

$$= \log_b\left(q^{13}/y^4\right)$$

4. Solve for x in the equation: $2^{x-5} = 5^{2x+6}$

Take "ln" of both sides

$$\ln\left(2^{x-5}\right) = \ln\left(5^{2x+6}\right)$$

$$(x-5)\ln 2 = (2x+6)\ln 5$$

$$x \cdot \ln 2 - 5\ln 2 = x \cdot (2\ln 5) + 6\ln 5$$

$$x\left[\ln 2 - 2\ln 5\right] = 5\ln 2 + 6\ln 5$$

$$x\left[\ln 2 - \ln(5^2)\right] = \ln(2^5) + \ln(5^6)$$

$$x \cdot \ln\left(\frac{2}{5^2}\right) = \ln\left(2^5 \cdot 5^6\right)$$

$$\boxed{x = \frac{\ln(2^5 \cdot 5^6)}{\ln(2/5^2)} \cong -5.1955}$$

5. Solve for x in the equation

$$\log x - \log(6x+17) = 2$$

$$\Rightarrow \log\left(\frac{x}{6x+17}\right) = 2 \iff 10^2 = \frac{x}{6x+17}$$

$$\iff 100(6x+17) = x \quad \text{or} \quad 600x + 1700 = x$$

$$\iff 599x = -1700 \qquad \boxed{x = -\frac{1700}{599}}$$