WORKSHEET 4

1. Suppose $[H^+] = 6.7 \times 10^{-3}$ its pH is

$$pH = -\log(6.7 \times 10^{-3}) = -\log(6.7) - \log(10^{-3})$$
$$= 3 - \log(6.7) \cong \underline{\underline{2.1739}}$$

2. If $pH = 4.18$ then its hydrogen ion concentration is $\left(pH = -\log[H^+] \right)$

$$\rightsquigarrow 4.18 = -\log[H^+] \rightsquigarrow -4.18 = \log[H^+]$$

raise 10 to those exponents $10^{-4.18} = 10^{\log[H^+]}$

so $[H^+] = 10^{-4.18} = 10^{-5+0.82} = 10^{0.82} \cdot 10^{-5}$

$$= \underline{\underline{6.607 \cdot 10^{-5}}}$$

(note that $10^{-4.18} = 10^{-0.18} \cdot 10^{-4} = 0.6607 \cdot 10^{-4}$ is not good because the coefficient is < 1)

3. $y = 3.14 \times 10^{0.86x}$

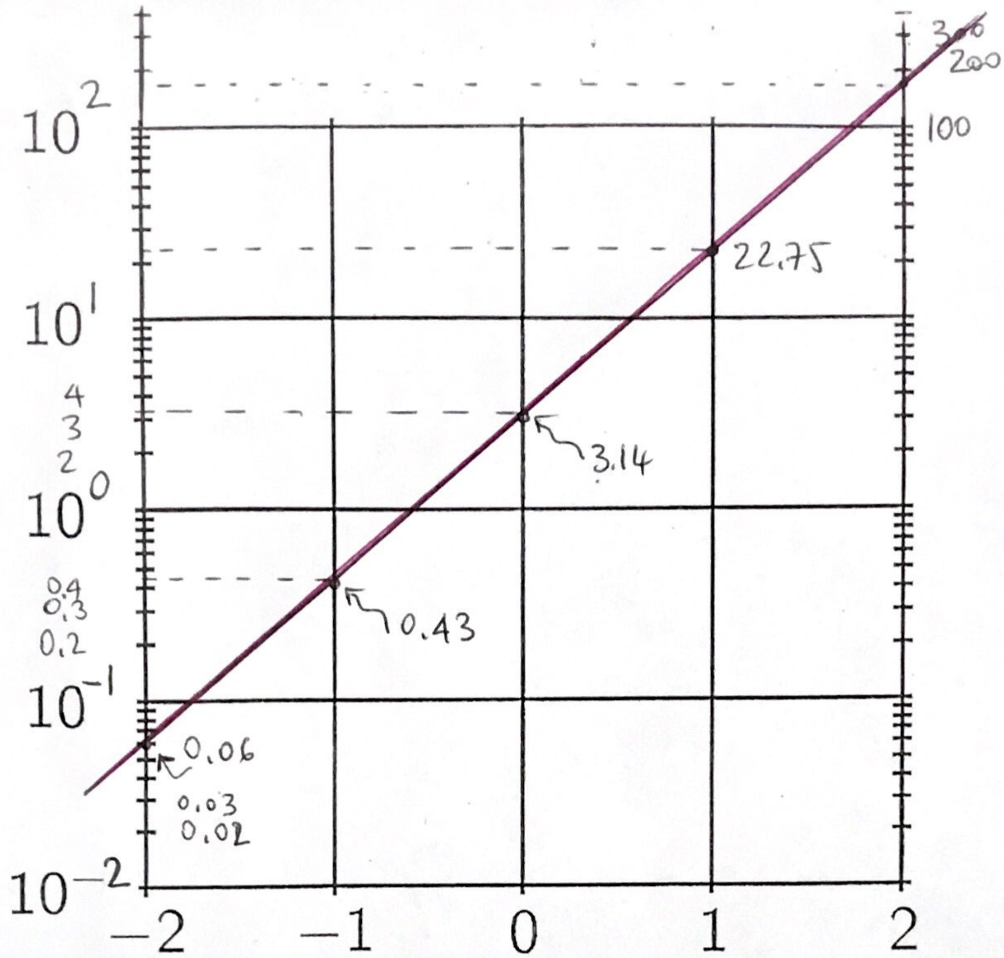
Actual values

x	y
-2	0.06
-1	0.43
0	3.14
1	22.75
2	164.79

Take $\log = \log_{10}$

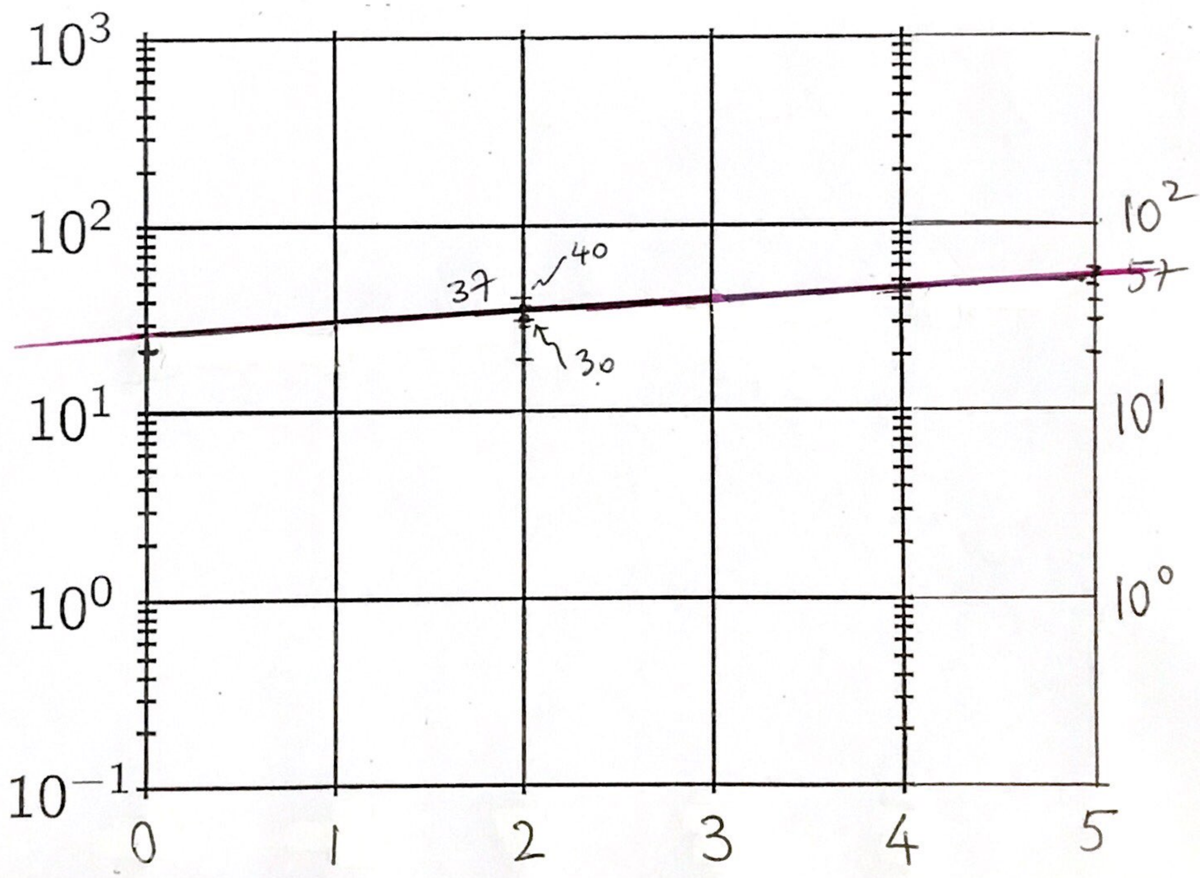
$$\log y = \log(3.14 \cdot 10^{0.86x})$$
$$\log y = \log(3.14) + \log(10^{0.86x})$$
$$\underbrace{\log y}_{Y} = 0.86x + \log(3.14)$$

$$\boxed{Y = 0.86x + \log(3.14)}$$



10^2
10^1
10^0
10^{-1}
10^{-2}
−2
−1
0
1
2
300
200
100
22.75
4
3
2
3.14
0.4
0.3
0.2
0.43
0.06
0.03
0.02

4.



10^3
10^2
10^1
10^0
10^{-1}
0
1
2
3
4
5
37
40
30
57

The straight line determined by the 2 points plotted in a semilog plot correspond to an exponential relation among the original quantities: $y = ab^x$. Substitute $(2, 37)$ and $(5, 52)$ to get

$37 = a \cdot b^2$ & $52 = a \cdot b^5$ Solve for a and equate: $\frac{37}{b^2} = a$ & $\frac{52}{b^5} = a$

$\rightsquigarrow \frac{37}{b^2} = \frac{52}{b^5} \rightsquigarrow b^5/b^2 = \frac{52}{37} \rightarrow b^3 = \frac{52}{37}$

$\boxed{b = \sqrt[3]{52/37}}$ hence $\boxed{a = \frac{37}{b^2}} = \frac{37}{\left(\sqrt[3]{\frac{52}{37}}\right)^2} \boxed{\approx 29.4894}$

≈ 1.1201

Thus $\boxed{y = 29.4894(1.1201)^x}$

$\boxed{2^{nd} \text{ method:}}$ the line through $(2, \log 37)$ and $(5, \log 52)$ has slope

$$m = \frac{\log 52 - \log 37}{5 - 2} = \frac{1}{3}\left(\log\left(\frac{52}{37}\right)\right) = \underline{\underline{\log\left(\left(\frac{52}{37}\right)^{1/3}\right)}}$$

The point-slope form of the line is

$$\boxed{Y - \log 37 = \log\left(\sqrt[3]{\frac{52}{37}}\right) \cdot (x - 2)}$$

Recall that Y represents $\log(y)$

So

$$\log y - \log 37 = \log\left(\sqrt[3]{\frac{52}{37}}\right) \cdot (x-2)$$

Can be rewritten as

$$\log\left(\frac{y}{37}\right) = (x-2) \cdot \log\left(\sqrt[3]{\frac{52}{37}}\right)$$

$$\log\left(\frac{y}{37}\right) = \log\left(\left(\sqrt[3]{\frac{52}{37}}\right)^{x-2}\right)$$

Since log is a one-to-one function

$$\frac{y}{37} = \left(\sqrt[3]{\frac{52}{37}}\right)^{x-2}$$

or $$y = 37 \cdot \left(\sqrt[3]{\frac{52}{37}}\right)^{x} \cdot \left(\sqrt[3]{\frac{52}{37}}\right)^{-2}$$

$$\underline{\underline{y}} = \underbrace{\frac{37}{\left(\sqrt[3]{\frac{52}{37}}\right)^{2}}}_{a} \cdot \left(\underbrace{\sqrt[3]{\frac{52}{37}}}_{b}\right)^{x} = \underline{\underline{29.4894(1.12)^{x}}}$$