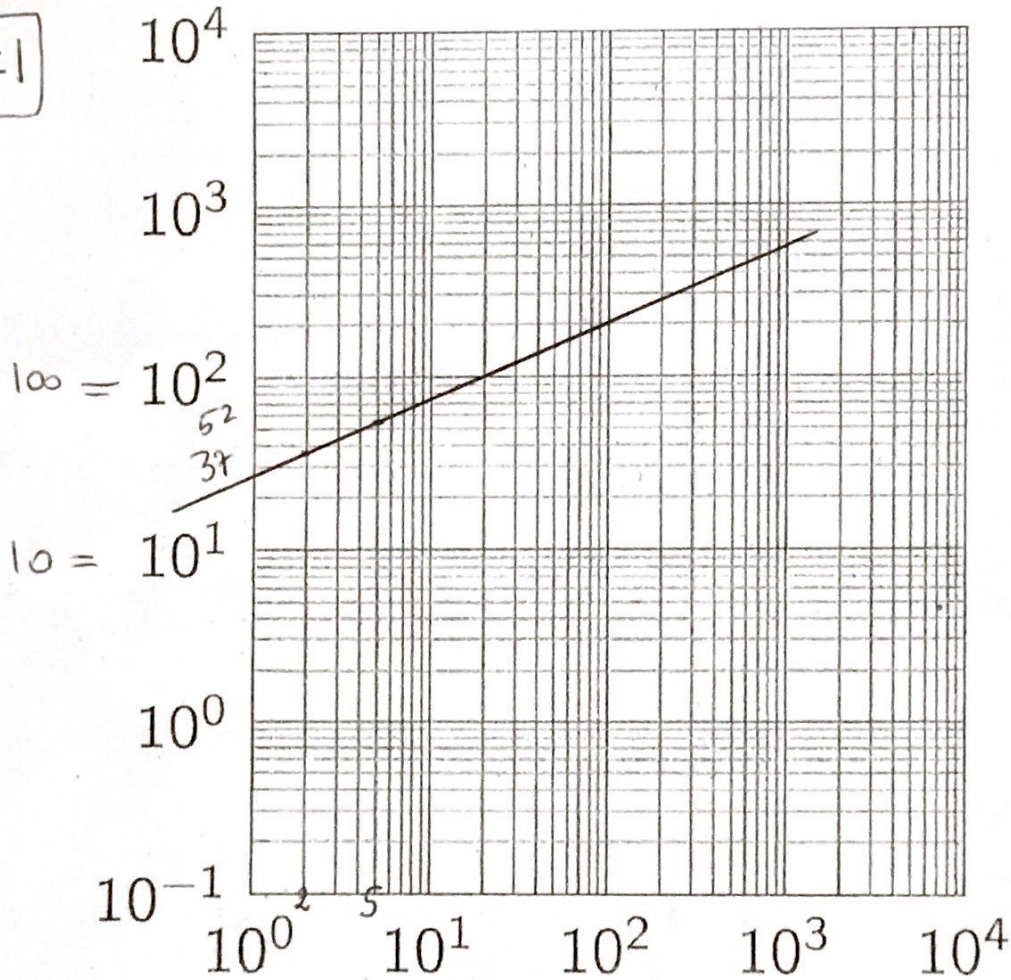


#1



That is approximately the line in the doublelog plot corresponding to the actual observations  $(2, 37)$  and  $(5, 52)$

The functional relationship between the actual quantities  $x$  and  $y$  is a power function relation:

$$y = C \cdot x^p$$

Substitute  $(2, 37)$  and  $(5, 52)$  in the expression to get

$$37 = C \cdot 2^p \quad \text{and} \quad 52 = C \cdot 5^p$$

Solve for  $C$  and equate:  $\frac{37}{2^p} = C = \frac{52}{5^p}$

or  $\frac{5^p}{2^p} = \frac{52}{37}$  or  $\left(\frac{5}{2}\right)^p = \frac{52}{37}$ . Take

log of both side to get:  $\log\left[\left(\frac{5}{2}\right)^p\right] = \log\left(\frac{52}{37}\right)$

$$p \cdot \log\left(\frac{5}{2}\right) = \log\left(\frac{52}{37}\right) \quad p = \frac{\log\left(\frac{52}{37}\right)}{\log\left(\frac{5}{2}\right)} = 0.37142$$

To get  $C$  substitute  $p$  in

$$C = \frac{37}{2^p} \approx \frac{37}{2^{0.37142}} \approx 28.60185$$

$$\therefore \boxed{y = 28.60185 \cdot x^{0.37142}}$$

Try to compute the equation of the line through  $(\log 2, \log 37)$  and  $(\log 5, \log 52)$ . For example the



slope of the line is

$$m = \frac{\log 52 - \log 37}{\log 5 - \log 2} = \frac{\log(52/37)}{\log(5/2)} \approx 0.37142$$

that is the power of the power function  
expression:  $y = 28.60185 x^{0.37142}$

The equation of the line is

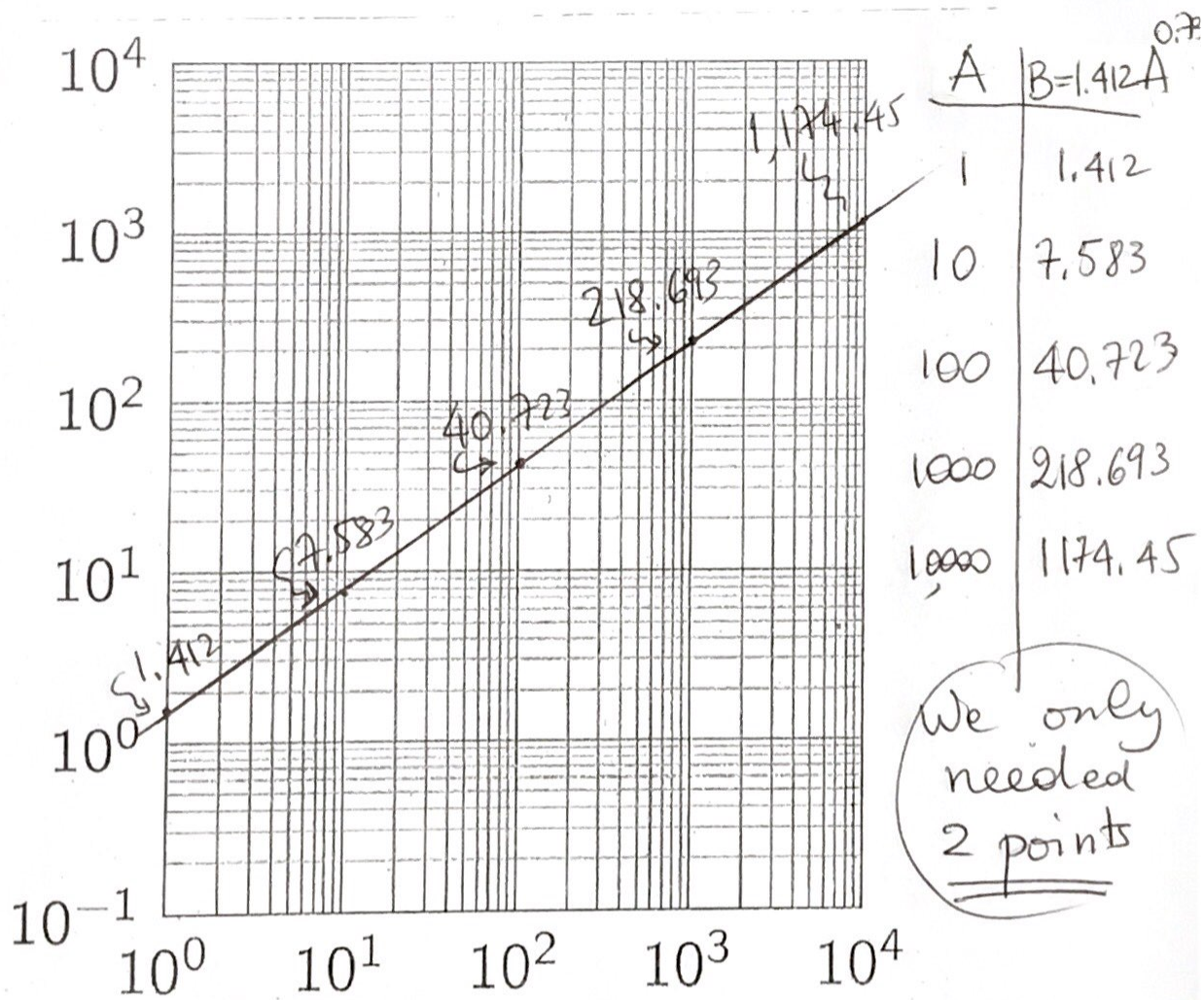
$$\log y - \log 5 = 0.37142 (\log x - \log 5)$$

Solve for  $y$  to get  $y = 28.60185 x^{0.37142}$

**#2** Given  $B = 1.412 \cdot A^{0.73}$ , then  
we obtain that the log of that equation  
is

$$\begin{aligned} \log B &= \log(1.412 \cdot A^{0.73}) \\ &= \log(1.412) + \log(A^{0.73}) \\ &= 0.73 \log A + \log(1.412) \end{aligned}$$

That is:  $y = 0.73 \cdot x + \log(1.412)$   
with  $y = \log B$  and  $x = \log A$



#3

$$a_1 = \frac{4}{10} \quad a_2 = \frac{9}{17} \quad a_3 = \frac{16}{26} \quad a_4 = \frac{25}{37}$$

$a_5 = \frac{36}{50}$  etc... the numerator is always a square. The denominator is a square + 1.

$$a_1 = \frac{2^2}{3^2+1}$$

$$a_2 = \frac{3^2}{4^2+1}$$

$$a_3 = \frac{4^2}{5^2+1}$$

$$a_4 = \frac{5^2}{6^2+1}$$

$$a_5 = \frac{6^2}{7^2+1}$$

$$a_n = \frac{(n+1)^2}{(n+2)^2+1}$$



#4

$$a_0 = 6 \quad a_{n+1} = \frac{1}{3}a_n + \frac{5}{6}$$

Write  $a_1, a_2, a_3, a_4 \dots$

$$a_1 = \frac{1}{3}a_0 + \frac{5}{6} = \frac{1}{3} \cdot 6 + \frac{5}{6} = 2 + \frac{5}{6} = \frac{17}{6}$$

$$a_2 = \frac{1}{3}a_1 + \frac{5}{6} = \frac{1}{3} \cdot \frac{17}{6} + \frac{5}{6} = \frac{17+15}{18} = \frac{32}{18}$$

$$a_3 = \frac{1}{3}a_2 + \frac{5}{6} = \frac{1}{3} \left( \frac{32}{18} \right) + \frac{5}{6} = \frac{32+45}{54} = \frac{77}{54}$$

$$a_4 = \frac{1}{3}a_3 + \frac{5}{6} = \frac{1}{3} \left( \frac{77}{54} \right) + \frac{5}{6} = \frac{77+135}{162} = \frac{106}{81}$$