

#1

$$\lim_{n \rightarrow \infty} \frac{(2+3n)^2}{4-n^2} = \lim_{n \rightarrow \infty} \frac{4 + 12n + 9n^2}{4 - n^2}$$

$$= \lim_{n \rightarrow \infty} \frac{(4 + 12n + 9n^2) \left(\frac{1}{n^2}\right)}{(4 - n^2) \left(\frac{1}{n^2}\right)} =$$

$$= \lim_{n \rightarrow \infty} \frac{\frac{4}{n^2} + \frac{12}{n} + 9}{\frac{4}{n^2} - 1}$$

$$= \frac{\lim_{n \rightarrow \infty} \left(\frac{4}{n^2} + \frac{12}{n} + 9 \right)}{\lim_{n \rightarrow \infty} \left(\frac{4}{n^2} - 1 \right)} =$$

$$= \frac{\lim_{n \rightarrow \infty} \left(\frac{4}{n^2} \right) + \lim_{n \rightarrow \infty} \left(\frac{12}{n} \right) + \left(\lim_{n \rightarrow \infty} 9 \right)}{\lim_{n \rightarrow \infty} \left(\frac{4}{n^2} \right) - \lim_{n \rightarrow \infty} 1}$$

$$= \frac{0 + 0 + 9}{0 - 1} = \boxed{-9}$$

#2

We solve this problem in a similar manner

$$\lim_{n \rightarrow \infty} \frac{(8+n)^2}{5n^2+3} = \lim_{n \rightarrow \infty} \frac{64 + 16n + n^2}{5n^2+3}$$

$$= \lim_{n \rightarrow \infty} \frac{(64 + 16n + n^2) \frac{1}{n^2}}{(5n^2 + 3) \frac{1}{n^2}}$$

$$= \lim_{n \rightarrow \infty} \frac{\left(\frac{64}{n^2} + \frac{16}{n} + 1\right)}{5 + \frac{3}{n^2}}$$

$$= \frac{\lim_{n \rightarrow \infty} \frac{64}{n^2} + \lim_{n \rightarrow \infty} \frac{3}{n} + \lim_{n \rightarrow \infty} 1}{\lim_{n \rightarrow \infty} 5 + \lim_{n \rightarrow \infty} \frac{3}{n^2}} = \boxed{\frac{1}{5}}$$

#3

$$\lim_{n \rightarrow \infty} \frac{3n^3 + \frac{1}{3^n}}{4n^2 + n} =$$

$$= \lim_{n \rightarrow \infty} \frac{\left(3n^3 + \frac{1}{3^n}\right) \frac{1}{n^2}}{(4n^2 + n) \frac{1}{n^2}} =$$

$$= \lim_{n \rightarrow \infty} \frac{\left(3n + \frac{1}{3 \cdot n^2}\right)}{4 + \frac{1}{n}} =$$

$$= \frac{\lim_{n \rightarrow \infty} (3n) + \lim_{n \rightarrow \infty} \frac{1}{3 \cdot n^2}}{\lim_{n \rightarrow \infty} 4 + \lim_{n \rightarrow \infty} \frac{1}{n}} = \frac{\infty}{4} = \infty$$

Diverges