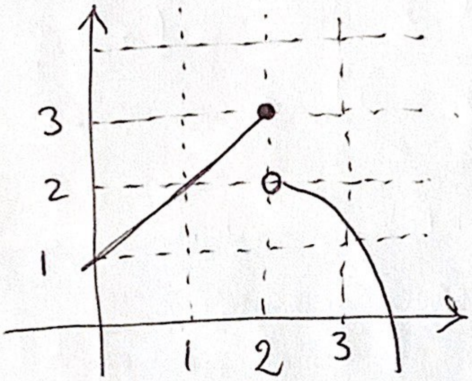
WORKSHEET #8

1.



Note that from the picture we have

$\boxed{f(2)=3}$

$\boxed{\lim_{x\to 2^-} f(x) = 3 = f(2)}$

(from the left of 2)

$\boxed{\lim_{x\to 2^+} f(x)=2}$

(from the right of 2)

$\boxed{\lim_{x\to 2} f(x) = \text{does not exist}}$

as the left limit is different from the right limit

2.

$$\lim_{x\to 0} \frac{2e^x - \sin(x)}{7x+3} =$$

$$= \frac{\lim_{x\to 0}\left[2e^x - \sin(x)\right]}{\lim_{x\to 0}\left[7x+3\right]}$$

we use the four (4) properties of limits

$$= \frac{\lim_{x\to 0}[2e^x] - \lim_{x\to 0}[\sin(x)]}{\lim_{x\to 0}[7x] + \lim_{x\to 0}[3]} =$$

$$= \frac{2\lim_{x\to 0} e^x - \lim_{x\to 0} \sin(x)}{7\lim_{x\to 0} x + 3}$$

$$= \frac{2\cdot e^0 - 0}{7(0)+3} = \boxed{\frac{2}{3}}$$

$$\lim_{x\to 0} \frac{\tan(x) - x}{x^3} = \frac{\lim_{x\to 0}\tan(x) - \lim_{x\to 0} x}{\left[\lim_{x\to 0} x\right]^3}$$

$$= \frac{0-0}{0^3} = \frac{0}{0}$$ Let's construct

a table of values

$\longrightarrow$ $\longleftarrow$

x	-0.1	-0.01	-0.001	0.001	0.01
$\frac{\tan x - x}{x^3}$	0.334672	0.3333467	0.33333	0.33333	0.333346

Thus the table suggests that

$$\boxed{\lim_{x\to 0} \frac{\tan(x) - x}{x^3} = 1/3 = 0.33333}$$

$\boxed{3.}$ Suppose $\lim\limits_{x\to 0} f(x) = 4$ and

$\lim\limits_{x\to 0} g(x) = 7$

then

$$\lim_{x\to 0} \frac{f(x)\sqrt{2+g(x)}}{[g(x)]^2 - f(x)} = \frac{\lim\limits_{x\to 0}\left[f(x)\sqrt{2+g(x)}\right]}{\lim\limits_{x\to 0}\left[[g(x)]^2 - f(x)\right]}$$

property 4 of limits

$$= \frac{\left[\lim\limits_{x\to 0} f(x)\right]\left[\lim\limits_{x\to 0}\sqrt{2+g(x)}\right]}{\lim\limits_{x\to 0}[g(x)]^2 - \lim\limits_{x\to 0} f(x)} =$$

properties 3 and 1

properties 1 & 3

$$= \frac{\left[\lim_{x\to 0} f(x)\right] \cdot \sqrt{\lim_{x\to 0} 2 + \lim_{x\to 0} g(x)}}{\left[\lim_{x\to 0} g(x)\right]^2 - \lim_{x\to 0} f(x)}$$

using the given values

$$= \frac{4 \cdot \sqrt{2+7}}{(7)^2 - 4} = \frac{4 \cdot 3}{49-4} = \frac{12}{45} = \boxed{\frac{4}{15}}$$