1. Because $f$ and $g$ are continuous functions so is $\frac{7g}{f}$. Thus

$$\lim_{{x \to 3}} \left[ \frac{7g(x)}{f(x)} \right] = \frac{7g(3)}{f(3)}$$

By assumption, $\frac{7g(3)}{f(3)} = 14$ and $f(3) = 6$

Thus $g(3) = 14 \cdot \frac{f(3)}{7} = 2 \cdot 6 = 12$

2. $f(x) = \begin{cases} \frac{x^2-4}{x-2} & x < 2 \\ (c^2-c)x-8 & x \geq 2 \end{cases}$

$f(x)$ is continuous for all values $x < 2$ as it is given by either a rational function for $x < 2$ or
by a polynomial for \( x > 2 \).

The issue is the continuity at \( x = 2 \).

We need

\[
\lim_{x \to 2} f(x) = f(2)
\]

In particular we need that the left and right limits exist and are equal.

\[
\lim_{x \to 2^-} f(x) = \lim_{x \to 2^+} f(x)
\]

\[
\Rightarrow \quad \lim_{x \to 2^-} \frac{x^2 - 4}{x - 2} = \lim_{x \to 2^+} (c^2 - c)x - 8
\]

\[
\Rightarrow \quad \lim_{x \to 2^-} (x + 2) = \lim_{x \to 2^+} (c^2 - c)x - 8
\]

\[
\Rightarrow \quad 2 + 2 = (c^2 - c)2 - 8
\]

\[
\Rightarrow \quad 2(c^2 - c) - 8 - 4 = 0
\]

\[
\Rightarrow \quad c^2 - c - 6 = 0 \quad (c - 3)(c + 2) = 0
\]

\[
\Rightarrow \quad c = 3 \quad \text{or} \quad c = -2
\]
3. \( f(x) = \begin{cases} \frac{x^2 - 4}{x - 2} & x \neq 2 \\ 5 & x = 2 \end{cases} \)

Has a graph that looks like \( f(x) \approx x + 2 \)

This is a removable discontinuity. If we set \( f(2) = 4 = \lim_{x \to 2} \frac{x^2 - 4}{x - 2} \) then \( f \) becomes continuous.

\( g(x) = \begin{cases} \frac{|x|}{x} & x \neq 0 \\ 0 & x = 0 \end{cases} \)

Has a discontinuity that cannot be removed by changing the value of \( g \) at \( x = 0 \).

\( h(x) = \frac{1}{x} \)

Has \( \lim_{x \to 0^-} \frac{1}{x} = -\infty \) \( \lim_{x \to 0^+} \frac{1}{x} = +\infty \).