WORKSHEET 9

$\boxed{1}$ Because f and g are continuous functions so is $\frac{7g}{f}$. Thus

$$\lim_{x\to 3}\left[\frac{7g(x)}{f(x)}\right] = \frac{7g(3)}{f(3)}$$

By assumption, $\frac{7g(3)}{f(3)} = 14$ and $f(3)=6$

Thus $g(3) = \frac{14 \cdot f(3)}{7} = 2 \cdot 6 = 12$

$\boxed{2}$

$$f(x) = \begin{cases} \frac{x^2-4}{x-2} & x<2 \\ (c^2-c)x-8 & x \geq 2 \end{cases}$$

$f(x)$ is continuous for all values $x \neq 2$ as it is given by either a rational function for $x<2$ or by a polynomial for $x>2$.

The issue is the continuity at $x=2$.

We need

$$\lim_{x\to 2} f(x) = f(2)$$

In particular we need that the left and right limits exist and are equal.

$$\lim_{x\to 2^-} f(x) = \lim_{x\to 2^+} f(x)$$

$$\iff \lim_{x\to 2^-} \frac{x^2-4}{x-2} \overset{\downarrow}{=} \lim_{x\to 2^+} (c^2-c)x-8$$

$$\iff \lim_{x\to 2^-} (x+2) = \lim_{x\to 2^+} (c^2-c)x-8$$

$$\iff 2+2 = (c^2-c)2-8$$

$$\iff 2(c^2-c)-8-4=0$$

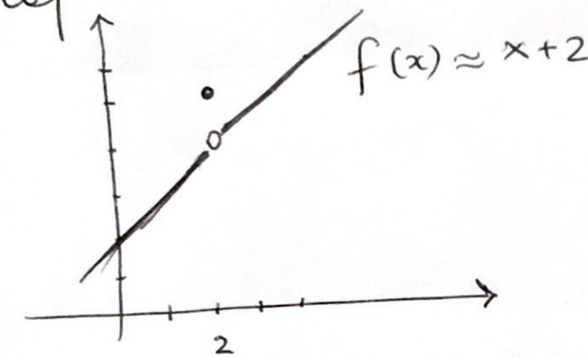
$\therefore$ $c=3$ OR $c=-2$

$$\iff c^2-c-6=0 \qquad (c-3)(c+2)=0$$

$\boxed{3}$

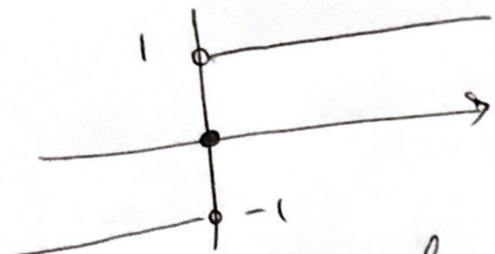
⊛ $$f(x) = \begin{cases} \dfrac{x^2-4}{x-2} & x \neq 2 \\ 5 & x = 2 \end{cases}$$

Has a graph that looks like

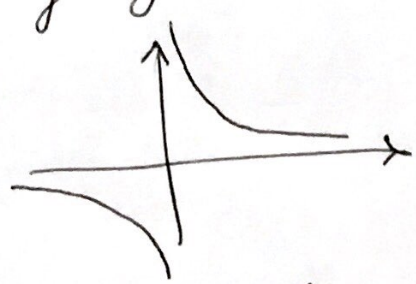


This is a removable discontinuity. If we set $f(2) = 4 = \lim_{x \to 2} \dfrac{x^2-4}{x-2}$ Then f becomes continuous.

⊛ $$g(x) = \begin{cases} \dfrac{|x|}{x} & x \neq 0 \\ 0 & x = 0 \end{cases}$$



Has a discontinuity that cannot be removed by changing the value of g at $x = 0$.



the limit <u>DNE</u>

⊛ $h(x) = \dfrac{1}{x}$

has $\lim_{x \to 0^-} \dfrac{1}{x} = -\infty$ $\lim_{x \to 0^+} \dfrac{1}{x} = +\infty$