MA 137 Worksheet #12

Sections 4.1 & 4.2
9/24/20

1. For the following functions, (a) find the difference quotient and then (b) use this to determine the derivative $f'(x)$ by the definition of the derivative. (c) Also find the tangent line of $y = f(x)$ at $x = 3$.

- $f(x) = x^3 - 5x^4 + 32$
- $f(x) = 2 - \sqrt{7x^2 - 3}$
- $f(x) = \pi^4 - 17x^{-3/2}$

2. Assume that $f(x)$ is everywhere continuous and you are given

$$\lim_{x \to -6} \frac{f(x) - \frac{1}{2}}{x + 6} = -\pi.$$

It follows that _____ is the equation of the tangent line to $y = f(x)$ at the point $(\_, \_)$.

3. The limit below represents a derivative $f'(a)$. Find $f(x)$ and a.
(**Hint:** Use the definition of the derivative in general.)

$$\lim_{h \to 0} \frac{4\sqrt[3]{8 + h} - 8}{h}$$

4. What needs to be true for a function to be differentiable at a point? How is this different than continuity?

5. Find all values of c such that the function

$$f(x) = \begin{cases} ax^2 + bx + c & \text{if } x \leq 0 \\ x^2 + 1 & \text{if } x > 0 \end{cases}$$

is continuous and differentiable everywhere.