MA 137 Worksheet #23

Sections 5.5 and 5.7
11/5/20

1. Determine whether L'Hôpital's Rule can be used to evaluate the following limits. If so, then use L'Hôpital's Rule to evaluate the limit.

   (a) $\lim_{x \longrightarrow 4} \dfrac{(4-x)^2}{16-x^2}$

   (b) $\lim_{x \longrightarrow \infty} \dfrac{\sqrt{8-x+4x^2}}{5x+3}$

   (c) $\lim_{x \longrightarrow 0} \dfrac{e^x - x - 1}{x^2}$

   (d) $\lim_{x \longrightarrow 0^+} x^{3\sin(2x)}$

2. Use the stability criterion to characterize the stability of the equilibria (fixed points) of the following recursive sequences:

   (a) $x_{n+1} = \dfrac{1}{4}x_n^2 + x_n - \dfrac{1}{4}$, $n = 0, 1, 2, \ldots$

   (b) $x_{t+1} = \dfrac{5x_t^2}{4 + x_t^2}$, $t = 0, 1, 2, \ldots$
   Use cobwebbing to decide to which value x_t converges as $t \longrightarrow \infty$ if
   (i) $x_0 = 0.5$ and **(ii)** $x_0 = 3$.

   (c) We consider density-dependent population growth models of the form $N_{t+1} = R(N_t)N_t$. The function $R(N) = rN^{1-\gamma}$ describes the per capita growth. Find all nontrivial fixed points $\widehat{N}$ (i.e., $\widehat{N} > 0$) and determine the stability as a function of the parameter values. We assume that the function parameters are $r > 0$ and $\gamma > 1$.