1 The domain and range of $f(x)$ are $[2,4]$ and $[5,11]$, respectively. Find the domain and range of the following functions:

(a) $g(x) = 3f(x-2)$

$X - 2 = 2 \Rightarrow X = 4$

$X - 2 = 4 \Rightarrow X = 6$

Domain of $g(x) =$ $[4, 6]$

Range of $g(x) =$ $[15, 33]$

$3 \times [5, 11] \Rightarrow [15, 33]$

(b) $h(x) = f(2x) + 3$

$2X = 2 \Rightarrow X = 1$

$2X = 4 \Rightarrow X = 2$

Domain of $h(x) =$ $[1, 2]$

Range of $h(x) =$ $[8, 14]$

$[5, 11] + 3 \Rightarrow [8, 14]$

2 Find the equation of the line perpendicular to $3x + 2y = 4$ that goes through the point $(4, 3)$.

$3x + 2y = 4 \rightarrow y = -\frac{3}{2}x + 2$ Slope $= -\frac{3}{2} \Rightarrow$ new slope $= \frac{2}{3}$

$$\boxed{y - 3 = \frac{2}{3}(x - 4)}$$

3 Use the Intermediate Value Theorem to explain why there is a value of x such that $e^{-x} = x$.

Let $f(x) = e^{-x} - x$. A solution to $e^{-x} = x$ is a root of $f(x)$.

As $f(0) = 1$ and $f(1) = \frac{1}{e} - 1 \approx -.63$,

and f is continuous on $[0, 1]$,

by IVT there is a root of f on $[0, 1]$.

(a) Solve $\log_2(\log_3(x)) = 2$ for x.

$\log_2(\log_3(x)) = 2 \rightarrow \log_3(x) = 2^2 = 4$

$\rightarrow \boxed{x = 3^4 = 81}$

A population of *E. coli* grows according to the equation $N(t) = 350e^t$, where t is given in hours.

(b) How long until the population has quadrupled?

Solve $4 \times 350 = 350e^t$ for t

$4 = e^t \rightarrow \boxed{t = \ln 4}$

(c) Antibiotics then devastate the bacterial population to 10% of it's original size. How long does it take to reach the population from part (b) again?

$4 \times 350 = .1 \times 350\, e^t$

$\rightarrow 40 = e^t \rightarrow \boxed{t = \ln(40)}$

(a) For what value of c is the following function continuous everywhere?

$$f(x) = \begin{cases} x^3 - cx + 1 & x \leq 2 \\ x^2 - 2x + c & x > 2 \end{cases}$$

We need the pieces to agree at $x = 2$

$2^3 - 2c + 1 = 2^2 - 4 + c$

$9 - 2c = c \Rightarrow 9 = 3c \Rightarrow \boxed{c = 3}$

(b) Where is the function $f(x) = \dfrac{\sqrt{49 - x^2}}{1 - \cos(x)}$ continuous?

f is continuous anywhere it is defined.

Numerator: need $49 - x^2 \geq 0 \rightarrow |x| \leq 7$ or $-7 \leq x \leq 7$

Denominator: need $1 - \cos(x) \neq 0 \Rightarrow \cos(x) \neq 1$

$\Rightarrow x \neq 0, \pm 2\pi, \pm 4\pi, \pm 6\pi, \ldots$

Note that $\pm 4\pi, \pm 6\pi, \ldots$ are outside the interval $-7 \leq x \leq 7$.

So: $f(x)$ is continuous when $-7 \leq x \leq 7$ and $x \neq 0, \pm 2\pi$

6 Find the following limits or explain why they do not exist.

(a) $\lim_{x\to\infty} x - \sqrt{1+x^2}$ $\cdot \dfrac{x+\sqrt{1+x^2}}{x+\sqrt{1+x^2}} = \lim_{x\to\infty} \dfrac{x^2-(1+x^2)}{x+\sqrt{1+x^2}}$

$$= \lim_{x\to\infty} \frac{-1}{x+\sqrt{1+x^2}} = \boxed{0}$$

(b) $\lim_{z\to 3} \dfrac{4}{(z-3)^3}$

$$\left.\begin{array}{l} \lim_{z\to 3^+} \dfrac{4}{(z-3)^3} = \infty \\ \lim_{z\to 3^-} \dfrac{4}{(z-3)^3} = -\infty \end{array}\right\} \boxed{\text{Limit DNE}}$$

(c) $\lim_{x\to a^+} \dfrac{x^2-a^2}{x^4-a^4}$

$$= \lim_{x\to a^+} \frac{x^2-a^2}{(x^2-a^2)(x^2+a^2)} = \lim_{x\to a^+} \frac{1}{x^2+a^2} = \boxed{\frac{1}{2a^2}}$$

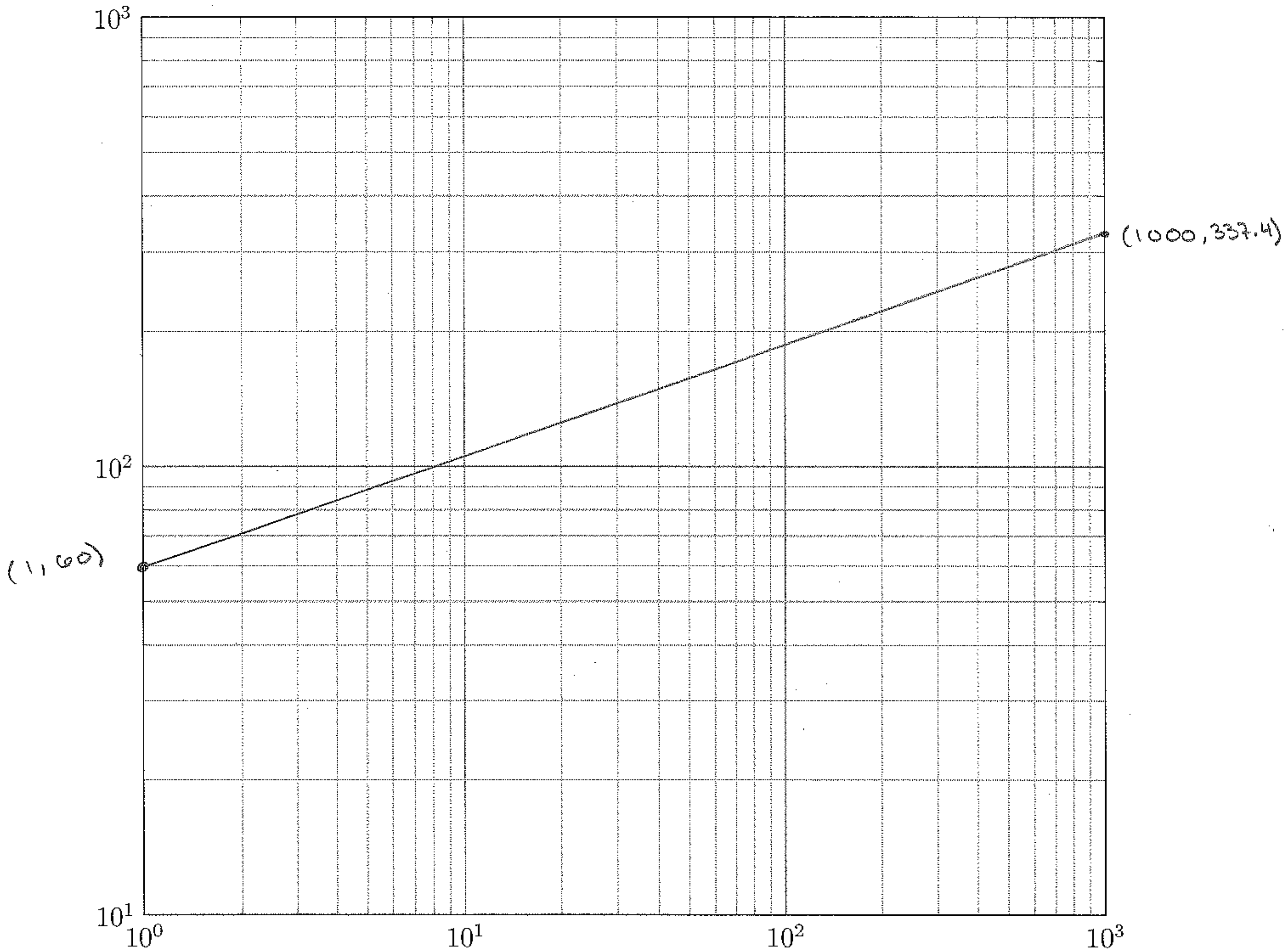
(d) $\lim_{x\to 0} \dfrac{\sin(3x)}{\sin(2x)}$

$$= \lim_{x\to 0} \frac{\sin(3x)}{3x} \cdot \frac{2x}{\sin(2x)} \cdot \frac{3x}{2x} = 1 \cdot 1 \cdot \frac{3}{2} = \boxed{\frac{3}{2}}$$

7 Many studies have shown that the number of species (S) on an island increases with the area (A) of the island. Frequently this relationship is approximated by

$$S = CA^z, \qquad S = 60A^{.25}$$

where z is a constant that depends on the species and habitat in the study. For a particular species, suppose that $z = .25$ and $C = 60$. Plot this function on the log-log graph below. **Clearly label at least 2 points on your plot.**



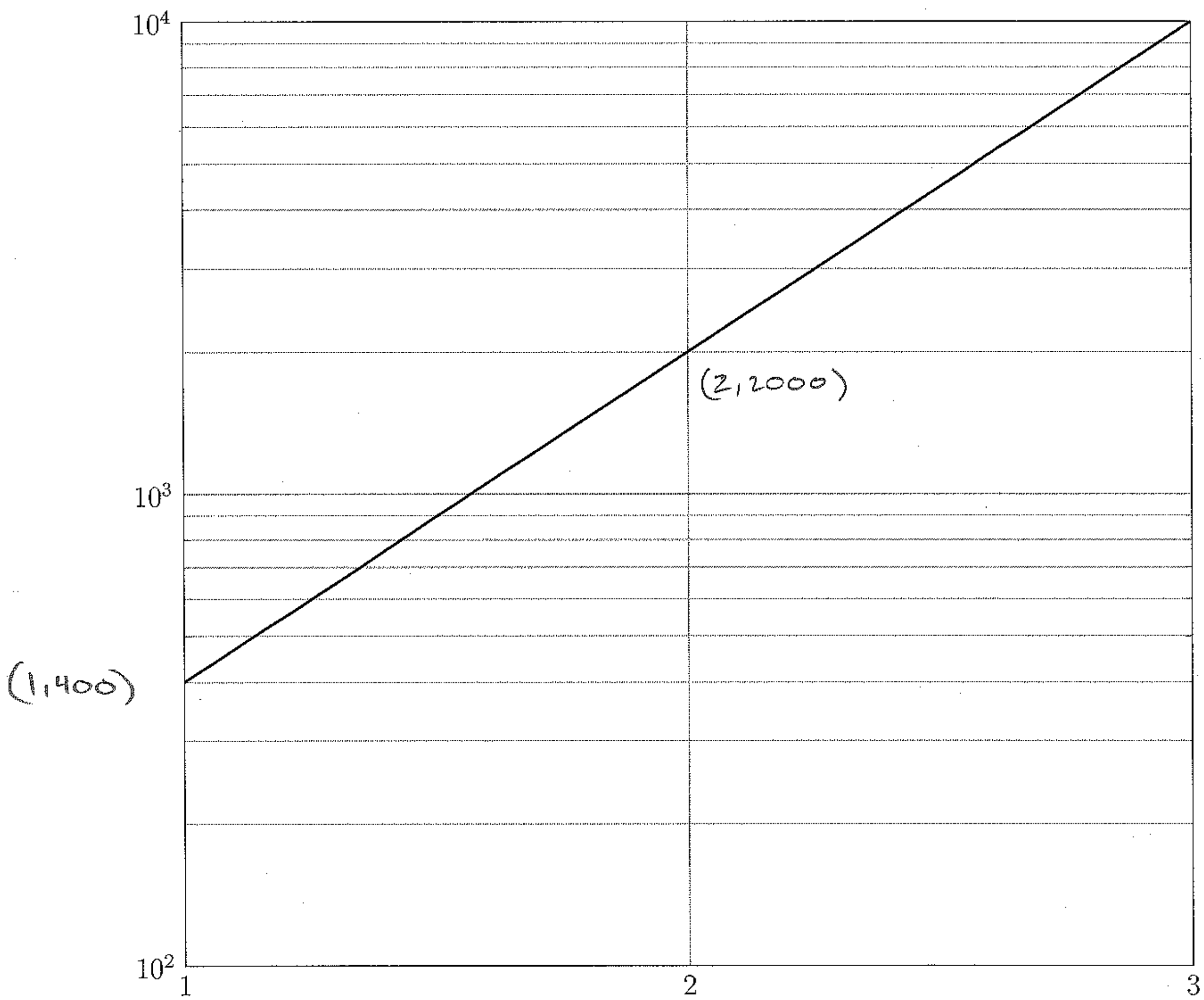
What is the slope of the line in your plot?

$$\ln(S) = \ln(60) + .25\ln(A)$$

$$\uparrow$$

slope $= .25$

8 Determine the relationship between x and y from the plot below.



$y =$ $80 \cdot 5^x$

We expect y to be exponential, as this is a semilog plot.

$y = ba^x$

$400 = b \cdot a^1 = ba \Rightarrow b = \frac{400}{a}$

$2000 = b \cdot a^2 \longrightarrow 2000 = \frac{400}{a} \cdot a^2 \Rightarrow a = 5 \Rightarrow b = \frac{400}{5} = 80$